New delay-dependent stability conditions for neutral systems with nonlinear perturbations

Lianglin Xiong, Xiuyong Ding, and Shouming Zhong

Abstract—In this paper, the problem of asymptotical stability of neutral systems with nonlinear perturbations is investigated. Based on a class of novel augment Lyapunov functionals which contain free-weighting matrices, some new delay-dependent asymptotical stability criteria are formulated in terms of linear matrix inequalities (LMIs) by using new inequality analysis technique. Numerical examples are given to demonstrate the derived condition are much less conservative than those given in the literature.

Keywords—Asymptotical stability, neutral system, nonlinear perturbation, Delay-dependent, linear matrix inequality(LMI)

I. INTRODUCTION

It is well known that neutral systems are frequently encountered in various engineering systems, including population ecology, distributed networks containing lossless transmission lines, heat exchangers, and repetitive control [13], [9], [18]. There are many reports about stability conditions for neutral systems in the literature, such as [16], [7], [10], [11], [12], [17], [8], and the references therein. Currently efforts on the problem for stability of neutral systems can be divided into two categories, namely delay-dependent stability criteria and delay-independent stability criteria. Generally speaking, the delay-dependent stability conditions are less conservative than the delay-independent stability conditions for the neutral systems with small time delay.

In recent decades, the problem of robust stability of time-delay systems with nonlinear perturbations has also received considerable attention. To deal with the stability of systems with time varying delays and nonlinear perturbations, Cao and Lam proposed a model transformation technique [1]. By using a descriptor transformation method combined with a matrix decomposition approach, [4] presented the stability conditions for uncertain systems including time-varying delays, and both nonlinear perturbations and norm-bounded uncertainties are considered. The results in [4] were less conservative than those of [1],[4]. In order to reduce the conservatism, some free-weighting matrices (slack matrices) were introduced together with a descriptor transformation method [23]. Using the Lyapunov functional technique combined with matrix inequality technique, [14] presented a novel asymptotic stability criterion for neutral systems with nonlinear perturbations. [6] also studied the neutral systems with nonlinear parameter perturbations with a model transformation technique, by constructing Lyapunov-functionals. To reduce the conservatism, a new integral inequality which is particularly suitable for the analysis of the stability of the neutral systems was introduced in [21]. However, both the results of time-delay bounds in [6] and [21] are so small that can be improved with another novel approach, and some novel integrate inequalities which were introduced in [22] might also be considered into the stability of neutral systems with nonlinear perturbation, all of which motivates this paper.

In this paper, the delay-dependent asymptotic stability for uncertain neutral systems with nonlinear perturbations is studied. Owing to a class of novel augmented Lyapunov-Krasovskii functionals, improved delay-dependent asymptotical stability criteria for the neutral systems are derived by using the inequalities analysis technique and introducing some free weighting matrices. Note that these advantages are not obtained at the cost of high computational complexity. Finally, numerical examples are given to illustrate the superiority of present result to those in the literature.

II. PROBLEM STATEMENT

Nomenclature

\( R^n \) n-dimensional real space
\( R_0 \) set of all real n by n matrices
\( x^T \) or \( A^T \) transpose of vector x (or matrix A)
\( P \geq 0 \) (respectively, \( P < 0 \)) matrix \( P \) is symmetric positive (respectively, negative) definite
\( P \geq 0 \) (respectively, \( P \leq 0 \)) matrix \( P \) is symmetric positive (respectively, negative) semi-definite
* the elements below the main diagonal of a symmetric block matrix.

