Self-Adaptive Differential Evolution Based Power Economic Dispatch of Generators with Valve-Point Effects and Multiple Fuel Options

R. Balamurugan and S. Subramanian

Abstract—This paper presents the solution of power economic dispatch (PED) problem of generating units with valve point effects and multiple fuel options using Self-Adaptive Differential Evolution (SDE) algorithm. The global optimal solution by mathematical approaches becomes difficult for the realistic PED problem in power systems. The Differential Evolution (DE) algorithm is found to be a powerful evolutionary algorithm for global optimization in many real problems. In this paper the key parameters of control in DE algorithm such as the crossover constant CR and weight applied to random differential F are self-adapted. The PED problem formulation takes into consideration of nonsmooth fuel cost function due to valve point effects and multi fuel options of generator. The proposed approach has been examined and tested with the numerical results of PED problems with thirteen-generation units including valve-point effects, ten-generation units with multiple fuel options neglecting valve-point effects and ten-generation units including valve-point effects and multiple fuel options. The test results are promising and show the effectiveness of proposed approach for solving PED problems.

Keywords—Multiple fuels, power economic dispatch, self-adaptive differential evolution and valve-point effects.

I. INTRODUCTION

The main objective of economic power dispatch problem is to determine the optimal combination of power outputs for all generating units, which minimizes the total fuel cost of thermal power plants while satisfying load demand and operating constraints of a power system. This makes the PED problem a large-scale non-linear constrained optimization problem. Conventional techniques offer good results but when the search space is non-linear and it has discontinuities they become very complicated with a slow convergence ratio and not always seeking to the optimal solution. New numerical methods are needed to cope with these difficulties, especially those with high-speed search to the optimal and not being trapped in local minima.

The stochastic search algorithms such as genetic algorithm (GA) [1], evolutionary programming (EP) [2,3], simulated annealing (SA) [4], and particle swarm optimization (PSO) [5],[6], may prove to be very effective in solving nonlinear PED problems without any restriction on the shape of the cost curves. Although these heuristic methods do not always guarantee discovering the globally optimal solution in finite time, they often provide a fast and reasonable solution (sub-optimal nearly global optimal). SA is applied in many power system problems. But, the setting of control parameters of the SA algorithm is a difficult task and convergence speed is slow when applied to a real system [7]. The Tabu Search (TS) method have been applied solve to the PED problem [8]. Though the GA methods have been employed successfully to solve complex optimization problems, recent research has identified some deficiencies in GA performance. This degradation in efficiency is apparent in applications with highly epistatic objective functions. Moreover the premature convergence of GA degrades its performance and reduces its search capability that leads to a higher probability toward obtaining a local optimum [9]. EP seems to be good method to solve optimization problems. When applied to problems consisting of more number of local optima the solutions obtained from EP method is just near global optimum one. In addition, GA and EP take long simulation time in order to obtain solution for such problems. Therefore, hybrid methods combining two or more optimization methods were introduced [10]-[13].

The generation cost function for fossil fired plants can be represented as a segmented piecewise quadratic function. The generating units, particularly those that are supplied with multi-fuel sources (coal, nature gas, or oil), lead to the problem of determining the most economic fuel to burn. Lin and Vivani [14] have discussed such a problem using the hierarchical method (HM) to find incremental fuel cost for subsystems comprising groups of units. The solution searches for optimal Lagrangian multiplier ($\lambda$) for various choices of fuel and generation range of units iteratively. J.H.Park et al. [15] proposed to apply a Hopfield Neural Network (HNN) to PED problem for a piecewise quadratic cost function. Lee et al. [16] presented an improved adaptive Hopfield neural network (AHNN) approach for finding solution of PED with multiple fuel options. It is well known that HNN converges very slowly and normally takes large number of iterations.

Manuscript received February 23, 2007.
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Baskar et al. [17] discussed a hybrid real coded genetic algorithm (HGA) for solving PED problem with multiple fuel options. An improved GA [18] has been applied for PED problems including valve-point effects and multiple fuels.

Differential evolution developed by Storn and Price is one of the most excellent evolutionary algorithms. DE is a robust statistical method for cost minimization, which does not make use of a single nominal parameter vector but instead uses a population of equally important vectors. The fittest of an offspring competes one to one with of the corresponding parent, which is different from the other evolutionary algorithms [19]. The SDE algorithm has been successfully tested on three sample test systems of PED problems. In the proposed SDE algorithm, the control parameters F and CR are not required to be pre-defined and self-adaptive mechanism is used to change these control parameters during the evolution process. These control parameters are applied at the individual levels in the population. The better values of these control parameters lead to better individuals which, in turn, are more likely to survive and produce offspring and, hence, propagate these better values. The comparison of results from the previously methods shows the proposed method provides global optimal or near global optimal solutions for realistic PED problems with reasonable execution time.

