Multi-objective Optimization with Fuzzy Based Ranking for TCSC Supplementary Controller to Improve Rotor Angle and Voltage Stability


Abstract—Many real-world optimization problems involve multiple conflicting objectives and the use of evolutionary algorithms to solve the problems has attracted much attention recently. This paper investigates the application of multi-objective optimization technique for the design of a Thyristor Controlled Series Compensator (TCSC)-based controller to enhance the performance of a power system. The design objective is to improve both rotor angle stability and system voltage profile. A Genetic Algorithm (GA) based solution technique is applied to generate a Pareto set of global optimal solutions to the given multi-objective optimisation problem. Further, a fuzzy-based membership value assignment method is employed to choose the best compromise solution from the obtained Pareto solution set. Simulation results are presented to show the effectiveness and robustness of the proposed approach.

Keywords—Multi-objective Optimisation, Thyristor Controlled Series Compensator, Power System Stability, Genetic Algorithm, Pareto Solution Set, Fuzzy Ranking.

I. INTRODUCTION

POWER system oscillations and system voltage profile are the two important criteria which define the performance of a power system subjected to a disturbance [1]. There has been much research interest in developing new control methodologies for increasing the performance of the power system. Recent development of power electronics introduces the use of Flexible AC Transmission Systems (FACTS) controllers in power systems. Thyristor Controlled Series Compensator (TCSC) is one of the important members of FACTS family that is increasingly applied with long transmission lines by the utilities in modern power systems [2-8]. The majority of the control methodologies presented in literature concerns improvement of only one type of stability performance; either improving the oscillatory stability performance (reflected in the deviation in generator speed) or the system voltage profile and minimization of a single objective function is employed to get the desired performance. Multi-objective genetic algorithm approach has been applied to design a TCSC controller [9], where the three objectives are closely related and the voltage deviations are not taken into account. Further, the procedure to obtain the best compromise solution from the obtained Pareto set is not addressed. Obviously, the main purpose of design of any controller is to enable it to improve both oscillatory stability and system voltage profile. Design of such kind of controller is inherently a multi-objective optimisation problem.

There are two general approaches to multiple objective optimisations. One approach to solve multi-objective optimisation problems is by combining the multiple objectives into a scalar cost function, ultimately making the problem single-objective prior to optimisation. However, in practice, it can be very difficult to precisely and accurately select these weights as small perturbations in the weights can lead to very different solutions. Further, if the final solution found cannot be accepted as a good compromise, new runs of the optimiser on modified objective function using different weights may be needed, until a suitable solution is found. These methods also have the disadvantage of requiring new runs of the optimiser every time the preferences or weights of the objectives in the multi-objective function change [10]. The second general approach is to determine an entire Pareto optimal solution set or a representative subset. Pareto optimal solution sets are often preferred to single solutions because they can be practical when considering real-life problems, since the final solution of the decision maker is always a trade-off between crucial parameters [11].

The main motivation for using Genetic Algorithm (GA) to solve multi-objective optimisation problems is because GAs deal simultaneously with a set of possible solutions (the so-called population) which allows the user to find several members of the Pareto optimal set in a single run of the algorithm, instead of having to perform a series of separate runs as in the case of the traditional mathematical programming techniques. The Pareto optimal solutions are ones within the search space whose corresponding objective vector components cannot be improved simultaneously. Additionally, GAs are less susceptible to the shape or continuity of the Pareto front as they can easily deal with
discontinuous and concave Pareto fronts, whereas these two issues are known problems with mathematical programming [12].

In this paper, the design problem of a TCSC is formulated as a multi-objective optimisation problem. GA based multi-objective optimisation method is adapted for generating Pareto solutions in designing a TCSC-based controller. The design objective is to improve the oscillatory stability and system voltage profile of a power system following a disturbance. Further a fuzzy based membership function value assignment method is employed to choose the best compromise solution from the obtained Pareto set. Simulation results are presented at various loading conditions to show the effectiveness and robustness of the proposed approach.

The reminder of the paper is organized in five major sections. Power system modeling with the proposed TCSC-based supplementary damping controller is presented in Section II. The proposed design approach and the objective function are presented in section III. In Section IV, an overview of multi-objective optimization has been presented. The results are presented and discussed in Section V. Finally, in Section VI conclusions are given.

