Mathematical Expression for Machining Performance

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Abstract—In electrical discharge machining (EDM), a complete and clear theory has not yet been established. The developed theory (physical models) yields results far from reality due to the complexity of the physics. It is difficult to select proper parameter settings in order to achieve better EDM performance. However, modelling can solve this critical problem concerning the parameter settings. Therefore, the purpose of the present work is to develop mathematical models to predict performance characteristics of EDM on Ti-5Al-2.5Sn titanium alloy. Response surface method (RSM) and artificial neural network (ANN) are employed to develop the mathematical models. The developed models are verified through analysis of variance (ANOVA). The ANN models are trained, tested, and validated utilizing a set of data. It is found that the developed ANN and mathematical model can predict performance of EDM effectively. Thus, the model has found a precise tool that turns EDM process cost-effective and more efficient.

Keywords—Analysis of variance, artificial neural network, material removal rate, modelling, response surface method, surface finish.

I. INTRODUCTION

Although the utility of titanium alloys is increasing, it has become a great challenge to produce parts with superior quality and high productivity. With conventional machining process, it is very difficult to machine titanium alloys due to their poor machinability [1]. On the other hand, this difficult to cut titanium material can be machined effectively with a non-traditional machining process, EDM [1]-[3]. However, high machining time and parameter selection are the significant deficiencies of this EDM technique. High machining time is introduced because of inadequate selection of machining parameters. Usually, the selection of EDM parameters depends on operator’s experience and conservative technological data provided by the manufacturers. Ultimately it causes inconsistent machining performance [3], [4]. Data provided by the manufacturers are only suitable for some common steel materials that cannot be utilized for machining titanium alloy material. Thus, the correct selection of parameters is most important in order to achieve better EDM performance. Using the physical properties of the electrode materials, several attempts have been made by previous researchers to develop theoretical model. Longfellow et al. proposed an equation for wear ratio [5]. However, this equation shows a poor fit with experimental values [6], [7]. The experimental wear ratio was found 15–19 times as high as the calculated wear ratios. Dibitonto et al. presented a theoretical cathode erosion model for EDM performance measures [8]. In the model validation test, the predicted value was found 1.5–46 times higher than the experimental values. A non-linear, transient, thermophysical model of die-sinking EDM was developed using the finite element method (FEM) to predict the shape of the crater cavity and the material removal rate (MRR) [9]. The result confirmed that the analytical results over-predict the MRR compared with the experimental results. Their numerical model was also validated by experimental result, and the model’s predicted value was 1.5–14 times higher than experimental results. The growth of the plasma channel, energy sharing between electrodes, the process of vaporization, formation of a recast layer, plasma-flushing efficiency and the thermal properties of the work material are a few physical phenomena that make the machining process highly complex and stochastic. It is very difficult to consider all these complex phenomena in mathematical form. Therefore, mathematical prediction of process characteristics shows wide variation compared with experimental results. On the other hand, experiment-based modelling of EDM helps to give a good understanding of the complex process [6]. Experimentalis have tried to formulate regression model for the EDM process by employing several techniques such as RSM, ANN and so forth.

Through RSM, regression model was developed to evaluate the surface quality and MRR in terms of machining parameters [10]-[14]. Modelling for MRR was performed for EDM on silicon-infiltrated silicon carbide, aluminium-silicon carbide, metal matrix composite (MMC) copper steel (EN-8), tungsten carbide and cobalt composites, FW4 steel material [12], [13], [15]-[18]. Study was carried out to model surface finish of aluminium-silicon carbide, F-1110, alumina-based ceramic composite, Ck60 steel, MMC, EN-8, tungsten carbide and cobalt composites, FW4 steel work piece [10], [11], [13]-[18]. In recent years, the ANN has been transformed into a very useful tool for modelling complex systems [19]. A neural network (NN) is able to model non-linear processes by catching the desired input and output vectors. Cybenko and Nielsen showed that such a network is capable of estimating any non-linear function with desired accuracy [20]. Mahdavinejad used artificial intelligent to distinguish the EDM pulse type [21]. NN models were proposed in order to assess MRR for HE15, 15CDV6, and M250, C40 steel, nickel-based alloy [22]-[24]. NN was employed to model surface finish of HE15, 15CDV6, and M250, nickel-based alloy, mild steel (St 37), alloyed steels and high strength low alloyed (HSLA) steels such as a micro alloyed (Mic/al 1) steel.
It is evident from the prior work that the mathematical model was developed for particular work-tool combination, and for particular electrode polarity. Moreover, it is revealed that mathematical model with titanium alloy, specifically Ti-5Al-2.5Sn, in EDM process is still lagging. In this context, an effort has been made to begin regression equation and NN model in order to predict MRR and surface finish of Ti-5Al-2.5Sn titanium alloy. In outstanding, distinct electrode materials such as copper (Cu), copper-tungsten (Cu-W), and graphite (Gr), and all polarities (positive and negative) of electrode have been considered for modeling in this work. Besides, a comparison has been made between regression equation and NN model.