II. FORMULATION OF PED PROBLEM

The PED problem is to schedule the outputs of the online generating units so that the total fuel cost of generation can be minimized while simultaneously satisfying all unit and system equality and inequality constraints. The objective function can be formulated as

\[
\min F_T = \sum_{i=1}^{n} F_i(P_i)
\]

where \(F_i(P_i)\) is the fuel cost function of \(i\)th generator (in $/hr), \(P_i\) is the power output of \(i\)th unit, \(n\) is the number of generating units in the system. The fuel cost for the \(i\)th generator is defined by [20]

\[
F_i(P_i) = a_i P_i^2 + b_i P_i + c_i
\]

where \(a_i, b_i\) and \(c_i\) are the cost coefficients of generator \(i\).

While minimizing the total generation cost (TGC), the total generation should be the same as the total system demand. The equality constraint of the power balance is represented by

\[
\sum_{i=1}^{n} P_i = P_D
\]

where \(P_D\) is the total system demand in MW.

The generation of each generator should lie between maximum and minimum limits and this inequality constraints is represented by

\[
p_i^{\text{min}} \leq P_i \leq p_i^{\text{max}}
\]

where \(p_i^{\text{min}}\) and \(p_i^{\text{max}}\) are the minimum and maximum power outputs of the \(i\)th unit in MW.

In reality, the objective function of PED problem has non-differential points according to valve-point loadings and multiple fuels. Therefore, the objective function should be composed of a set of non-smooth cost functions. In this paper, three cases of cost functions are considered.

First is the case with the valve-point loading problem where the objective function is generally described as the superposition of sinusoidal functions and quadratic functions.

Second is the case with the multiple fuels problem where the objective function is expressed as the piecewise quadratic cost function. The other is the case with both valve-point effects and multiple fuels for the realistic PED operation, where the objective function is represented as a set of piecewise superposition of sinusoidal functions and quadratic functions.

A. PED Problem with Valve-Point Effects

The generator cost function is obtained from a data point taken “heat run” tests, when input and output data are measured as the slowly varies through its operating region. Wire drawing effects, which occur as each steam admission valve in a turbine starts to open, produce a rippling effect on the unit curve. Ref. [1] has shown the input-output performance curve for a typical thermal unit with many valve points.

Smooth quadratic function approximations of the generating unit input-output characteristics provide the basis for most classical economic dispatch techniques. Thus, the valve-point effects are ignored. This introduces some inaccuracy into the resulting dispatch. To consider the accurate cost curve of each generating unit, the valve-point effects must be included in the cost model. Therefore, the sinusoidal function is incorporated into the quadratic function [1],[2].

The fuel cost functions taking into account the valve-point effects were expressed as

\[
F_i(P_i) = a_i P_i^2 + b_i P_i + c_i + |f_i(P_i^{\text{min}} - P_i)|
\]

where \(c_i\) and \(f_i\) are the fuel cost coefficients of generator \(i\) reflecting valve point effects.

B. PED Problem with Multiple Fuels

A piecewise quadratic function is used to represent the input-output curve of a generator with multiple fuel option. A generator with \(k\) fuel options the cost curve is divided into \(k\) discrete regions between lower and upper bounds. The economic dispatch problem with piecewise quadratic function is defined as

\[
\begin{align*}
F_i(P_i) =  \begin{cases} 
   a_{1i} P_i^2 + b_{1i} P_i + c_{1i}, & \text{fuel 1}, \quad P_i^{\text{min}} \leq P_i \leq P_i^{\text{max}} \\
   a_{2i} P_i^2 + b_{2i} P_i + c_{2i}, & \text{fuel 2}, \quad P_i^{\text{min}} \leq P_i \leq P_i^{\text{max}} \\
   \vdots \\
   a_{ki} P_i^2 + b_{ki} P_i + c_{ki}, & \text{fuel k}, \quad P_i^{\text{min}} \leq P_i \leq P_i^{\text{max}} 
\end{cases}
\end{align*}
\]

where \(F_i(P_i)\) is the fuel cost function of \(i\)th unit, \(P_i\) is the power output of \(i\)th unit, and \(a_{ki}, b_{ki}\) and \(c_{ki}\) are cost functions.
coefficients of the \(i\)th unit using the fuel type \(k\).

