Computational Networks for Knowledge Representation

Nhon Van Do

Abstract—In the artificial intelligence field, knowledge representation and reasoning are important areas for intelligent systems, especially knowledge base systems and expert systems. Knowledge representation methods have an important role in designing the systems. There have been many models for knowledge such as semantic networks, conceptual graphs, and neural networks. These models are useful tools to design intelligent systems. However, they are not suitable to represent knowledge in the domains of reality applications. In this paper, new models for knowledge representation called computational networks will be presented. They have been used in designing some knowledge base systems in education for solving problems such as the system that supports studying knowledge and solving analytic geometry problems, the program for studying and solving problems in Plane Geometry, the program for solving problems about alternating current in physics. These applications have been implemented by using programming tools and computer algebra systems such as C++, JAVA, and MAPLE. They are very easy to use for students in studying knowledge, to solve automatically problems and give human readable solutions agree with those written by teachers and students. Problems are also modeled easily using the computational networks, together with the algorithms for solving problems automatically and propose a simple language for specifying them.

Keywords—Artificial intelligence, artificial intelligence and education, knowledge engineering, knowledge representation.

I. INTRODUCTION

In artificial intelligence science, models and methods for knowledge representation play an important role in designing knowledge base systems and expert systems. Nowadays there are many various knowledge models which have already been suggested and applied. In the books [1], [2], [3], and [4] we have found popular methods for knowledge representation in designing knowledge base systems (KBS) such as predicate logic, semantic nets, frames, deductive rules. Many new methods and techniques were presented in [11], [12], [13], and [14]. Among these methods neural networks and fuzzy logic can be used for computational intelligence. Some methods are suitable for representing and processing semantics such as conceptual graphs in [8], [9] and [10].

The above methods are very useful for designing intelligent systems, and for solving complex problems. However, they are not suitable to represent knowledge in the domains of reality applications in many cases, especially the systems that can solve problems in practice based on the knowledge base. The ontology called COKB-ONT presented in [7] is also a good and useful tool for developing knowledge base systems in practice. This ontology was used to construct some intelligent systems in education, and these systems were introduced in [5], [6] and [15]. Although COKB-ONT is very useful and suitable for representing knowledge, it is not strong enough for representing knowledge in the domains of reality applications. Therefore, it is needed to develop new models to represent problems with knowledge. In this paper, we present the models for knowledge representation that are called computational networks. They have been used in designing some knowledge base systems in education for solving problems such as the system that supports studying knowledge and solving analytic geometry problems, the program for studying and solving problems in Plane Geometry, the program for solving problems about alternating current in physics. These applications have been implemented by using programming tools and computer algebra systems such as C++, JAVA, and MAPLE. They are very easy to use for students in studying knowledge, to solve automatically problems and give human readable solutions agree with those written by teachers and students. Problems are also modeled easily using the computational networks, together with the algorithms for solving problems automatically and propose a simple language for specifying them.

II. COMPUTATIONAL NETWORKS WITH SIMPLE VALUED VARIABLES

In this part a simple model of computational nets will be presented together related problems and techniques for solving them. Although this model is not very complicated, but it is a very useful tool for designing many knowledge base systems in practice.

A. Definitions

Definition 2.1: A computational network (CN) with simple valued variables is a pair \((M, F)\), in which \(M = \{x_1, x_2, ..., x_n\}\) is a set of variables with simple values (or unstructured values), and \(F = \{f_1, f_2, ..., f_m\}\) is a set of computational relations over the variables in the set \(M\). Each computational relation \(f \in F\) has the following form:

(i) An equation over some variables in \(M\), or
(ii) Deductive rule \(f: u(f) \rightarrow v(f)\), with \(u(f) \subseteq M\), \(v(f) \subseteq M\), and there are corresponding formulas to determine (or to compute) variables in \(v(f)\) from variables in \(u(f)\). We also define the set \(M(f) = u(f) \cup v(f)\).

Remark: In many applications equations can be represented as deduction rules.

Example 2.1: The computational knowledge over elements of a triangle named ABC can be represented by a computational networks \((M, F)\) with \(M = \{A, B, C, a, b, c, R, S, p, ...\}\) (the set of all attributes of triangle ABC) and
Given a computational net $K = (M, F)$. Let $f_1, f_2, ..., f_m$ be a good solution of the problem $H \rightarrow G$ on computational net $(M, F)$. Step 1: NewS $\leftarrow \emptyset$; $V \leftarrow G$; 
Step 2: for $i := k$ downto 1 do 
If $(v(f_i) \cap V \neq \emptyset)$ then 
Begin 
Insert $f_i$ at the beginning of NewS; 
$V \leftarrow (V - v(f_i)) \cup (u(f_i) - H)$; 
End 
Step 3: NewS is a good solution.

