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The direct updating of damping and gyroscopic matrices using incomplete complex test data

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Abstract—In this paper we develop an efficient numerical method for the finite-element model updating of damped gyroscopic systems based on incomplete complex modal measured data. It is assumed that the analytical mass and stiffness matrices are correct and only the damping and gyroscopic matrices need to be updated. By solving a constrained optimization problem, the optimal corrected symmetric damping matrix and skew-symmetric gyroscopic matrix complied with the required eigenvalue equation are found under a weighted Frobenius norm sense.

Keywords—model updating, damped gyroscopic system, partially prescribed spectral information.

I. Introduction

DAMPED gyroscopic systems are important class of nonproportionally damped systems. They correspond to spinning structures where the Coriolis inertia forces are taken into account. Examples of such systems include helicopter rotor blades and spin-stabilized satellites with flexible elastic appendages such as solar panels or antennas[12, 16]. Using finite element techniques, a damped gyroscopic system with finitely many degrees-of-freedom can be modelled by a vector differential equation in the second-order form given by

$$M_a\ddot{x}(t) + (C_a + G_a)\dot{x}(t) + K_a x(t) = 0, \qquad (1)$$

where M_a, C_a, G_a, K_a are $n \times n$ analytical mass, damping, gyroscopic and stiffness matrices, respectively. The gyroscopic matrix G_a is always skew-symmetric and, in many practical applications, the mass matrix M_a is symmetric and positive definite($M_a > 0$), and K_a, D_a are symmetric matrices. If the gyroscopic force is not present, then the system is called non-gyroscopic.

It is well-known that all solutions of the differential equation of (1) can be obtained via the algebraic equation

$$(\lambda^2 M_a + \lambda(D_a + G_a) + K_a)\phi = 0. \qquad (2)$$

Complex numbers λ and nonzero vectors ϕ for which this relation holds are, respectively, the eigenvalues and eigenvectors of the system. It is known that the equation of (2) has $2n$ finite eigenvalues over the complex field, provided that the leading matrix coefficient M_a is nonsingular.

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Finite element model updating, at its ambitious, is used to correct inaccurate analytical model by measured data, such as natural frequencies, damping ratios, mode shapes and frequency response function, which can usually be obtained by physical vibration test. The need to solve the finite element model updating problem arises from the fact that very often natural frequencies and mode shapes (eigenvalues and eigenvectors) of a finite element model described by (1) do not match very well with experimentally measured frequencies and mode shapes obtained from a real-life vibrating structure. Thus, a vibration engineer needs to update the theoretical finite element model to ensure its validity for future use. Because of its immense practical importance, finite element model updating problem has been well-studied in the past twenty years, Baruch[1, 2], Berman[5, 6] and Wei[13-15] considered variant aspects of finite element model updating by using measured data for the case that $C_a = 0$ and $D_a = 0$. In view of in analytical model (1) for structure dynamics, the mass and stiffness are, in general, clearly defined by physical parameters. However, the effect of damping and Coriolis forces on structural dynamic systems is not well understood because it is purely dynamics property that can not be measured statically. Many works have been done about the damping matrix adjustment using measured data(see[3, 8, 9, 10]). Recently, Datta and Sarkissian[7, 11] have considered the partial eigenvalue and eigenstructure assignment problems using feedback control technique for the undamped gyroscopic systems. However, the problem of updating the damping and gyroscopic matrices simultaneously hasn't been considered as yet. In this paper we develop an efficient numerical method for the finite-element model updating of damped gyroscopic system based on incomplete complex modal measured data. It is assumed that the analytical mass matrix M_a and stiffness matrix K_a are correct and the damping and gyroscopic matrices need to be updated. By solving a constrained optimization problem, the optimal corrected symmetric damping matrix and skew-symmetric gyroscopic matrix complied with the required eigenvalue equation are found under a weighted Frobenius norm sense. That is, we deal with the following problem:

Problem P. Let $\Lambda \in \mathbf{C}^{m\times m}$ and $\Phi \in \mathbf{C}^{n\times m}$ be the measured eigenvalue and eigenvector matrices in the form

$$\Lambda = \text{diag}\{\lambda_1, \lambda_2, \cdots, \lambda_{2l-1}, \lambda_{2l}, \lambda_{2l+1}, \cdots, \lambda_m\} \in \mathbf{C}^{m\times m} \tag{3}$$

and

$$\Phi = [\phi_1, \phi_2, \cdots, \phi_{2l-1}, \phi_{2l}, \phi_{2l+1}, \cdots, \phi_m] \in \mathbf{C}^{n\times m}, \tag{4}$$

where Λ and Φ are closed under complex conjugation in the sense that $\lambda_{2j} = \bar{\lambda}_{2j-1} \in \mathbf{C}, \phi_{2j} = \bar{\phi}_{2j-1} \in \mathbf{C}^n$ for $j = 1, \cdots, l$, and $\lambda_k \in \mathbf{R}, \phi_k \in \mathbf{R}^n$ for $k = 2l+1, \cdots, m$, find real-valued symmetric matrix C and real-valued skew-symmetric matrix G such that the penalty function

$$J = \|W^{-\frac{1}{2}}(C - C_a)W^{-\frac{1}{2}}\|^2 + \|W^{-\frac{1}{2}}(G - G_a)W^{-\frac{1}{2}}\|^2 \tag{5}$$

is minimized, subject to

$$M_a\Phi\Lambda^2 + (C+G)\Phi\Lambda + K_a\Phi = 0 \tag{6}$$

and

$$C = C^T, \quad G = -G^T,$$

where W is a symmetric positive definite weighting matrix.

In this paper we shall adopt the following notation. $\mathbf{C}^{m\times n}, \mathbf{R}^{m\times n}$ denote the set of all $m \times n$ complex and real matrices, respectively. $\bar{\alpha}$ denotes the conjugate of the complex number α, A^T, A^+ and tr$A$ denote the transpose, the Moore-Penrose generalized inverse and the trace of the matrix A, respectively. I_n denotes the $n \times n$ identity matrix, and $\|\cdot\|$ stands for the matrix Frobenius norm.

II. Solving Problem P

To begin with, we introduce a lemma(see [4]).

Lemma 1: If $T \in \mathbf{R}^{m\times m}$, $S \in \mathbf{R}^{n\times m}$ then $ZT = S$ has a solution $Z \in \mathbf{R}^{n\times m}$ if and only if $ST^+T = S$. In this case, the general solution of the equation can be described as $Z = ST^+ + L(I_m - TT^+)$, where $L \in \mathbf{R}^{n\times m}$ is an arbitrary matrix.

Let $\alpha_i = \text{Re}(\lambda_i)$ (the real part of the complex number λ_i), $\beta_i = \text{Im }(\lambda_i)$ (the imaginary part of the complex number λ_i), $y_i = \text{Re}(\phi_i)$, $z_i = \text{Im }(\phi_i)$ for $i = 1, 3, \cdots, 2l-1$. Define

$$\tilde{\Lambda} = \text{diag}\left\{\begin{bmatrix} \alpha_1 & \beta_1 \\ -\beta_1 & \alpha_1 \end{bmatrix}, \cdots, \begin{bmatrix} \alpha_{2l-1} & \beta_{2l-1} \\ -\beta_{2l-1} & \alpha_{2l-1} \end{bmatrix}, \lambda_{2l+1}, \cdots, \lambda_m\right\} \in \mathbf{R}^{m\times m}, \tag{7}$$

$$\tilde{\Phi} = [y_1, z_1, \cdots, y_{2l-1}, z_{2l-1}, \phi_{2l+1}, \cdots, \phi_m] \in \mathbf{R}^{n\times m}. \tag{8}$$

Then the equation of (6) can be written equivalently as

$$M_a\tilde{\Phi}\tilde{\Lambda}^2 + (C+G)\tilde{\Phi}\tilde{\Lambda} + K_a\tilde{\Phi} = 0. \tag{9}$$

In order to ensure that $C = C^T, G = -G^T$, we write $C = A + A^T$ and $G = B - B^T$, where A, B are arbitrary real-valued matrices. From the function J and the equation of (9), the following Lagrange function is constructed

$$\begin{aligned} f = &\frac{1}{4}\text{tr}\{W^{-1}(A + A^T - C_a)W^{-1}(A + A^T - C_a)\} \\ &+\frac{1}{4}\text{tr}\{W^{-1}(B^T - B + G_a)W^{-1}(B - B^T - G_a)\} \\ &+\text{tr}\{\Psi^T(M_a\tilde{\Phi}\tilde{\Lambda}^2 + (A + A^T + B - B^T)\tilde{\Phi}\tilde{\Lambda} + K_a\tilde{\Phi})\}. \end{aligned}$$

The first two terms on the right are the Frobenius norm in "trace" form (the $\frac{1}{4}$ was added for convenience) and Ψ is the Lagrange multiplier matrix.