II. MATHEMATICAL MODELLING

Mathematical models were developed for predicting MRR and surface finish, mainly surface roughness (SR), considering first-order and second-order polynomial equation using RSM. In addition, feed-forward multilayer perceptron (MLP) NN model was set up in order to evaluate MRR and SR through ANN. Henceforward, the confirmation test is performed for model validation. The peak current ($I_p$) through ANN. Henceforward, the confirmation test is performed for model validation. The peak current ($I_p$), pulse-on time ($T_{on}$), pulse-off time ($T_{off}$), and servo-voltage ($S_v$) were selected as the process variables for the present research. Positive and negative polarity of the Cu, Cu-W, and Gr electrode were considered.

A. Regression Equation

RSM describes the correlation between responses and quantitative factors. The process parameters can be presented as [25]:

$$Y = f(X_1, X_2, X_3, \ldots X_n) + \varepsilon$$  \hspace{1cm} (1)

where, $Y$ is the response, $f$ is the response function, $X_1, X_2, X_3, \ldots, X_n$ are factors, and $\varepsilon$ is the experimental error. In support of the present work, the response MRR and surface roughness can be shown by (2) and (3) respectively:

$$MRR = f(I_p, T_{on}, T_{off}, S_v) + \varepsilon$$  \hspace{1cm} (2)

$$SR = f(I_p, T_{on}, T_{off}, S_v) + \varepsilon$$  \hspace{1cm} (3)

Single-order response surface mathematical models can be developed using (2) and (3):

$$MRR = A_0 + \sum_{i=1}^{4} A_i X_i + \varepsilon$$  \hspace{1cm} (4)

$$SR = B_0 + \sum_{i=1}^{4} B_i X_i + \varepsilon$$  \hspace{1cm} (5)

where $X_i$ is the process variables ($I_p, T_{on}, T_{off}$, and $S_v$); the term $A_0$, and $B_0$ are the constants. The term $A_i$, and $B_i$ are the single-order regression coefficient for MRR and SR respectively. Neglecting the error ($\varepsilon$), (4) and (5) can be written as

$$MRR = A_0 + A_1 I_p + A_2 T_{on} + A_3 T_{off} + A_4 S_v$$  \hspace{1cm} (6)

$$SR = B_0 + B_1 I_p + B_2 T_{on} + B_3 T_{off} + B_4 S_v$$  \hspace{1cm} (7)

where the parameters $A_0$, $A_1$, $A_2$, $A_3$, and $A_4$ are the regression coefficients of the linear effect of the connecting factor for MRR; and $B_0$, $B_1$, $B_2$, $B_3$, and $B_4$ are the regression coefficients which represent the linear effect of the connecting factor for SR.

A second-order response surface model for MRR and SR consists of the following terms:

$$MRR = A_0 + \sum_{i=1}^{4} A_i X_i + \sum_{i=1}^{4} A_{ii} X_i^2 + \sum_{i=1}^{4} \sum_{j=1, j \neq i}^{4} \sum_{i=1}^{4} A_{ij} X_i X_j + \varepsilon$$ \hspace{1cm} (8)

$$SR = B_0 + \sum_{i=1}^{4} B_i X_i + \sum_{i=1}^{4} B_{ii} X_i^2 + \sum_{i=1}^{4} \sum_{j=1, j \neq i}^{4} \sum_{i=1}^{4} B_{ij} X_i X_j + \varepsilon$$ \hspace{1cm} (9)

where the parameters $A_0$, $A_1$, $A_2$, $A_3$, and $A_4$ are the regression coefficients for MRR; $B_0$, $B_1$, $B_2$, $B_3$, and $B_4$ are the regression coefficients for SR. Therefore, the second-order polynomial models, while neglecting the experimental error, can be written as

$$MRR = A_0 + A_1 I_p + A_2 T_{on} + A_3 T_{off} + A_4 S_v + A_1 I_p^2$$

$$+ A_2 T_{on}^2 + A_3 T_{off}^2 + A_4 S_v^2 + A_{11} I_p T_{on} + A_{12} I_p T_{off} + A_{13} I_p S_v$$ \hspace{1cm} (10)

$$+ A_{22} T_{on}^2 + A_{23} T_{off} S_v + A_{33} S_v^2 + A_{12} T_{off} S_v + A_{13} T_{off} S_v + A_{23} T_{on} S_v$$ \hspace{1cm} (10)

