Performance of Coded Multi-Line Copper Wire for G.fast Communications in the Presence of Impulsive Noise

Israa Al-Neami, Ali J. Al-Askery, Martin Johnston, Charalampos Tsimenidis

Abstract—In this paper, we focus on the design of a multi-line copper wire (MLCW) communication system. First, we construct our proposed MLCW channel and verify its characteristics based on the Kolmogorov-Smirnov test. In addition, we apply Middleton class A impulsive noise (IN) to the copper channel for further investigation. Second, the MIMO G.fast system is adopted utilizing the proposed MLCW channel model and is compared to a single line G.fast system. Second, the performance of the coded system is obtained utilizing concatenated interleaved Reed-Solomon (RS) code with four-dimensional trellis-coded modulation (4D TCM), and compared to the single line G-fast system. Simulations are obtained for high quadrature amplitude modulation (QAM) constellations that are commonly used with G-fast communications, the results demonstrate that the bit error rate (BER) performance of the coded MLCW system shows an improvement compared to the single line G-fast systems.

Keywords—G.fast, Middleton Class A impulsive noise, mitigation techniques, copper channel Model.

I. INTRODUCTION

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VER time, digital subscriber line (DSL) technology has evolved through generations to face the demand for high bandwidth data transmission over telephone transmission lines. Generally, DSL communication can be referred to as multiple independent system having a single transmitter coupled to a single receiver by a twisted pair [1]. Due to the higher reliability of data transmission it offers and the potential to dramatically increase the capacity of wireless channels, multiple-input multiple-output (MIMO) technology has been identified as one of the most practical methods in wireless communication systems to increase the transmission capacity and improve the link reliability [2]. This technology can be utilized in wire-line communications by considering multiple copper line systems bundled together in the same cable binder [3]. One potential method to achieve a high bit rate is to use a bundle of twisted pairs to transmit the bit stream rather than one pair of wires. Clearly, a system employing a channel as a single MIMO channel (two pairs of wires) with matrix transfer function \( \mathbf{H} \) would increase the bit error rate over a system using one set of transceivers assuming that the pairs of wires are treat as two sets of wideband channels [4]. The concept of the vectoring method was presented in [5] as a crosstalk reduction technique. In the DSL literature, it is common to describe DSL systems employing such methods as a vectored system. Most of these methods are based on multiple twisted pairs which are represented as a MIMO channel in a vectored system. In [6], discrete multi-tone modulation (DMT) with orthogonal frequency-division multiplexing (OFDM) was considered over MIMO cable bundles and was presented alongside a mathematical description of the transmission line.

Impulsive noise (IN) has been a limiting factor in DSLs, so the interest in techniques for IN reduction has increased and drawn attention [5]. A well-known IN model in communication systems, namely Middleton’s A (MCA) noise, has been utilized to model impulse noise in this study. The MCA model is a special case of the popular multi-component Gaussian mixture model [7].

An overview of impulsive noise cancellation for the G.fast DSL standard with multi-line copper wire is presented herein. This letter enumerates the bit error rate (BER) performance of a G.fast system over our multi-line copper wire (MLCW) channel in the presence of IN modeled as an MCA noise source.

In [7], the simulation results showed that Zhidkov’s algorithm outperforms the blanking and clipping methods, demonstrating that it is more appropriate to be used at high order quadrature amplitude modulation (QAM) constellations, such as those used in G.fast systems. This study therefore presents a practical algorithm for impulsive noise cancellation in OFDM receivers using Zhidkov’s algorithm [8] for MLCW G.fast systems. The key point of this algorithm is to compensate for IN in the frequency domain after OFDM demodulation and channel equalization, instead of in the time domain before OFDM demodulation as in conventional methods.

This paper is organized as follows: Section II-A describes the proposed MLCW G.fast system model. Section II-B demonstrates the construction and verification of the MLCW channel model. In Section III, the Zhidkov algorithm is presented as a solution to the IN effect. Simulation results are detailed in Sections IV and V concludes this work.

