Application of Data Envelopment Analysis to Assess Quality Management Efficiency

Chuen Tse Kuah, Kuan Yew Wong, and Farzad Behrouzi

Abstract—This paper is aimed to give an illustration on the application of Data Envelopment Analysis (DEA) as a tool to assess Quality Management (QM) efficiency. A variant of DEA, slack based measure (SBM) is used for this purpose. From this study, it is found that DEA is suitable to measure QM efficiency and give improvement suggestions to the inefficient QM.

Keywords—Quality Management, Data Envelopment Analysis, Slack Based Measure, Efficiency Measurement.

I. INTRODUCTION

Quality Management (QM) has been defined as a philosophy or an approach to management, made up of a set of mutually reinforcing principles, each of which is supported by a set of practices and techniques [1]. QM started to draw researchers’ and practitioners’ attention since around three decades ago. Since then, many firms have been practicing QM to improve their business performance and remain competitive in their industries.

Traditionally, managers used financial measures, such as itemized quality cost reporting and analysis of quality cost components, to evaluate performance of organizations, but these financial measures were proved to be insufficient to capture some limitations and therefore non-financial measures, such as percentage of product reworks and total number of customer complaints were suggested as some performance measures. Uyar [2] has demonstrated that both financial and non-financial measures should be utilized in a balanced way to evaluate quality performance of organizations. However, integrating these multiple performance measures has made the measurement of QM performance becomes more challenging than before. Thus, there is a need for a tool that can evaluate quality performance of organizations based on both financial and non-financial measures.

The main objective of this paper is to develop a model to assess QM efficiency in firms by using Data Envelopment Analysis (DEA). DEA is a simple yet powerful tool used to assess the relative efficiencies of multiple-input multiple-output decision making units (DMUs) without prior weights on the inputs and outputs [3].

In this study, QM of firms will be treated as DMUs, which convert inputs (e.g. quality improvement expenditures and percent of employees involved in quality improvement), into outputs (e.g. quality and financial performances). The model developed is expected to be able to provide more and useful QM improvement suggestions for firms according to industries and size of the companies.

This paper will be structured as follows: in Section II, we shall identify the existing performance measurement methods in QM through a review of the literature. Next, a brief review on DEA follows in Section III. Section IV will be explaining the DEA model developed to assess the QM efficiencies of firms. Later, in Section V, an illustration example on the application of the DEA model will be provided; followed by a discussion on the implications of the results. Lastly, the paper ends with some recommendations for future work.

II. MEASURES TO ASSESS QM

QM helps firms to improve their business performance in many ways. For examples, QM allows firms to obtain a high degree of differentiation, satisfy customers’ demands, improve brand image, reduce costs, improve processes, and etc [4]. Both researchers and practitioners have studied QM, and identified critical factors for its successful implementation. The common critical factors for effective QM as observed in the literature are such as:

- customer focus
- management leadership and commitment,
- training,
- employee empowerment,
- human resource management,
- process management,
- quality planning,
- continuous improvement, and
- supplier management.

In practice, formal evaluation models are used as a guide for evaluation and implementation of QM. These formal evaluation models include: Malcolm Baldrige National Quality Award, European Quality Award, Deming Application Prize, and Kanji’s Business Excellence Model. On the other hand, QM has also been evaluated through empirical research [4]-[11]. These formal models and studies have yielded a valid, reliable measurement tool to evaluate a firm’s QM and help managers to make decisions related to QM.
Assessment of QM can either be done objectively, for instance, by examining unexpected changes in published financial results [11]; or in a subjective way, for example, by measuring respondents’ perception [4]-[10]. Most assessments on QM are the latter category due to the difficulty to obtain objective measures for QM. This difficulty is basically due to the unknown and complex relationships between the factors and also with the performance measures [12].