II. MODELING THE POWER SYSTEM WITH TCSC

The single-machine infinite-bus (SMIB) power system installed with a TCSC, shown in Figure 1 is considered in this study. In the figure, $X_T$ and $X_L$ represent the reactance of the transformer and the transmission line respectively; $V_T$ and $V_B$ are the generator terminal and infinite bus voltage respectively.

TCSC is one of the most important and best known FACTS devices, which has been in use for many years to increase line power transfer as well as to enhance system stability. Basically, a TCSC consists of three main components: capacitor bank $C$, bypass inductor $L$ and bidirectional thyristors $SCR_1$ and $SCR_2$. The firing angles of the thyristors are controlled to adjust the TCSC reactance in accordance with a system control algorithm, normally in response to some system parameter variations. According to the variation of the thyristor firing angle or conduction angle, this process can be modeled as a fast switch between corresponding reactance offered to the power system.

A. Non-Linear Equations

The non-linear differential equations of the SMIB system with TCSC are derived by neglecting the resistances of all components of the system (generator, transformer and transmission lines) and the transients of the transmission lines and transformer. The non-linear differential equations are [13]:

\[
\delta = \omega_o \Delta \omega \\
\omega_o = \frac{1}{M} [P_m - P_e] \\
E_q = \frac{1}{T_{do}} [-E_q + E_{fd}] \\
E_{fd} = \frac{K_A}{1 + sT_A} [V_R - V_T]
\]

where,

\[
P_e = \frac{E_q V_B}{X_{\Sigma \omega}} \sin \delta - \frac{V_B^2 (X_q - X_{\omega})}{2X_{\Sigma \omega}} \sin 2\delta
\]

\[
E_q = \frac{X_{\Sigma \omega} E_q}{X_{\Sigma \omega}} - \frac{(X_q - X_{\omega})}{X_{\Sigma \omega}} V_B \cos \delta
\]

\[
V_{td} = \frac{X_q V_B}{X_{\omega}} \sin \delta
\]

\[
V_{td} = \frac{X_{\Sigma \omega} E_q}{X_{\Sigma \omega}} + \frac{V_B X_{\omega}}{X_{\Sigma \omega}} \cos \delta
\]

\[
V_T = \sqrt{\left( V_{td}^2 + V_{td}^2 \right)}
\]

\[
X_{\Sigma \omega} = X_{\omega} + X_{TS} - X_{\Sigma \omega} (\alpha)
\]

\[
X_{\Sigma \omega} = X_{\omega} + X_{TS} \quad X_{\Sigma \omega} = X_{\omega} + X_{TS}
\]

\[
X_{\Sigma \omega} = X_{\omega} + X_{TS}
\]

\[
V_{R} + \sum V_{T} = \frac{K_A}{1 + sT_A} [E_{fd} - E_{min}]
\]

Fig. 1 Single machine infinite bus power system with TCSC

Fig. 2 Simplified IEEE type ST 1A excitation system
The simplified IEEE Type-ST1A excitation system is considered in this work. The diagram of the IEEE Type-ST1A excitation system is shown in Fig. 2. The inputs to the excitation system are the terminal voltage \( V_T \) and reference voltage \( V_R \). The gain and time constants of the excitation system are represented by \( K_p \) and \( T_1 \) respectively.

### B. Linearized Equations

In the design of electromechanical mode damping stabilizer, a linearized incremental model around an operating point is usually employed. The Phillips-Heffron model of the power system with FACTS devices is obtained by linearizing the set of equations (1) – (5) around an operating condition of the system. The linearized expressions are as follows:

\[
\Delta \delta = \omega_c \Delta \omega \tag{6}
\]

\[
\Delta \omega = \left[ -K_1 \Delta \delta - K_2 \Delta E_q' - K_p \Delta \sigma - D \Delta \omega \right] / M \tag{7}
\]

\[
\Delta E_q' = \left[ -K_1 \Delta E_q' - K_5 \Delta \delta - K_6 \Delta \sigma + \Delta E_{fd}' \right] / T_{d0} \tag{8}
\]

\[
\Delta E_{fd}' = \left[ -K_5 (K_4 \Delta \delta + K_6 \Delta \sigma) - \Delta E_{fd}' \right] / T_{dA} \tag{9}
\]

where,

\[
K_1 = \frac{\partial P}{\partial \delta}, \quad K_2 = \frac{\partial P}{\partial E_q'}, \quad K_p = \frac{\partial P}{\partial \sigma}
\]

\[
K_3 = \frac{\partial E_q'}{\partial \delta}, \quad K_4 = \frac{\partial E_q'}{\partial \sigma}, \quad K_5 = \frac{\partial V_f}{\partial \delta}, \quad K_6 = \frac{\partial V_f}{\partial \sigma}
\]