II. G.fast SYSTEM OVER COPPER CHANNEL

A. System Model

This paper considers a MLCW system with \( N_r \times N_t \) receivers and transmitters with a blanking non-linear compensator, as presented in Fig. 1. The data symbols are generated such that \( x \in \text{GF}(2^s) \), where \( \text{GF}(\cdot) \) is a Galois field with \( s \) being positive integer. Then, the data symbols are encoded by a Reed-Solomon (RS (255,223)) encoder

Israa Al-Neami is staff member with the University of Technology in Iraq (e-mail: I.al-shaikhli1@newcastle.ac.uk).
defined in \( GF(2^8) \) as an outer code that produces \( N = 255 \) coded symbols from \( K = 223 \) information symbols. Next, the coded symbols are interleaved and encoded with the inner four-dimensional trellis-coded modulation (4-D TCM) producing the 4D, 32-QAM signal mapper. Utilizing the constellation mapper from [9], the codewords are mapped into 4-D 32-QAM symbols and fed to the DMT modulator to be transmitted through the MLCW.

The channel is modeled as two parallel pairs of copper wires that carry the encoded data to the users, taking into consideration the effect of the far-end cross-talk (FEXT) effect between the pairs \( i \) and \( j \). This channel will be further described in the next section to cover the important characteristics of this model. The received signal per symbol \( r_k \in C^{N_k \times 1} \) can be written as

\[
    r_k = H_k d_k + w_k, \tag{1}
\]

where, \( H_k \in C^{N_k \times N_i} \) represents the channel parameter matrix per symbol, \( d_k \in C^{N_i \times 1} \) is the transmitted vector per symbol, and \( (w_k; i + w_k) \in C^{N_k \times 1} \) is the additive white Gaussian noise (AWGN) or the AWGN plus IN vector per symbol, respectively. The received signals \( r_k \) are first demodulated using DMT and equalized based on the zero-forcing equalizer (ZFE). Following that, each receiver will apply the decoding process on the equalized received vector in reverse order to fully reconstruct the transmitted vectors.

**B. Channel Model**

This section considers a MLCW channel with two pairs \( i \) and \( j \) as direct lines with the FEXT effect of pair \( i \) on pair \( j \) and vice-versa. The near-end cross-talk (NEXT) effect will not be taken into consideration for this channel since we are assuming long cables [3] and the effect of the NEXT will be negligible compared to the FEXT coefficients. The direct channels are implemented based on Chen’s model, such that [9]

\[
    h_{1i} = e^{-L \gamma(f)}, \tag{2}
\]

where \( L \) is the cable length in meters, \( \gamma(f) = \alpha(f) + j \beta(f) \) is the propagation constant with \( \alpha(f) \) and \( \beta(f) \) as the attenuation and the phase constants respectively, which can be calculated using

\[
    \alpha(f) = k_1 \sqrt{f} + k_2 f, \\
    \beta(f) = k_3 f, \tag{3}
\]

where, \( k_1, k_2, \) and \( k_3 \), represent Chen model constants and \( f \) is the frequency [10]. The mutual channels (i.e. between link \( i \) and \( j \)) are implemented utilizing the FEXT model shown in [3], which represent the American wire gauge (AWG) with a 0.4 mm loop. First, the insertion loss is calculated using

\[
    H_{IL}(f, L) = e^{-L/L_{m}k_L \dagger}\sqrt{L/k_Lf - j L/L_{m}k_L}, \tag{4}
\]

where \( L_{m} = 1609.344m, k_{L1} = 4.8 \times 10^{-3}, k_{L2} = -1.709 \times 10^{-3}, \) and \( k_{L3} = 4.907 \times 10^{-3}. \) Based on (4), the FEXT model can be written as

\[
    H_{FEXT}(f, l) = k_{sf} f / f_{0} \sqrt{L/L_{0}H_{IL}(f, l)}, \tag{5}
\]

where \( f_{0} = 1\text{MHz}, L_{0} = 1\text{km}, \) and \( k_{sf} = 10^{-45/20}. \)

The channel length is \( L_{11} = 100 \) and \( L_{22} = 200 \) and the transfer function for the FEXT channels \( H_{12} \) and \( H_{22} \) and the corresponding FEXT ATIS model [11] are presented in Fig. 2. It is very important to check the independence of the individual channels to design the relevant detector. By applying the Kolmogrov-Smirnov (KS) test of independence [12] at 5% significance level, the result was 0 implying that the two vectors exhibit the same distribution and the channels are independent.