On a computational net $(M, F)$, in many cases the problem $H \rightarrow G$ has a solution $S$ in which there are relations producing some redundancy variables. At those situations, we must determine necessary variables of each step in the problem solving process. The following theorem shows the way to analyse the solution to determine necessary variables to compute at each step.

Theorem 2.2: Given a computational net $K = (M, F)$. Let $f_1, f_2, ..., f_m$ be a good solution of the problem $H \rightarrow G$. Denote $A_0 = H$, $A_i = [f_1, f_2, ..., f_i](H)$, with $i=1, ..., m$. Then there exists a list $[B_{m}, B_{m-1}, B_{m-2}, ..., B_{1}]$ satisfying the following conditions:

1. $B_m = G$.
2. $B_i \subseteq A_i$, with $i=0, 1, ..., m$.
3. For $i=1, ..., m$, $[f_i]$ is a solution of the problem $B_{i-1} \rightarrow B_i$ but not to be a solution of the problem $B \rightarrow B_i$, with $B$ is any proper subset of $B_{i-1}$.

Example 2.3: For the computational problem $A \rightarrow \{S\}$, we can apply the algorithm to the following system of relations:

\[ F = \{ f_1 : A + B + C = \pi, \quad f_2 : \frac{a}{\sin(A)} = \frac{b}{\sin(B)} \}, \]
\[ f_3 : \frac{c}{\sin(C)} = \frac{b}{\sin(B)}, \quad f_4 : \frac{a}{\sin(A)} = \frac{c}{\sin(C)}, \]
\[ f_5 : p = (a+b+c)/2, \quad f_6 : S = a.h_a / 2, \quad f_7 : S = b.h_b / 2, \]
\[ f_8 : S = c.h_c / 2, \quad f_9 : S = a.b \sin(C) / 2, \ldots \} \]
\[ \begin{align*}
\text{Solution_found} & \leftarrow \text{false}; \\
\text{Step 3: Repeat} & \\
\text{Hold} & \leftarrow H; \\
\text{Select} & \ f \in F; \\
\text{while} & \text{not Solution_found} \text{ and } (f \text{ found}) \text{ do} \\
\text{begin} & \\
\text{if} & (\text{applying } f \text{ from } H \text{ produces new facts}) \\
\text{then} & \\
\text{begin} & \\
H & \leftarrow H \cup M(f); \\
\text{Add } f \text{ to Solution}; \\
\text{end}; \\
\text{if} & G \subseteq H \text{ then} \\
\text{Solution_found} & \leftarrow \text{true}; \\
\text{Select new } & f \in F; \\
\text{end}; \{ \text{while} \} \\
\text{Until} & \text{ Solution_found or } (H = \text{Hold}); \\
\text{Step 4: if} & \text{ not Solution_found then} \\
\text{There is no solution found;} \end{align*} \]
is not a good solution because there exists a redundancy relation such as $f_5$. From the solution $\text{Sol}$ the algorithm 2.2 will give the new solution $\text{NewSol} = \{f_1, f_2, f_3\}$, and the process to solve the problem is as follows:

Step 1: Compute $A$ by applying $f_1$;
Step 2: Compute $b$ by applying $f_2$;
Step 3: Compute $S$ by applying $f_3$;

III. NETWORKS OF COMPUTATIONAL OBJECTS

In many problems we usually meet many different kinds of objects. Each object has attributes and internal relations between them. Therefore, it is necessary to consider an extension of computational nets in which each variable is a computational object.

Definition 3.1: A computational object (or Com-object) has the following characteristics:

1. It has valued attributes. The set consists of all attributes of the object $O$ will be denoted by $M(O)$.
2. There are internal computational relations between attributes of a Com-object $O$. These are manifested in the following features of the object:
   - Given a subset $A$ of $M(O)$. The object $O$ can show us the attributes that can be determined from $A$.
   - The object $O$ will give the value of an attribute.
   - It can also show the internal process of determining the attributes.

Example 3.1: A triangle with some knowledge (formulas, theorems, etc ...) is an object. The attributes of a “triangle” object are 3 edges, 3 angles, etc A “triangle” object can also answer some questions such as “Is there a solution for the problem that to compute the surface from one edge and two angles?”.