There is another important area of research in QM, which studies how and to what extent QM practices affect firm performance. Garvin [13] suggested two ways for which quality can improve the profitability of firms. As shown in Fig. 1, firstly, improved reliability or conformance in the manufacturing processes improves internal process quality, and drives down both manufacturing costs and service costs. Secondly, as shown in Fig. 2, improvements in the product quality cause better reputation, increased sales (market share), and higher prices of products; both of these lead to increased profit and operational performance [13].

The relationships between QM and firm performances, such as quality performance, financial performance, and operational performance, are generally recognized and have been verified through a number of studies [8], [12], [14]-[18]. These studies have found that QM practices are related to the firm performance.

Through the previous studies, the relationship between QM and firm performance has been confirmed. In addition, QM has been assessed using both formal evaluation models and empirical models. However, several limitations of previous studies should be noted:

- most of the studies were done subjectively, which did not evaluate QM based on the actual performances of the firms,
- the relationships among the factors and relationships between the factors and the firm performances were disregarded,
- the companies were evaluated without considering their sizes (number of employees); while size of a company might be affecting their QM practices and results,
- there is a lack of benchmarking tools to compare the QM of companies in the same industry,

The efficiency of QM is hard to be assessed because QM itself is a multiple-factor system and the relationships between the factors are complex and difficult, if not impossible, to determine. A benchmarking efficiency measure for QM is important for a firm to estimate to what extent its QM systems can be improved, by comparing itself to the other firms, which have similar size in the particular industry.

Through reviewing the literature of operations management, DEA has been identified as a suitable tool to evaluate the effectiveness of QM, by treating a firm’s QM as a DMU which consumes inputs (resources) and produces outputs (performances). The next section will give a brief review on DEA.

![Fig. 1 Increased profits through cost savings [12]](image1)

![Fig. 2 Increased profits through market gains [12]](image2)

### III. REVIEW ON DEA

Since DEA was first introduced by Charnes, Cooper, and Rhodes in 1978 [3], the simple yet powerful method has been vastly developed and used to assess the relative efficiencies of multiple-input multiple-output DMUs. The popularity of DEA is due to its ability to measure relative efficiencies of multiple-input and multiple-output DMUs without prior weights on the inputs and outputs. To date, DEA is still widely researched and is being applied as internal/external benchmarking tools in many areas and domains. Such as in banking industry [19], information technology and information system [20], education [21], airline [22], airline industry [23], power plant [24], sport [25], stock market [26], government [27], supply chain [28], and many more. Readers should be noted that the coverage of this paper is not meant to be complete as the volume of literature is immense. Readers who are interested in a thorough discussion on the various topics of DEA are advised to refer to a thorough literature review on...
DEA by Cook and Seiford [29] and Kuah, Wong, and Behrouzi [30]. From the past literature, DEA models have been widely applied in various industries and areas; however, thus far, no study investigating the application of DEA in QM efficiency measurement has been reported. Therefore, it is sensible to extend the traditional DEA models into QM. This study aims to develop a model to assess QM efficiency of firms by using DEA.

IV. DEA Model for QM

This section will firstly present the conceptual model which views QM as a DMU. Then, the DEA model developed to measure QM efficiency will be presented.

A. Conceptual Model for QM

In this study, QM is viewed as a DMU which converts multiple inputs and produces multiple outputs. Fig. 3 shows the conceptual model of a QM system.

![Fig. 3 QM as a system](image)

The inputs and costs for the system include costs incurred for QM activities (quality costs) and percent of employees of the company who are involved in quality improvement activities (quality worker).

The performance measures considered in this study include key areas such as quality, operational, and financial measures. The performance measures can be further broken down into more specific variables, which are summarized in Table 1.

Some notes regarding the measurements are - firstly, quality products are measured by the total number of units produced minus defective, reworked, and scrapped units; and secondly, customer satisfaction rate is measured by number of sales minus number of service calls received.

B. Conceptual Model for QM Efficiency Assessment

The methodology used in this study is DEA. It is suitable to be used in measuring QM efficiency because it can handle multiple inputs and outputs and it does not require prior unrealistic assumptions on the variables. Fig. 4 shows the conceptual model used in this study. Specifically, a DEA model will be constructed to measure QM efficiency. It will use the input and output data from the firms under evaluation to calculate the QM efficiency of each firm.