The modified Phillips-Heffron model of the single-machine infinite-bus (SMIB) power system with TCSC-based damping controller is obtained using linearized equations (6)-(9) as shown in Fig. 3.

### III. The Proposed Approach

#### A. Structure of Proposed TCSC-based Supplementary Damping Controller

The commonly used lead–lag structure is chosen in this study as TCSC-based supplementary damping controller as shown in Fig. 4. The structure consists of a gain block; a signal washout block and two-stage phase compensation block. The phase compensation block provides the appropriate phase-lead characteristics to compensate for the phase lag between input and the output signals. The signal washout block serves as a high-pass filter which allows signals associated with oscillations in input signal to pass unchanged. Without it steady changes in input would modify the output.

The damping torque contributed by the TCSC can be considered to be in two parts. The first part \( K_p \), which is referred as the direct damping torque, is directly applied to the electromechanical oscillation loop of the generator. The second part \( K_v \), named as the indirect damping torque, applies through the field channel of the generator. The damping torque contributed by TCSC controller to the electromechanical oscillation loop of the generator is:

\[
\Delta T_D = T_D \omega_0 \Delta \omega \approx K_p K_T K_D \Delta \omega \tag{10}
\]

The transfer functions of the TCSC controller is:

\[
u_{TCSC} = K_T \left( \frac{sT_{sT} + 1}{sT_{sT}} \right) \left( 1 + sT_1 \right) \left( 1 + sT_2 \right) \left( 1 + sT_3 \right) \left( 1 + sT_4 \right) y \tag{11}
\]

Where, \( u_{TCSC} \) is the output signal of TCSC controller and \( y \) is the input signal. The input signal of the proposed TCSC-based controller is the speed deviation \( \Delta \omega \) and the output is the change in conduction angle \( \Delta \sigma \). During steady state conditions \( \Delta \sigma = 0 \) and so the effective reactance \( X_{Eff} \) is given by: \( X_{Eff} = X_T + X_{TL} - X_{TCSC} (\alpha_0) \). During dynamic conditions the series compensation is modulated for damping system oscillations. The effective reactance in dynamic conditions is given by: \( X_{Eff} = X_T + X_{TL} - X_{TCSC} (\alpha) \), where \( \sigma = \sigma_0 + \Delta \sigma \) and \( \sigma = 2(\pi - \alpha) \), \( \alpha_0 \) and \( \sigma_0 \) being initial value of firing and conduction angle respectively.

From the viewpoint of the washout function the value of washout time constant is not critical in lead-lag structured
controllers and may be in the range 1 to 20 seconds [1]. In the present study, washout time constant of $T_{W}$ is 10 s is used. The controller gains $K_T$ and the time constants $T_1$, $T_2$, $T_3$ and $T_4$ are to be determined.

B. Objective Function

It is worth mentioning that the TCSC controller is designed to damp power system oscillations and improve the system voltage profile after a disturbance. A multi-objective function based on $\Delta \omega$ and $\Delta V_T$ is used as an objective function in the present study. The objective can be formulated as the minimisation of function $F$ given by:

$$ F = (F_1, F_2) $$

Where,

$$ F_1 = \int_0^T |\Delta \omega| \cdot t \cdot dt $$

and

$$ F_2 = \int_0^T |\Delta V_T| \cdot t \cdot dt $$

In the above equations, $|\Delta \omega|$ and $|\Delta V_T|$ denote the absolute values of rotor speed and terminal voltage deviations following a disturbance and $t$ is the time range of the simulation. For the objective function calculation, the time-domain simulation of the power system model is carried out for the simulation period.