C. PED Problem with Both Valve-Point Effects and Multiple Fuels

The accurate and practical modeling of PED problem should include the valve-point effects and multiple fuel options in the problem formulation [18]. Therefore, the total cost function should combine (5) with (6) and is formulated as follows:

\[
\begin{align*}
F_j(P_i) &= \begin{cases} 
    a_{1i}P_i^2 + b_{1i}P_i + c_{1i} + e_{1i} \sin f_{1i}(P_{\text{min}}^i - P_i), & \text{fuel1, } P_i^{\text{min}} \leq P_i \leq P_i^1 \\
    a_{2i}P_i^2 + b_{2i}P_i + c_{2i} + e_{2i} \sin f_{2i}(P_i^{\text{min}}^2 - P_i^2), & \text{fuel2, } P_i^2 < P_i \leq P_i^2 \\
    a_{ki}P_i^2 + b_{ki}P_i + c_{ki} + e_{ki} \sin f_{ki}(P_{\text{min}}^i - P_i^k), & \text{fuelk, } P_i^k < P_i \leq P_i^{\text{max}} 
    \end{cases}
\end{align*}
\]

The practical PED problem including valve-point effects and multiple fuel options makes the complication to find global optimum solution.

III. DIFFERENTIAL EVOLUTION

DE is a GA like numerical algorithm, but it differs from GA with respect to the mechanics of mutation, crossover and selections are performed. DE is a population-based approach to function optimization, whose main strategy is to generate a new position for an individual by calculating vector differences between other randomly selected members of the population. The algorithm then proceeds to manipulate the population until a termination criterion is met. The following are the outline of DE algorithm.

1. Initialize all the vector population randomly in the given upper and lower bound
2. Evaluate the fitness of each vector in the population
3. Generate a new population where each candidate individual is generated in parallel according to:
   i. For each vector \(X_i\) (target vector), select three distinct vectors \(X_a, X_b, X_c\) (select five, if two vector differences are to be used) randomly from the current population other than vector \(X_i\).
   ii. Generate a new population vector on the formula
   \[
   T = X_a + F(X_b - X_c)
   \] (8)
   iii. Perform crossover CR for target vector with its noisy vector to create a trial vector.
   iv. Evaluate the candidate.
   v. Use the candidate in the new generation if it is at least as good the current individual.
4. Loop to 3 unless the termination criterion is met.

F and CR are the control parameters. F guides the amplitude of the influence of the difference vector and CR the amount of the candidate solution that is used.

IV. SELF-ADAPTIVE DIFFERENTIAL EVOLUTION BASED PED

1. Initialization and structure of individuals: DE uses NP D-dimensional parameter vectors

\[
P_k,G, \quad k = 1, 2, \ldots, NP
\]

in a generation \(G\), with \(NP\) being constant over the entire optimization process. At the start of the procedure, i.e., generation \(G = 1\), the population vectors have to be generated randomly within the limits. These initial individual values are chosen at random from within user-defined bounds. The structure of an individual for PED problem is composed of set of elements (i.e., generation output of units) An array of control variable vectors or positions of the each agent can be represented as

\[
P_{k,G} = \left[ \left( P_{1k,1} P_{1k,2} P_{1k,3} \ldots P_{1k,n} \right) \right]
\]

(10)

where \(k = 1, 2, \ldots, NP\) is the individual’s index of population and \(n\) is the number of generators.

2. Mutation: Mutation is an operation that adds a vector differential to a population vector of individuals. For the following generation \(G+1\), new vectors \(V_{k,G+1}\) are generated according to the following mutation scheme

\[
V_{k,G+1} = P_{k,G} + F \cdot (P_{1,G} - P_{2,G})
\]

(11)

The integers \(r1\) and \(r2\) are chosen randomly over \([1, NP]\) and should be mutually different from the running index \(k\). Under certain circumstances, the index \(k\) will be exchanged by an arbitrary random number \(r3 \in [1, NP]\). \(F\) is a scaling factor, which controls the amplification of the difference between two individuals so as to avoid search stagnation. A self-adaptive control mechanism is used to change the control parameter \(F\) during the run. At generation \(G = 1\), the amplification factor \(F_{k,G}\) for each individual in the population are randomly generated within the range \([0.1, 1.0]\).

\[
\begin{array}{c|c|c|c|c|c}
P_{1,G} & F_{1,G} \\
P_{2,G} & F_{2,G} \\
P_{3,G} & F_{3,G} \\
\vdots & \vdots \\
P_{NP,G} & F_{NP,G} \\
\end{array}
\]