The partial derivatives of f with respect to A, B and Ψ are as follows

$$\frac{\partial f}{\partial A} = W^{-1}(A + A^T - C_a)W^{-1} + \Psi(\tilde{\Phi}\tilde{\Lambda})^T + \tilde{\Phi}\tilde{\Lambda}\Psi^T = 0, \tag{10}$$

$$\frac{\partial f}{\partial B} = W^{-1}(B - B^T - G_a)W^{-1} + \Psi(\tilde{\Phi}\tilde{\Lambda})^T - \tilde{\Phi}\tilde{\Lambda}\Psi^T = 0, \tag{11}$$

$$\frac{\partial f}{\partial \Psi} = M_a\tilde{\Phi}\tilde{\Lambda}^2 + (A + A^T + B - B^T)\tilde{\Phi}\tilde{\Lambda} + K_a\tilde{\Phi} = 0. \tag{12}$$

Notice that A and B appear in the form $A + A^T$ and $B - B^T$ in the equations (10), (11) and (12) so $C = A + A^T$ and $G = B - B^T$ are used in the remainder of this section. adding equation (10) and (11) yields

$$\Psi(\tilde{\Phi}\tilde{\Lambda})^T = -\frac{1}{2}W^{-1}(C + G - C_a - G_a)W^{-1}. \tag{13}$$

Substituting equation (13) into (12) leads to

$$M_a\tilde{\Phi}\tilde{\Lambda}^2 + (C_a + G_a)\tilde{\Phi}\tilde{\Lambda} + K_a\tilde{\Phi} = 2W\Psi(\tilde{\Phi}\tilde{\Lambda})^T W\tilde{\Phi}\tilde{\Lambda}. \tag{14}$$

For convenience, we shall denote

$$S = M_a\tilde{\Phi}\tilde{\Lambda}^2 + (C_a + G_a)\tilde{\Phi}\tilde{\Lambda} + K_a\tilde{\Phi},\ T = (\tilde{\Phi}\tilde{\Lambda})^T W\tilde{\Phi}\tilde{\Lambda}. \tag{15}$$

By Lemma 1, the equation of (14) with respect to unknown matrix $\Psi \in R^{n\times m}$ has a solution if and only if

$$ST^+T = S. \tag{16}$$

In this case, the general solution of (14) can be written as

$$\Psi = \frac{1}{2}W^{-1}ST^+ + L(I_m - TT^+) \tag{17}$$

where $L \in \mathbf{R}^{n\times m}$ is an arbitrary matrix. Substituting equation (17) into (10) and (11), and recalling that $(I_m - TT^+)(\tilde{\Phi}\tilde{\Lambda})^T = 0$, leads to the solution to problem P as

$$C = C_a - \frac{1}{2}ST^+(\tilde{\Phi}\tilde{\Lambda})^T W - \frac{1}{2}W\tilde{\Phi}\tilde{\Lambda}T^+S^T, \tag{18}$$

$$G = G_a - \frac{1}{2}ST^+(\tilde{\Phi}\tilde{\Lambda})^T W + \frac{1}{2}W\tilde{\Phi}\tilde{\Lambda}T^+S^T. \tag{19}$$

Notice that if rank$(\Phi\Lambda) = m$, then the matrix T is nonsingular, in this case, the condition (16) is always satisfied and the unique solution to problem P can be expressed as

$$C = C_a - \frac{1}{2}ST^{-1}(\tilde{\Phi}\tilde{\Lambda})^T W - \frac{1}{2}W\tilde{\Phi}\tilde{\Lambda}T^{-1}S^T, \tag{20}$$

$$G = G_a - \frac{1}{2}ST^{-1}(\tilde{\Phi}\tilde{\Lambda})^T W + \frac{1}{2}W\tilde{\Phi}\tilde{\Lambda}T^{-1}S^T. \quad (21)$$

Based on the above discuss, we can state the following algorithm.

Algorithm 1(An algorithm for solving Problem P).

1) Input $M_a,\ K_a,\ C_a,\ G_a,\ \Lambda,\ \Phi$.
2) Separate matrices Λ and Φ into real parts and imaginary parts resulting $\tilde{\Lambda}$ and $\tilde{\Phi}$ given as in (7) and (8).
3) Select weighting matrix W.
4) Compute S, T according to (15).
5) If (16) holds, then continue, otherwise, go to 1).
6) According to (18) and (19) calculate C and G.