It is assumed that the channel is perfectly estimated at the receiver. Now, the received signal \( R \) can be equalized using a frequency domain (FD) equalizer and expressed as

\[
    R^{eq} = RH^{\dagger} = D + WH^{\dagger} + IH^{\dagger} \tag{6}
\]
III. MIDDLETON CLASS A DISTRIBUTIONS

In this work, we consider the same IN model that is used in [7], [8] which has a probability density function (PDF) defined as

$$p(x) = \sum_{s=0}^{\infty} \frac{e^{-A}A^s}{s!} \frac{1}{\sqrt{2\pi\sigma_s^2}} e^{-\frac{(x-s)^2}{2\sigma_s^2}},$$

(7)

with $$\sigma_s^2 = \sigma^2 \left( \frac{1}{1 + \Gamma} \right)$$, $$\sigma^2 = \sigma^2_W + \sigma^2_I$$ and $$\Gamma = \frac{\sigma_W^2}{\sigma_I^2}$$. A is the impulsive index, $$\sigma^2$$ is the total noise power, $$\Gamma$$ is the AWGN power-to-IN power ratio, $$\sigma^2_W$$ and $$\sigma^2_I$$ are Gaussian and IN power, respectively. The IN compensation algorithm is defined as follows: where, $$T$$ represents the probability of

false detection and is used as a threshold value, $$R^{(\text{comp})} = [R_0^{(\text{comp})}, R_1^{(\text{comp})}, \ldots, R_{M-1}^{(\text{comp})}]$$ is the compensated signal in the frequency domain by means of a FFT as shown in Fig. 1. Procedure 1 is repeated over several iterations for better IN cancellation and to improve the BER performance.

Algorithm 1: IN compensation algorithm [10]

1: Demapping and pilot estimation \( \hat{D} \)
2: Estimation of total noise \( \hat{S} = H(R_{eq} - \hat{D}) \)
3: \( \hat{s} = \text{IDFT}(\hat{S}) \)
4: \( \sigma^2 = \frac{1}{M} \sum_{i=0}^{M-1} |\hat{s_i}|^2 \)
5: \( \hat{i} = \left\{ \begin{array}{ll}
& \hat{s_i}, \text{if } |\hat{s_i}| > T \sigma^2, \text{ for } l = 0, 1, \ldots, M-1, \\
& 0, \quad \text{Otherwise}
\end{array} \right. \)
6: Samples detection \( \hat{I} = \text{DFT}(\hat{i}) \)
7: \( \mathbf{R}^{(\text{comp})} = \mathbf{R}^{(\text{eq})} - \mathbf{I} \times \mathbf{H}^{-1} \)

IV. SIMULATION RESULTS

In this section, we discuss the simulation results for the proposed system showing the bit error rate (BER) for a G.fast system over MLCW affected by Middleton class A impulsive noise. Fig. 3 presents the performance of the G.fast MLCW with different M-QAM constellations. It can be observed that as the constellation order increases, the capacity of transmission increases while the BER performance decreases.

In Fig. 4, the BER performance of the coded G.fast single-line copper wire system is compared to the G.fast coded MLCW for two cases, one in the presence of IN and one in the absence of IN. It is observed that G.fast MLCW model has improved performance compared to the single line G.fast scenario regardless of the presence of IN with a gain of 4dB.

Fig. 5 presents the BER performance for the proposed G.fast MLCW and for the single-line copper wire, utilizing Zhidkov’s algorithm as the IN mitigation method. The G.fast MLCW shows an improvement in BER performance of 2 dB after the second iteration compared to the single-line system. While the performance of the proposed system utilizing Zhidkov’s algorithm has improved the performance by almost 5 dB compared to the proposed system with IN.

V. CONCLUSION

This work investigated the performance of G.fast on our proposed MLCW in the presence of IN. The designed MLCW...
channel has been verified and tested with the KS test and shows the independence property between the individual channels. The simulation results of this channel combined with the MCA have been presented in Section IV and they show that the G.fast MLCW has an improved performance compared to the G.fast single-line copper wire system. In addition, the simulations are verified for different M-QAM constellations and for high M values to demonstrate how the proposed system behaves with different constellations.

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