![Fig. 4 Conceptual model to measure QM efficiency](image)

In brief, DEA is a mathematical programming technique that calculates the relative efficiencies of various DMUs. A DMU’s efficiency is calculated by comparing with all the DMUs under evaluation. Efficiency in DEA is generally defined as the weighted sum of outputs divided by the weighted sum of inputs. The set of weights for a DMU is computed in DEA with the objective to give the highest possible relative efficiency score for the DMU, while keeping the efficiency scores of other DMUs in the range of 0 to 1 under the same set of weights. Efficient DMUs have the score of 1; the other DMUs which score less than 1 are considered as inefficient.

There are many variations of DEA models, and the model chosen for this study is a DEA variant called slack based measure (SBM). Tone [31] introduced SBM, which is able to deal directly with the input excesses and the output shortfalls of the DMU under evaluation. SBM is invariant to the units of measurement and is monotone decreasing with respect to each input and output slack.

As mentioned earlier in this paper, the evaluation of QM should take the company size into consideration. Hence, in this study, companies will be grouped based on their sizes. When the DMU under evaluation is in a group, we want to ensure that it is evaluated only relative to those DMUs in the same group; therefore the other DMUs which belong to other categories should be excluded from the evaluation. To fulfill this requirement, a constraint has been added to the SBM to exclude DMUs from the evaluation which are not in the same group; therefore the other DMUs which belong to other categories should be excluded from the evaluation. To fulfill this requirement, a constraint has been added to the SBM to exclude DMUs from the evaluation which are not in the same group with the DMU under evaluation.

Consider there are \( n \) DMUs: \( \text{DMU}_1, \text{DMU}_2, \ldots, \text{DMU}_n \). Each \( \text{DMU}_j \) \((j = 1, 2, \ldots, n)\) uses \( m \) inputs \( x_{ij} \) \((i = 1, \ldots, m)\) and generates \( s \) outputs \( y_{rj} \) \((r = 1, \ldots, s)\). Let the input slacks (surpluses) be \( s_i^+ \) \((i = 1, \ldots, m)\) and the output slacks (shortfalls) be \( s_r^- \) \((r = 1, \ldots, s)\). Let the DMU \( j \) to be evaluated on any trial be designated as \( \text{DMU}_0 \) \((0 = 1, 2, \ldots, n)\). The efficiency of each \( \text{DMU}_0 \), \( p_0 \), is thus found by solving the modified SBM model shown in (1).

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**Table I**

<table>
<thead>
<tr>
<th>Performance Measures</th>
<th>Output Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality Performance</td>
<td>Quality products (%)</td>
</tr>
<tr>
<td>Operational Measures</td>
<td>Customer satisfaction rate (%)</td>
</tr>
<tr>
<td>Measures</td>
<td>On-time delivery rate (%)</td>
</tr>
<tr>
<td>Financial Performance</td>
<td>Revenue (million USD)</td>
</tr>
</tbody>
</table>
\[
\min p_0 = \frac{1 - \frac{1}{m} \sum_{i=0}^{m-1} s_i^-}{1 + \frac{1}{s} \sum_{i=0}^{s-1} s_i^+}
\]

\[
\text{S.t. } \sum_{j} \lambda_j x_{ij} + s_i^- = x_{io}
\]

\[
\sum_{j} \lambda_j y_{rij} - s_r^+ = y_{rio}
\]

\[
\sum_{j} \lambda_j = 1
\]

\[
\lambda_j, s_i^-, s_r^+ \geq 0
\]  

(1)

where \( \lambda \) is a nonnegative vector and \( f \) is the group of DMUs under consideration. The constraint \( f \in F \) is excluding those DMUs from the evaluation which are not in the same group with DMU\(_0\).