C. Optimization Problem

In this study, it is aimed to minimize the proposed objective functions $F$. The problem constraints are the TCSC Controller parameter bounds. Therefore, the design problem can be formulated as the following optimization problem:

Minimize $F$

Subject to

$$ K_T^\text{min} \leq K_T \leq K_T^\text{max} $$

$$ T_1^\text{min} \leq T_1 \leq T_1^\text{max} $$

$$ T_2^\text{min} \leq T_2 \leq T_2^\text{max} $$

$$ T_3^\text{min} \leq T_3 \leq T_3^\text{max} $$

$$ T_4^\text{min} \leq T_4 \leq T_4^\text{max} $$

The proposed approach employs genetic algorithm to solve this optimization problem and search for optimal set of the TCSC Controller parameters.

IV. MULTI-OBJECTIVE OPTIMIZATION

A. Multi-objective optimization

A multi-objective optimization problem (MOP) differs from a single-objective optimization problem because it contains several objectives that require optimization. In case of single objective optimization problems, the best single design solution is the goal. But for multi-objective problems, with several and possibly conflicting objectives, there is usually no single optimal solution. Therefore, the decision maker is required to select a solution from a finite set by making compromises. A suitable solution should provide for acceptable performance over all objectives.

A general formulation of a MOP consists of a number of objectives with a number of inequality and equality constraints. Mathematically, the problem can be written as [14]:

$$ \min \text{max} f(x) \quad \text{for} \quad i = 1, 2, ..., n. $$

Subject to constraints:

$$ g_j(x) \leq 0 \quad j = 1, 2, ..., J $$

$$ h_j(x) \leq 0 \quad k = 1, 2, ..., K $$

Where

$$ f(x) = \{ f_1(x), ..., f_n(x) \} $$

$$ n $$

is number of objectives or criteria to be optimized

$$ x = \{ x_1, ..., x_p \} $$ is a vector of decision variables

$$ p $$

is number of decision variables

There are two approaches to solve the MOP. One approach is the classical weighted-sum approach where the objective function is formulated as a weighted sum of the objectives. But the problem lies in the correct selection of the weights or utility functions to characterise the decision-makers preferences. In order to solve this problem, the second approach called Pareto-optimal solution can be adapted. The MOPs usually have no unique or perfect solution, but a set of non-dominated, alternative solutions, known as the Pareto-optimal set. Assuming a min minimisation problem, dominance is defined as follows:

$$ \forall \ \text{if and only if} \ \text{if} \quad u \text{ is partially less than } v, \quad u < v, $$

$$ \forall \ i \in \{1, ..., n\}, u_i \leq v_i \land \exists \ i \in \{1, ..., n\}; \ u_i < v_i $$

A solution $x_a \in U$ is said to be Pareto-optimal if and only if there is no $x_a \in U$ for which $v = f(x_a) = (v_1, ..., v_n)$ dominates $u = f(x_a) = (u_1, ..., u_n)$.

B. Pareto-optimal solutions

Pareto-optimal solutions are also called efficient, non-dominated, and noninferior solutions. The corresponding objective vectors are simply called non-dominated. The set of all non-dominated vectors is known as the non-dominated set, or the trade-off surface, of the problem. A Pareto optimal set is a set of solutions that are non-dominated with respect to each other. While moving from one Pareto solution to another, there is always a certain amount of sacrifice in one objective to achieve a certain amount of gain in the other. The elements in the Pareto set has the property that it is impossible to further reduce any of the objective functions, without increasing, at least, one of the other objective functions.
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C. GA method for generating Pareto solutions

The ability to handle complex problems, involving features such as discontinuities, multimodality, disjoint feasible spaces and noisy function evaluations reinforces the potential effectiveness of GA in optimisation problems. Although, the conventional GA is also suited for some kinds of multi-objective optimisation problems, it is still difficult to solve those multi-objective optimisation problems in which the individual objective functions are in the conflict condition.

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A. Application of Genetic Algorithm

The objective function given by equation (12) is evaluated by simulating the system dynamic model considering a 10% step increase in mechanical power input ($\Delta P_m$) at $t = 1.0$ sec. Optimization is terminated by the prespecified number of generations. While applying GA, a number of parameters are required to be specified. An appropriate choice of the parameters affects the speed of convergence of the algorithm. Table I shows the specified parameters for the GA algorithm. One more important factor that affects the optimal solution more or less is the range for unknowns.