Definition 3.2: A computational relation $f$ between attributes of certain objects is called a relation between the objects. A network of Com-objects will consists of a set of $\mathcal{O} = \{O_1, O_2, ..., O_n\}$ and a set of computational relations $\mathcal{F} = \{f_1, f_2, ..., f_m\}$. This network of Com-objects is denoted by $(\mathcal{O}, \mathcal{F})$. The following are some notations:

$M(f) = \bigcup_{i=1}^{m} M(f_i)$.

$M(O) = \bigcup_{i=1}^{n} M(O_i)$.

$M = \text{the set of attributes of C-objects considered in certain problem.}$

$M_{i} = M \cap M(O_i)$, for $i = 1, 2, ..., m$. By the above notations, $M_i$ is the set of attributes considered of the object $O_i$.

On the network of Com-objects $(\mathcal{O}, \mathcal{F})$, we consider the problem that to determine (or compute) attributes in set $G$ from given attributes in set $H$. The problem will be denoted by $H \rightarrow G$.

Example 3.2: In the figure 1 below, suppose that $AB = AC$, the values of the angle $A$ and the edge BC are given (hypothesis). ABDE and ACFG are squares. Compute EG.
Step 2: if \( G \subseteq H \) then
begin
Solution_found \leftarrow \text{true};
goto \text{step 5};
end
else
Solution_found \leftarrow \text{false};
Step 3: Repeat
Hold \leftarrow H;
Select \( f \in F; \)
while not Solution_found and (f found) do
begin
if (applying \( f \) from \( H \) produces new facts)
then
begin
\( H \leftarrow H \cup M(f); \)
Add \( f \) to Solution;
end;
if \( G \subseteq H \) then
Solution_found \leftarrow \text{true};
Select new \( f \in F; \)
end;
end;
\text{Until} Solution_found or (H = Hold);
Step 4: if not Solution_found then
begin
Select \( O_i \in O \) such that \( O_i(H) \neq H; \)
if (the selection is successful) then
begin
\( H \leftarrow O_i(H); \)
Add \( O_i \) to Solution;
if \( (G \subseteq H) \) then
begin
Solution_found \leftarrow \text{true};
goto \text{step 5};
end;
else
Solution_found \leftarrow \text{false};
\text{goto} \text{ step 3};
end;
end;
Step 5: if not Solution_found then
There is no solution found;
else
Solution is a solution of the problem;

The above algorithms will produce the solution
\[ D = \{ f_0, O_1, f_1, f_2, f_3, f_4, O_2 \}, \]

And the process of extending the set of attributes as follows:

\[
\begin{align*}
A_0 & \xrightarrow{f_1} A_1 \quad O_1 \quad A_2 \quad f_1 \\
A_3 & \xrightarrow{f_3} A_4 \quad f_5 \quad A_5 \quad f_4 \\
A_6 & \xrightarrow{O_2} A_7 
\end{align*}
\]

Where
\[
A_0 = A = \{ O_1.a, O_1.A \},
A_1 = \{ O_1.a, O_1.A, O_2.A \},
A_3 = \{ O_1.a, O_1.A, O_2.A, O_1.b, O_1.c \},
A_4 = \{ O_1.a, O_1.A, O_2.A, O_1.b, O_1.c, O_2.a \},
A_5 = \{ O_1.a, O_1.A, O_2.A, O_1.b, O_1.c, O_2.a, O_2.b \},
A_6 = \{ O_1.a, O_1.A, O_2.A, O_1.b, O_1.c, O_2.a, O_2.b, O_2.c \},
A_7 = \{ O_1.a, O_1.A, O_2.A, O_1.b, O_1.c, O_2.a, O_2.b, O_2.c, O_2.a \}.
\]

IV. EXTENSION OF COMPUTATIONAL NETWORKS

Computational Networks with simple valued variables and networks of computational objects can be used to represent knowledge in many domains of knowledge. The basic components of knowledge consist of a set of simple valued variables and a set of computational relations over the variables. However, there are domains of knowledge based on a set of elements, in which each element can be a simple valued variables or a function. For example, in the knowledge of alternating current the alternating current intensity \( i(t) \) and the alternating potential \( u(t) \) are functions. It requires considering some extensions of computational networks such as extensive computational networks and extensive computational objects networks that are defined below.