Fig. 1 Self-adapting: encoding aspect

New control parameters \(F_{k,G+1}\) were calculated as follows:

\[
F_{k,G+1} = \begin{cases} 
    F_{1} + \text{rand1} \times F_u \quad \text{if } \text{rand2} < \tau_1 \\
    F_{k,G} \quad \text{otherwise}
    \end{cases}
\]

(12)

and this produce new scaling factors \(F\) in a new parent vectors. \(\text{rand}\) are uniform random values within the range \([0,1.0]\). \(\tau_1\) represent probability to adjust control parameter \(F\). \(F_1, F_u, \tau_1\) were taken fixed values 0.1, 0.9, 0.8 respectively. The new \(F\) takes a value from \([0.1, 1.0]\) in a random manner. The encoding aspect of scaling factor is shown in Fig. 1.

3. Crossover operation: Each gene of \(i\)th individual is
replaced from the mutant vectors $V_{k,G+1}$ and the present individual $P_{k,G}$. That is
\[ U_{k,G+1} = P_{k,G} \times (1 - CR_{k,G+1}) + V_{k,G+1} \times CR_{k,G+1} \]

The crossover factor $CR_{k,G}$ is randomly taken from the interval $[0, 1]$ for each individual vector in the initial population. New crossover factor $CR_{k,G+1}$ for each individual during evolution process are calculated by
\[ CR_{k,G+1} = \begin{cases} CR_1 + rand_1 \times CR_u & \text{if } rand_2 < \tau_2 \\ CR_{k,G} & \text{otherwise.} \end{cases} \]

$\tau_2$ represent probability to adjust control parameter $CR$. $CR_1$, $CR_u$, $\tau_2$ were taken fixed values 0.1, 1.0, 0.7 respectively.

4. Evaluation of each agent: Each individual in the population is evaluated using the fitness function of the problem to minimize the fuel cost function. The power balance constraint is augmented with the objective to form a generalized fitness function $f_k$ as given below
\[ f_k = \sum_{i=1}^{n} f_i(P_i) + \mu \left( \sum_{i=1}^{n} P_i - P_D \right)^2 \]

where $\mu$ is penalty parameter. The penalty term reflects the violation of the equality constraint and assigns a high cost of penalty function to candidate point far from feasible region. The upper and lower generation limit of generating unit is violated then it can be fixed in the bound range by forcing it to lower/upper limit.

5. Estimation and selection: The parent is replaced by its child if the fitness of the child is better than that of its parent. Explicitly, the parent is retained in the next generation if the fitness of the child is worse than that of its parent. DE selection scheme is based on local competition only, i.e., a child $U_{k,G+1}$ will compete against one population member $P_{k,G}$ and survivor will enter the new population. The number NT of children which may be produced to compete against $P_{k,G}$ should be chosen sufficiently high so that sufficient number of child will enter the new population. If $U_{k,G+1}$ is worse than that of its parent, the vector generation process defined by (11) and (13) is repeated up to NT times. If $U_{k,G+1}$ still worse than that of its parent, $P_{k,G+1}$ will be set to $P_{k,G}$. An insufficient number NT leads to survival of too many old population vectors that may induce stagnation. To prevent a vector $P_{k,G}$ from surviving indefinitely, DE employs the concept of aging. NG defines how many generations a population vector may survive before it has to be replaced due to excessive age. To this end $P_{k,G}$ in (9) is checked first for how many generations it has already lived. If $P_{k,G}$ has an age of less than $N_G$ generations it remains unaltered, otherwise $P_{k,G}$ is replaced by $P_{r3,G}$ with $r3 \neq i$ being a randomly chosen integer $r3 \in [1, NP]$. In short, if $P_{k,G}$ is too old it may not serve as a parent vector any more but will be replaced by a randomly chosen member of the current generation $G$.

6. Stopping Criterion: The above iterative process of mutation, crossover, and selection on the population will continue until there is no appreciable improvement in the minimum fitness value or predefined maximum number of iterations reached.

V. NUMERICAL SIMULATION RESULTS AND DISCUSSION

In order to verify the effectiveness of the proposed SDE approach for solving PED problems, it has been applied to the three different kinds of PED problems with nonsmooth objective functions. The proposed SDE algorithm has been implemented in Matlab 6.5 programming language and executed on Pentium IV 2.8-GHZ computer. The solutions obtained through the SDE are compared with results reported in the literature.