In the present study, both the first-order and the second-order polynomial models were studied using the experimental results. The ANOVA was carried out for first-order and second-order polynomial models to determine the adequacy of the fitted model. The model adequacy checking includes testing for significance of the regression model (mainly $R^2$, $R^2$-adjusted), testing for significance of model coefficients, and testing for lack-of-fit [10], [26].

$$SR = B_0 + B_1 I_p + B_2 T_{on} + B_3 T_{off} + B_4 S_v +$$

$$+ B_{11} I_p^2 + B_{22} T_{on}^2 + B_{33} T_{off}^2 + B_{44} S_v^2 +$$

$$+ B_{12} I_p T_{on} + B_{13} I_p T_{off} + B_{14} I_p S_v +$$

$$+ B_{23} T_{on} T_{off} + B_{24} T_{on} S_v + B_{34} T_{off} S_v$$ \hspace{1cm} (11)
B. ANN

NN are able to model non-linear processes through catching the desired input and output vectors [19]. The distinct architectures of NN are studied and eventually, MLP NN is developed owing to its more appropriate result. The general network with four input variables and two responses is supposed to be 4-j-2 for the network with one hidden layer. In MLP NN with one hidden layer, the net input to unit \( j \) in the hidden layer, and the net input to unit \( o \) in the output layer are expressed in (12) and (13) respectively:

\[
\text{(net input)}_{\text{hidden}} = \sum_{i=1}^{I} w_{ij} y_{i} + b_{j} \\
\text{(net input)}_{\text{output}} = \sum_{j=1}^{J} w_{jo} h_{j} + b_{o}
\]

where \( w_{ij} \) is the weight between the input neurons and hidden neurons; \( w_{jo} \) is the weight between the hidden and output neurons; \( y_{i} \) is the value of the input as \( x_{i}^{Ip}, x_{i}^{Tin}, x_{i}^{Ton}, x_{i}^{Toff} \); \( h_{j} \) is the value of the output for hidden nodes; \( b_{j} \) is the bias on the hidden nodes; and \( b_{o} \) is the bias on the output nodes.

The output for hidden nodes \( (h) \) and the output for output nodes \( (y_{o}) \) can be shown as (14) and (15):

\[
h_{j} = f(\text{(net input)}_{\text{hidden}}) \\
y_{o} = f(\text{(net input)}_{\text{output}})
\]

where \( f \) is the transfer function and \( y_{o} \) is the ANN predicted output, namely MRR and SR; the output for hidden nodes \( (h) \) and the output from the output nodes \( (y_{o}) \) with the sigmoid function can be written as (16) and (17):

\[
h_{j} = \frac{1}{1 + e^{-\sum_{i=1}^{I} w_{ij} y_{i} + b_{j}}} \\
y_{o} = \frac{1}{1 + e^{-\sum_{j=1}^{J} w_{jo} h_{j} + b_{o}}}
\]

Hence, the NN model for MRR and SR with one hidden layer can be presented as

\[
y_{o} = \frac{1}{1 + e^{-\sum_{j=1}^{J} w_{jo} h_{j} + b_{o}}}
\]

Likewise, the network structure with four input variables and two responses can be defined as 4-j-k-2 for the network with two hidden layers. The values of the output for first hidden layer \( (h) \), and for second hidden layer \( (z) \) with sigmoid function can be presented as (19) and (20) respectively:

\[
h_{j} = \frac{1}{1 + e^{-\sum_{i=1}^{I} w_{ij} y_{i} + b_{j}}} \\
z_{k} = \frac{1}{1 + e^{-\sum_{j=1}^{J} w_{jk} h_{j} + b_{k}}}
\]

where \( w_{ij} \) is the weight between the input neurons and first hidden neurons; \( w_{jk} \) is the weight between first hidden layer and second hidden layer; \( b_{j} \) is the bias on the first hidden layer; and \( b_{k} \) is the bias on the second hidden layer. Thus, the NN output \( (y_{o} = f(z_{k})) \) for MRR and SR with two hidden layers can be expressed as (21):

\[
y_{o} = \frac{1}{1 + e^{-\sum_{k=1}^{K} w_{ko} z_{k} + b_{o}}}
\]

Several networks were trained with different numbers of neurons in the hidden layer. A trial-and-error approach was used to ascertain the optimal structure. It is obvious that the weight of the optimal NN will be transformed for distinct electrode-polarity combination.