**Definition 4.1:** An extensive computational network is a structure \((M, R)\) consisting of two following sets:

- \( M = M_v \cup M_f \) is a set of attributes or elements, with simple valued or functional valued.
  \( M_v = \{x_{v1}, x_{v2}, \ldots, x_{vk}\} \) is the set of simple valued variables.
  \( M_f = \{x_{f1}, x_{f2}, \ldots, x_{fn}\} \) is the set of functional valued elements.
- \( R = R_{vv} \cup R_{fv} \cup R_{vf} \cup R_{vvf} \) is the set of deduction rules, and \( R \) is the union of four subsets of rules
  \( R_{vv}, R_{fv}, R_{vf}, R_{vvf} \). Each rule \( r \) has the form \( r: u(r) \rightarrow v(r) \), with \( u(r) \) is the hypotheses of \( r \) and \( v(r) \) is the conclusion of \( r \). A rule is also one of the four cases below.

- **Case 1:** \( r \in R_{vv} \). For this case, \( u(r) \subseteq M_v \) and \( v(r) \subseteq M_v. \)
Case 2: \( r \in R_\ell \). For this case, \( u(r) \subseteq M_\ell \) and \( v(r) \subseteq M_\ell \).

Case 3: \( r \in R_{vf} \). For this case, \( u(r) \subseteq M_v \) and \( v(r) \subseteq M_v \).

Case 4: \( r \in R_{fvf} \). For this case, \( u(r) \subseteq M_v \), \( u(r) \cap M_f \neq \emptyset \), and \( v(r) \subseteq M_f \).

Each rule in \( R \) has the corresponding computational relation in the set \( F = F_{vv} \cup F_{vf} \cup F_{fvf} \).

**Definition 4.2:** An extensive computational Object (ECom-Object) is an object \( O \) has structure including:
1. A set of attributes \( \text{Attr}(O) = M_v \cup M_f \) with \( M_v \) is a set of simple valued variables; \( M_f \) is a set of functional variables. Between the variables (or attributes) there are internal relations, that are deduction rules or the computational relations.
2. The object \( O \) has behaviours of reasoning and computing on attributes of objects or facts such as:
   - Find the closure of a set \( A \subseteq \text{Attr}(O) \).
   - Find a solution of problems which has the form \( A \rightarrow B \), with \( A \subseteq \text{Attr}(O) \) and \( B \subseteq \text{Attr}(O) \).
   - Perform computations.
   - Consider determination of objects or facts.

**Definition 4.3:** An extensive computational objects network is a model \( (O, M, F, T) \) that has the components below.
1. \( O = \{O_1, O_2, \ldots, O_n\} \) is the set of extensive computational objects.
2. \( M \) is a set of object attributes. We will use the following notations:
   - \( M_v(O) \) is the set of simple valued attributes of the object \( O \),
   - \( M_f(O) \) is the set of functional attributes of \( O \),
   - \( M(O) = M(O_1) \cup M(O_2) \cup \ldots \cup M(O_n) \), and
   - \( M \subseteq M(O) \).
3. \( F = F(O) \) is the set of the computational relations on attributes in \( M \) and on objects in \( O \).
4. \( T = \{t_1, t_2, \ldots, t_k\} \) is a set of operators on objects.

On the structure \( (O, T) \), there are expressions of objects. Each expression of objects always has its attributes as if it is an object.

The extensions of the computational networks are more powerful in designing knowledge bases in reality.

V. CONCLUSION AND FUTURE WORKS

Computational Networks with simple valued variables and networks of computational objects are useful models. It can be used to represent knowledge in many domains of knowledge. The methods and techniques for solving the problems on the networks will be useful tool for designing intelligent systems, especially systems that can solve problems based on a knowledge base.

There are domains of knowledge with functional attributes such as knowledge of alternating current in physics. This motivates another extensions of the above computational networks presented in the previous sections. The new computational networks with its simple valued variables and functional variables will be considered. Also, the computational objects in future works will have functional attributes. On the network of computational objects, operators will be considered. Such future works on computational networks make them more powerful for representing knowledge in practice.

REFERENCES


Nhon Van Do is currently a senior lecturer in the faculty of Computer Science at the University of Information Technology, Ho Chi Minh City, Vietnam. He got his MSc and Ph.D. in 1996 and 2002 respectively, from The University of Natural Sciences – National University of Ho Chi Minh City. His research interests include Artificial Intelligence, computer science, and their practical applications, especially intelligent systems and knowledge base systems.