Example 1 Consider a five-DOF system modelled analytically with mass and stiffness matrices given by

$$M_a = \text{diag}\{1,\ 2,\ 5,\ 4,\ 3\},$$

$$K_a = \begin{bmatrix} 100 & -20 & 0 & 0 & 0 \\ -20 & 120 & -35 & 0 & 0 \\ 0 & -35 & 80 & -12 & 0 \\ 0 & 0 & -12 & 95 & -40 \\ 0 & 0 & 0 & -40 & 124 \end{bmatrix}.$$

The measured eigenvalue and eigenvector matrices Λ and Φ are given by

$$\Lambda = \text{diag}\{-0.0077 + 10.2676i,\ -0.0077 - 10.2676i,\ -0.0024 + 7.6320i,\ -0.0024 - 7.6320i\},$$

$$\Phi = \begin{bmatrix} 0.9670 + 0.0845i & 0.9670 - 0.0845i & -0.0673 - 0.4349i & -0.0673 + 0.4349i \\ -0.2207 - 0.0748i & -0.2207 + 0.0748i & 0.0205 - 0.8815i & 0.0205 + 0.8815i \\ 0.0139 + 0.0120i & 0.0139 - 0.0120i & -0.0402 + 0.1424i & -0.0402 - 0.1424i \\ -0.0045 - 0.0089i & -0.0045 + 0.0089i & 0.0314 - 0.0283i & 0.0314 + 0.0283i \\ -0.0069 + 0.0546i & -0.0069 - 0.0546i & -0.0718 + 0.0034i & -0.0718 - 0.0034i \end{bmatrix}.$$

The estimated analytical damping and gyroscopic matrices are

$$C_a = \begin{bmatrix} 0.0110 & -0.0080 & 0 & 0 & 0 \\ -0.0080 & 0.0140 & -0.0035 & 0 & 0 \\ 0 & -0.0035 & 0.0130 & -0.0078 & 0 \\ 0 & 0 & -0.0078 & 0.0135 & -0.0090 \\ 0 & 0 & 0 & -0.0090 & 0.0154 \end{bmatrix},$$

$$G_a = \begin{bmatrix} 0 & 0.5304 & -0.0276 & -0.0334 & -0.9247 \\ -0.5304 & 0 & 1.1740 & -0.4364 & 0.9274 \\ 0.0276 & -1.1740 & 0 & -1.7405 & 1.1363 \\ 0.0334 & 0.4364 & 1.7405 & 0 & 3.3130 \\ 0.9247 & -0.9274 & -1.1363 & -3.3130 & 0 \end{bmatrix}.$$

Let $W = M_a$, according to Algorithm 1, it is calculated that the condition (16) holds. Using the Software "MATLAB", we can figure out

$$C = \begin{bmatrix} 0.0120 & -0.0089 & -0.0030 & -0.0031 & -0.0020 \\ -0.0089 & 0.0166 & -0.0023 & 0.0123 & -0.0085 \\ -0.0030 & -0.0023 & -0.0671 & -0.0427 & -0.0986 \\ -0.0031 & 0.0123 & -0.0427 & 0.1155 & -0.1535 \\ -0.0020 & -0.0085 & -0.0986 & -0.1535 & -0.0116 \end{bmatrix},$$

$$G = \begin{bmatrix} -0.0000 & 0.4770 & -0.0264 & -0.0343 & -0.8310 \\ -0.4770 & 0.0000 & 1.0664 & -0.3868 & 0.8449 \\ 0.0264 & -1.0664 & -0.0000 & -1.6632 & 1.1087 \\ 0.0343 & 0.3868 & 1.6632 & -0.0000 & 3.1350 \\ 0.8310 & -0.8449 & -1.1087 & -3.1350 & 0.0000 \end{bmatrix}.$$

We define the residual as

$$\text{res}(\lambda_i, \phi_i) = \|(\lambda_i^2 M_a + \lambda_i(\hat{D} + \hat{G}) + K_a)\phi_i\|,$$

and show the numerical results

(λ_i, ϕ_i)	$\text{res}(\lambda_i, \phi_i)$
(λ_1, ϕ_1)	1.3402e-014
(λ_2, ϕ_2)	1.3402e-014
(λ_3, ϕ_3)	8.4423e-015
(λ_4, ϕ_4)	8.4423e-015

III. Concluding remarks

we have developed an efficient numerical method for the finite-element model updating of damped gyroscopic system based on incomplete complex modal measured data. Numerical example shows that this method can serve as a fast and reliable manner for updating the analytical model.

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