Consider the following uncertain nonlinear with mixed time-varying delay system:

\[
\begin{align*}
\dot{x}(t) - C \dot{x}(t - \tau_2) &= A x(t) + B x(t - \tau_1(t)) + f_1(t, x(t)) + f_2(t, x(t - \tau_1(t))) + f_3(t, \dot{x}(t - \tau_2)) \\
\dot{x}(t_0 + \theta) &= \varphi(\theta), \forall \theta \in [-\rho, 0]
\end{align*}
\]

where \( x(t) \in R^n \) is the state vector , the time-varying delays \( h(t) \) and \( \tau(t) \) satisfy

\[
0 \leq \tau_1(t) \leq \tau_1 < \infty, \tau_1 \leq \tau_4, \rho = \max \{\tau_1, \tau_2\}
\]

\( \varphi(\theta) \) is the initial condition function, \( A \in R^{n \times n}, B \in R^{n \times n}, C \in R^{n \times n} \) are uncertain matrices, and the function...
where \( \beta_1 \geq 0, \beta_2 \geq 0 \) and \( \beta_3 \geq 0 \) are given constants.

Constraint (2) can be rewritten as following:

\[
\begin{align*}
\dot{f}_1^T(t, x(t)) &= f_1(t, x(t)) \leq \beta_1 \|x(t)\|^T \leq \beta_1 \|x(t - \tau_1(t))\| \\
\dot{f}_2^T(t, x(t - \tau_1(t))) &= f_2(t, x(t - \tau_1(t))) \leq \beta_2 \|x(t - \tau_1(t))\| \\
\dot{f}_3^T(t, \dot{x}(t - \tau_2)) &= f_3(t, \dot{x}(t - \tau_2)) \leq \beta_3 \|\dot{x}(t - \tau_2)\|
\end{align*}
\]

for the sake of simplicity, let \( f_1 := f_1(t, x(t)), f_2 := f_2(t, x(t - \tau_1(t))), f_3 := f_3(t, \dot{x}(t - \tau_2)) \).

Lemma 1: [22] For any constant symmetric matrix \( Q \in R^{n \times n}, Q = Q^T > 0 \), and any appropriate dimensioned matrices, \( M_1 \in R^{n}, M_2 \in R^{n}, Z = \left( \begin{array}{cc}
Z_{11} & Z_{12} \\
Z_{12}^T & Z_{22}
\end{array} \right) \in R^{2n \times 2n}, Y = \left[ \begin{array}{cc}
M_1 & M_2
\end{array} \right] \in R^{n \times 2n}, \text{if} \left( \begin{array}{cc}
Q & Y \\
* & Z
\end{array} \right) > 0, \text{such that the integrations in the following are well defined, then}
\]

\[
-\int_{t-\tau}^{t} \dot{x}^T(s) Q \dot{x}(s) ds \leq \xi(t)^T \left( \begin{array}{cc}
M_{11} & M_{12} \\
M_{12}^T & M_{22}
\end{array} \right) \xi(t)
\]

where,

\[
\begin{align*}
M_{11} &= M_1 + M_1^T + \tau Z_{11}, \\
M_{12} &= -M_1^T + M_2 + \tau Z_{12}, \\
M_{22} &= -M_2 - M_2^T + \tau Z_{22}, \\
\xi(t) &= \text{col} \left( x(t), x(t - \tau(t)) \right).
\end{align*}
\]

Lemma 2: [5] For any constant symmetric matrix \( M \in R^{n \times n}, M = M^T > 0 \), scalar \( r > 0 \), vector function \( g : [0, r] \rightarrow R^n \), such that the integrations in the following are well defined, then

\[
r \int_0^r g^T(s) Mg(s) ds \geq \left[ \int_0^r g(s) ds \right]^T M \left[ \int_0^r g(s) ds \right]
\]

III. MAIN RESULTS

In general, the following assumption is satisfied as considering the stability of neutral systems.

A1. All the eigenvalues of matrix \( C \) are inside the unit circle.

For the asymptotically stability of systems described by (1), we have the following result.