For the very first execution of the program, a wider solution space can be given and after getting the solution one can shorten the solution space nearer to the values obtained in the previous iteration. The final Pareto solution surface is shown in Fig. 6 where the Pareto solutions are shown with the marker ‘o’.

B. Best Compromise Solution

In the present paper, a Fuzzy-based approach is applied to select the best compromise solution from the obtained Pareto set. The $j$-th objective function of a solution in a Pareto set $f_j$
It is clear from Table III that the open loop system is unstable at all the loading conditions because of negative damping of electromechanical mode ($s = 0.2655, 0.0278, 0.4864$ for nominal, light and heavy loading respectively). Without optimized TCSC controller parameters the system stability is maintained as the electromechanical mode eigenvalue shift to the left of the line in s-plane ($s = -0.0081, -1.1828, -0.0088$ for nominal, light and heavy loading respectively) for all loading conditions. It is also clear that MOGA optimized TCSC controller shifts substantially the electromechanical mode eigenvalue to the left of the line ($s = -3.6835, -3.8535, -2.3982$ for nominal, light and heavy loading respectively) in the s-plane, which enhances the system stability and improves the damping characteristics of electromechanical mode.

![Fig. 6 Pareto solution surface](image)

is represented by a membership function $\mu_j$ defined as [16]:

$$\mu_j = \begin{cases} 1, & f_j \leq f_{j_{\text{min}}} \\ \frac{f_{j_{\text{max}}} - f_j}{f_{j_{\text{max}}} - f_{j_{\text{min}}}}, & f_{j_{\text{min}}} < f_j < f_{j_{\text{max}}} \\ 0, & f_j \geq f_{j_{\text{max}}} \end{cases}$$ (22)

where $f_{j_{\text{max}}}$ and $f_{j_{\text{min}}}$ are the maximum and minimum values of the $j$-th objective function, respectively.

For each solution $i$, the membership function $\mu'_{i}$ is calculated as:

$$\mu'_{i} = \frac{\sum_{j=1}^{n} \mu_j}{\sum_{i=1}^{m} \sum_{j=1}^{n} \mu_j}$$ (23)

where, $n$ is the number of objectives functions and $m$ is the number of solutions. The solution having the maximum value of $\mu'_{i}$ is the best compromise solution.

Using the above approach the best compromise solution is obtained as:

$$K_T = 55.8577, \quad T_1 = 0.1684 \text{ s}, \quad T_2 = 0.0637 \text{ s}, \quad T_3 = 0.3126 \text{ s} \quad \text{and} \quad T_4 = 0.3126 \text{ s}$$

**C. Eigenvalue Analysis**

To assess the effectiveness and robustness of the proposed stabilizers, three different loading conditions given in Table II are considered. The system electromechanical mode eigenvalues without and with the proposed controllers are shown in Table III. Table III also shows the system electromechanical eigenvalues without optimized TCSC controller parameters. In this case the values are randomly chosen as:

$$K_T = 40, \quad T_1 = 0.2 \text{ s}, \quad T_2 = 0.05 \text{ s}, \quad T_3 = 0.25 \text{ s} \quad \text{and} \quad T_4 = 0.05 \text{ s}.$$
In this study, the performance improvement of a power system by optimal design of a TCSC-based controller is presented and discussed. The design objective is to improve both rotor angle stability and system voltage profile. A Genetic Algorithm (GA) based solution technique is applied to generate a Pareto set of global optimal solutions to the given multi-objective optimisation problem. Further, a fuzzy-based membership value assignment method is employed to choose the best compromise solution from the obtained Pareto solution set. Eigenvalue analysis and simulation results are presented under various loading conditions to show the effectiveness and robustness of the proposed approach. The proposed method is valuable for the design of the interactive decision making. The decision makers can choose from the solutions in the Pareto-optimal set to find out the best solution according to the requirement and needs as the desired parameters of their controllers. The results show that evolutionary algorithms are effective tools for handling multi-objective optimization where multiple Pareto-optimal solutions can be found in one simulation run.
REFERENCES


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