III. RESULTS AND DISCUSSION

A. Regression Equation

The collected experimental data have been analysed using response surface. Analyses have been accomplished through first-order as well as second-order regression equations. The result obtained through ANOVA revealed that the second-order mathematical model is more adequate and significant than the first-order model. Consequently, the formulated model of MRR for positive Cu, negative Cu, positive Cu-W, negative Cu-W, positive Gr, and negative Gr electrode based on (10) are presented by (22)-(27) respectively. Similarly, the formulated model of surface roughness for positive Cu, negative Cu, positive Cu-W, negative Cu-W, positive Gr, and negative Gr electrode based on (11) can be represented by (28)-(33) respectively.

The developed models are verified through confirmation test, and it is found that the average errors of the mathematical
models are in the range of 3.98–4.58%, and 3.55–5.17% for MRR and SR respectively. Thus, it is obvious that the accuracies of the developed models are satisfactory. Correspondingly, these models can predict the responses within an agreeable error (>6%).

\[
MRR = 2.195 + 0.0678T_{on} + 2.727 + 3.803 T_{off} - 0.042 S_v + 1.423 + 3.72 T_{on} + 1.19 + 0.6 T_{off} + 2.01 + 10^{-6} I_T^2 T_{on} + 1.93 + 10^{-6} I_T^2 T_{off} - 8.61 + 10^{-5} I_T T_{on} - 0.19 - 10^{-6} I_T T_{off} - 1.73 - 10^{-5} T_{on} S_v + 3.21 - 10^{-5} T_{off} S_v
\]

\[
SR = -2.639 + 0.8061 I_p + 0.0259 T_{on} + 3.787 T_{off} + 0.0025 S_v - 1.72 - 10^{-3} I_T^2 + 1.97 + 10^{-5} I_T^2 - 7.10 + 10^{-6} T_{on} + 2.51 - 10^{-4} I_T T_{on} - 6.49 - 10^{-5} I_T T_{off} + 6.25 + 10^{-3} I_T T_{on} + 7.28 - 10^{-6} T_{on} T_{off} + 2.39 + 10^{-4} T_{on} T_{off}
\]

\[
- 0.06 - 10^{-5} T_{on} S_v - 3.06 - 10^{-5} T_{off} S_v
\]

B. NN Model

A number of networks are constructed, altering the number of hidden neurons, maximum epochs, training repetition, learning step size, and momentum factor, and each of them is trained separately [23]. The best network was selected based on the accuracy of the predictions in the training and testing phase. The configurations shown in Table I give the best prediction for the performance measure (MRR and SR) for different electrode-polarity combinations. The developed NN
models with all settings are tested and validated. It is observed that the performance measures such as MSE and r-value are within the range of 0.0175–0.0513 and 0.9693–0.9962 respectively for MRR. Similarly, the values of MSE and r during testing are within the range of 0.0069–0.0981 and 0.9593–0.9963 respectively for SR. The high r-value (1) and small MSE (approaching zero) ensure that the NN model gives the best prediction. Thus, it is evident that the MSEs obtained are within the acceptable range, and the NN models are adequate. Thus, it is obvious that the error is within the agreeable limit and the accuracy of the developed model is satisfactory. Besides it seems that the accuracy of the ANN model is better than that of the RSM model.

### IV. CONCLUSION

This research work established the regression equation as well as NN model in order to evaluate the responses as MRR and surface finish for distinct electrode-polarity combination. The second-order regression equation evidenced more fitness and adequacy. The best NN models were set up with one hidden layer and two hidden layers that are linked with electrode-polarity combination. The developed models can predict the responses agreeably and effectively. In addition, the NN model is more accurate compared with RSM. It constructs the EDM process as cost-effective and efficient.

### ACKNOWLEDGMENT

The authors would like to thank Universiti Malaysia Pahang for providing laboratory facilities.

### REFERENCES


### TABLE I

<table>
<thead>
<tr>
<th>Electrode</th>
<th>Polarity</th>
<th>Network structure</th>
<th>No. of hidden layers</th>
<th>Learning rate</th>
<th>Momentum factor</th>
<th>No. of training repetitions</th>
<th>Max no. of epochs</th>
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