Theorem 1: Under A1, the systems (1) is asymptotically stable, if there exist matrices \( L = \left( \begin{array}{ccc}
L_{11} & L_{12} & L_{13} \\
L_{21}^T & L_{22} & L_{23} \\
L_{31}^T & L_{32}^T & L_{33}
\end{array} \right) \geq 0 \) with \( L_{11} > 0 \), \( R = \left( \begin{array}{ccc}
R_{11} & R_{12} & R_{13} \\
R_{21}^T & R_{22} & R_{23} \\
R_{31}^T & R_{32}^T & R_{33}
\end{array} \right) \geq 0 \), \( Q_1 > 0 \), \( Q_2 > 0 \), \( Q_3 > 0 \), \( N_{99} > 0 \) and any appropriate dimensioned matrices \( N_{ij} (i, j = 1, \ldots, 9), M_1 \in R^n, M_2 \in R^n, Y = \left[ \begin{array}{cc}
M_1 & M_2
\end{array} \right] \in R^{n \times 2n}, Z = \left( \begin{array}{cc}
Z_{11} & Z_{12} \\
Z_{12}^T & Z_{22}
\end{array} \right) \) and scalars \( \epsilon_1 > 0, \epsilon_2 > 0, \epsilon_3 > 0 \), such that the following LMIs holds:

\[
\begin{pmatrix}
Q_3 & Y \\
* & Z
\end{pmatrix} > 0
\]

\[
N = \left( \begin{array}{cccccc}
N_{11} & N_{12} & N_{13} & \cdots & N_{19} \\
* & N_{22} & N_{23} & \cdots & N_{29} \\
* & * & N_{33} & \cdots & N_{39} \\
* & * & * & \cdots & N_{49} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
* & * & * & \cdots & N_{99}
\end{array} \right) > 0
\]

where

\[
\begin{align*}
\phi_{11} &= (L_{11} + R_{11}) A + A^T (L_{11} + R_{12})^T + L_{13}^T + L_{13} + L_{13}^T + N_{19} + N_{19}^T + R_{11} + Q_1 + 7.2 Q_2 + \epsilon_1 \beta_1^2 I + \tau_1 N_{11}, \\
\phi_{12} &= A^T L_{12} + N_{29}^T - L_{23} + L_{24}^T + \tau_1 N_{12}, \\
\phi_{13} &= (L_{11} + R_{12}) C + L_{12} + N_{30}^T + \tau_1 N_{13}, \\
\phi_{14} &= (L_{11} + R_{12}) B + N_{29} - M_{11}^T + M_2 + \tau_1 Z_{12} + \tau_1 Z_{12} + \tau_1 N_{14}, \\
\phi_{15} &= L_{11} + R_{12} + N_{79} + \tau_1 N_{15}, \\
\phi_{16} &= L_{11} + R_{12} + N_{69} + \tau_1 N_{16}, \\
\phi_{17} &= L_{11} + R_{12} + N_{79} + \tau_1 N_{17}, \\
\phi_{18} &= T_2 A^T L_{13} + T_2 B^T C + N_{69} + \tau_1 N_{18}, \\
\phi_{19} &= A^T (\tau_1 N_{99} + R_{22} + \tau_1 Q_2), \\
\phi_{22} &= -L_{23} - L_{24}^T - R_{11} + \tau_1 N_{22}, \\
\phi_{23} &= L_{22} + L_{24}^T C - R_{12} + \tau_1 N_{23}, \\
\phi_{24} &= L_{12}^T B - N_{29} + \tau_1 N_{24}, \\
\phi_{25} &= L_{12} + \tau_1 N_{25}, \\
\phi_{26} &= L_{12}^T + \tau_1 N_{26}, \\
\phi_{27} &= L_{12}^T + \tau_1 N_{27}, \\
\phi_{28} &= -\tau_1 L_{33}^T + \tau_1 N_{28}, \\
\phi_{29} &= 0, \\
\phi_{33} &= R_{22} + \epsilon_3 \beta_2^2 I + \tau_1 N_{33}, \\
\phi_{34} &= -N_{39} + \tau_1 N_{34}, \\
\phi_{35} &= \tau_1 N_{35} (j = 5, 6, 7), \\
\phi_{38} &= \tau_1 C^T L_{13} + \tau_2 B^T C + \tau_1 N_{38}, \\
\phi_{39} &= \tau_1 C^T (\tau_1 N_{99} + R_{22} + \tau_1 Q_2), \\
\phi_{44} &= -\left( (1 - \tau_1 d) Q_1 - N_{49} - N_{49}^T - M_2 - \tau_1 Z_{22} + \epsilon_2 \beta_2^2 I + \tau_1 N_{14} \right)
\end{align*}
\]
\[ \phi_{45} = -N_{45} + \tau_1 N_{45}, \]
\[ \phi_{46} = -N_{46} + \tau_1 N_{46}, \]
\[ \phi_{47} = -N_{47} + \tau_1 N_{47}, \]
\[ \phi_{48} = -N_{48} + \tau_2 B^T L_{13} + \tau_1 N_{48}, \]
\[ \phi_{49} = B^T (\tau_1 N_{99} + R_{22} + \tau_1 Q_3), \]
\[ \phi_{55} = -\varepsilon_1 I + \tau_1 N_{55}, \]
\[ \phi_{56} = \tau_1 N_{56}, \quad \phi_{57} = \tau_1 N_{57}, \]
\[ \phi_{88} = \tau_2 L_{13} + \tau_1 N_{88}, \]
\[ \phi_{99} = \tau_1 N_{99} + R_{22} + \tau_1 Q_3, \]
\[ \phi_{66} = -\varepsilon_2 I + \tau_1 N_{66}, \]
\[ \phi_{67} = \tau_1 N_{67}, \quad \phi_{68} = \tau_2 L_{13} + \tau_1 N_{68}, \]
\[ \phi_{69} = \tau_1 N_{99} + R_{22} + \tau_1 Q_3, \]
\[ \phi_{77} = -\varepsilon_4 I + \tau_1 N_{77}, \]
\[ \phi_{78} = \tau_2 L_{13} + \tau_1 N_{78}, \]
\[ \phi_{99} = -\tau_2 Q_{2} + \tau_1 N_{88}, \quad \phi_{99} = -\tau_1 N_{99} - R_{22} - \tau_1 Q_3. \]