A. Case 1- PED Problem with Valve-Point Effects

The proposed algorithm is applied to a sample system consisting of thirteen generating units with valve-point effects. This PED problem includes one objective function, one equality constraint, that is, total generation of all the committed generating units should meet the given power demand, and twenty six inequality constraints, that is, power generation of each unit in the system should be within the minimum and maximum generation limits. To compare the results of the proposed approach to other reported approaches utilizes HSS [10], TSA [8], EP-SQP [13] and PSO-SQP [13], the same system data is considered. The generating unit operating data and cost data are obtained from [2] and is given in the Appendix-1. The expected power demand to be supplied by all the thirteen generating units is 2520 MW. The SDE simulation parameters chosen for this sample system are population size $NP = 50$; maximum number of generations $NG = 300$; number of trial per iteration NT=20 and number of generations a population vector may survive before it has to be replaced due to excessive age $NR=5$.

To show the consistency in getting optimal solutions, 50 runs were conducted with the total demand of 2520 MW. The best,

![Fig. 2 Cost curves of unit 3 with and without valve point effects](image-url)
average and worst operating costs found the proposed method are $24164.05, $24168.28 and $24200.05 respectively.

### TABLE I

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### TABLE III

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<td>526.3232</td>
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The best optimal dispatch of generators obtained using proposed method, HSSP[10], TSA[8], EP-SQP[13] and PSOSQP[13] are shown in Table I. It is clear that the proposed approach produces much better results compared to other methods reported in the literature. The average computation time taken by the proposed method is 28.5s. Figure 2 shows the input-output performance curves for a thermal unit-3 with many valve points. The data for the thirteen generating unit sample system is given in Appendix.

B. Case 2 - PED Problem with Multiple Fuels Neglecting Valve-point Effects

The ten-generator system, each unit with two or three fuel options is taken to illustrate the SDE solution to the PED problem including multiple fuel options and neglecting valve point effects. The fuel cost coefficients $e_i$ and $f_i$ of generator $i$ reflecting valve-point effects should be treated as zero for this case. The hierarchical system characteristics are shown in Appendix. Generation (Min) and (Max) are lower and upper limits of each generation unit. There are three different types of fuels : type 1, 2, and 3. The total system demand is 2700 MW. This economic dispatch problem includes one objective function with ten variable parameters ($P_1, P_2, ..., P_{10}$), one equality and twenty inequality constraints ie. power balance constraint, minimum and maximum limits of each generating unit. The system data and related constraints are taken from ref. [18] and the corresponding data for this example is given in Appendix neglecting valve point effects. The initial population of the SADE contains random choice of generation between minimum and maximum generation limits of each unit. The following control parameter has been chosen after study results proved that the proposed SDE algorithm converged to high quality solutions at the early iterations. The comparison of the results with other methods reported in the literature shows the superiority of the proposed algorithm. The average computation time taken by the proposed approach is 18.23 s.

C. Case 3 - PED Problem with Valve-point Effects and Multiple Fuels

In this case, a realistic PED problem with ten generating units considering both valve point effects and multiple fuels is studied. The unit curves have non-differential points according to valve-point loading and multiple fuel changes. The system data is given in the Appendix. Table III shows the best optimal dispatch of generating units for various system demands. The best, average and worst operating costs in 50 trials obtained are given in Table IV. Figure 3 shows the curve of unit 8 including valve-point effects and multiple fuel changes. Figure 4 shows the convergence characteristics of the SDE for a demand of 2700MW.

VI. CONCLUSION

A self-adaptive differential evolution algorithm for solving the PED problem with nonsmooth cost functions considering valve point effects and multiple fuels are presented in this paper. The feasibility of the proposed method for solving PED problems was demonstrated with various economic dispatch problems considering nonlinearities due to valve-point effects and multiple fuel options. The control variables F and CR are automatically adopted during the run, which avoids the complication of tuning the control parameters in the DE algorithm. The comparison of the results with other methods reported in the literature shows the superiority of the proposed method and its potential for solving non-smooth PED problems in a power system.
### APPENDIX

#### Table I

**SYSTEM DATA FOR 13-UNIT SYSTEM CONSIDERING VALVE-POINT EFFECTS**

<table>
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<th>Unit</th>
<th>Generation limits</th>
<th>Cost coefficients</th>
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<td>$P_i^{max}$</td>
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#### Table II

**SYSTEM DATA FOR 10 UNIT SYSTEM CONSIDERING VALVE-POINT EFFECTS AND MULTIPLE FUELS**

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ACKNOWLEDGMENT

The authors gratefully acknowledge the authorities of Annamalai University, Annamalainagar, Tamilnadu, India, for their continued support, encouragement, and the extensive facilities provided to carry out this research work.

REFERENCES