The optimum solution of this SBM model is 1, which can only be achieved when all the slacks equal to zero. A DMU is considered as efficient if and only if it gets an efficient score of 1; else it is considered as inefficient. Through an operation called SBM-projection, an inefficient DMU \((x_{io}, y_{rio})\) can be improved to become efficient. This is done by removing the input excess and augmenting the output shortfall as shown in (2).

\[
x_{io} \leftarrow x_{io} - s^- \\
y_{rio} \leftarrow y_{rio} + s^+
\]

These figures are the inputs and outputs efficient targets. This information is useful for managers to tune their inputs and outputs according to the efficient target level.

For more mathematical explanation on the SBM model, readers can refer to [31].

V. AN ILLUSTRATIVE EXAMPLE

In this section, we illustrate the application of the SBM model using an illustrative problem consisting of thirty DMUs, DMU\(_j\) (\( j = 1, \ldots, 30 \)), with two inputs \( x_i \) (\( i = 1, 2 \)) and four outputs \( y_r \) (\( r = 1, \ldots, 4 \)). Two additional variables have been included in the data, which are the number of employees and company size group. They are used to categorize the DMUs into groups based on the number of employees. For illustration purposes, only three groups are formed in this example - small (employees \( \leq 100 \)), medium (100 < employees \( \leq 500 \)), and large (employees > 500). The grouping can be refined if/when necessary. All the model variables are summarized in Table 2. The data set, which is generated by authors as a hypothetical example, is exhibited in Table 3.

As shown in Table 3, 10 DMUs belong to small company size group, 11 DMUs belong to medium company size group, and 9 DMUs belong to large company size group. The model was run to obtain the optimum efficiency scores for each DMU. DMUs with an efficiency score of 1 are considered as efficient; else are considered as inefficient. For those inefficient DMUs, SBM-projection was done using (2) to obtain the efficient input and output targets. The optimum efficiency scores and the input and output efficient targets are exhibited in Table 4 according to their groups.

These efficient targets are useful for managers in the sense of providing them with a guide on which areas and to what extent should the improvement be done. For instances, in Group: Small, the inefficient DMUs are DMUs 8, 25, 26, and 30, which have efficiency scores less than one. These DMUs have to adjust their inputs and outputs accordingly in order to be efficient. Take DMU 8 as an example, to tune its QM to be efficient, take DMU 8 as an example, to tune its QM to be efficient, take DMU 8 as an example, to tune its QM to be efficient.
efficient, it has to reduce its quality costs from 30 to 23 million USD and quality workers from 50 to 40 percent; increase its customer satisfaction rate from 92 to 93 percent and on-time delivery rate from 81 to 90 percent; while maintaining the percentage of quality product and the revenue.

VI. CONCLUSION

This study provides an insight into the use of DEA as a benchmarking tool to aid managerial decision making in assessing QM efficiency. DEA has been proven to be a suitable benchmarking tool in measuring relative efficiencies of QM. The information obtained from the developed model could help quality managers to identify the inefficient areas in their QM systems and to tune the QM variables to make their systems efficient. Future work of this study could include more variables into the model. For instance, more operational performance measures, such as lead time and inventory reduction, and undesirable outputs, such as warranty claim, could be included. In addition, instead of summing up all the quality costs, they can be further broken down into finer items to have a more detailed analysis of QM efficiency.

### TABLE IV

<table>
<thead>
<tr>
<th>DMU</th>
<th>Efficiency Scores</th>
<th>Input Efficient Targets</th>
<th>Output Efficient Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p*</td>
<td>s₁</td>
<td>s₂</td>
</tr>
<tr>
<td>Group: Small</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1</td>
<td>1</td>
<td>18</td>
<td>39</td>
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<tr>
<td>4</td>
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<td>19</td>
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</tr>
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<td>1</td>
<td>17</td>
<td>38</td>
</tr>
<tr>
<td>30</td>
<td>0.85</td>
<td>17</td>
<td>38</td>
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<tr>
<td>Group: Medium</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
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<td>6</td>
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<tr>
<td>29</td>
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<td>Group: Large</td>
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