**Proof.** Firstly, from (3), we obtain for any scalars \( \varepsilon_1 > 0, \) \( \varepsilon_2 > 0, \) \( \varepsilon_3 > 0, \)
\[ \varepsilon_1 [\beta_1 x^T(t) x(t) - \beta_1^2 (t, x(t)) f_1(t, x(t))] \geq 0 \] (7a)
\[ \varepsilon_2 [\beta_2 x^T(t - \tau_1) x(t - \tau_1) - \beta_2^2 f_2] \geq 0 \] (7b)
\[ \varepsilon_3 [\beta_3 x^T(t - \tau_2) x(t - \tau_2) - f_3^2 f_3] \geq 0 \] (7c)

Choose a class of augmented Lyapunov-Krasovskii functional candidate for systems (1) as following:
\[ V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t), \]
where,
\[ V_1(t) = \left( \begin{array}{c} x(t) \\ x(t - \tau_2) \end{array} \right)^T \left( \begin{array}{c} x(t) \\ x(t - \tau_2) \end{array} \right), \]
\[ V_2(t) = \int_{t-\tau_2}^{t} x(s) R x(s) ds, \]
\[ V_3(t) = \int_{t-\tau_1}^{t} x^T(s) Q_1 x(s) ds, \]
\[ V_4(t) = \int_{t-\tau_2}^{t} (\theta - t + \tau_2) x^T(\theta) Q_2 x(\theta) d\theta, \]
\[ V_5(t) = \int_{t-\tau_1}^{t} (\theta - t + \tau_1) x^T(\theta) Q_3 x(\theta) d\theta, \]
\[ V_6(t) = \int_{t-\tau_1}^{t} (\theta - t + \tau_1) x^T(\theta) N_{99} x(\theta) d\theta, \]
and \( L, Q_1, Q_2, Q_3, R \) and \( N_{99} \) are defined in theorem1.

The time derivative of \( V(t) \) along the trajectory of system (1) is given by:
\[ \dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) + \dot{V}_5(t) + \dot{V}_6(t), \]
where
\[ \dot{V}_1(t) = 2 \left( \begin{array}{c} x(t) \\ x(t - \tau_2) \end{array} \right)^T L \left( \begin{array}{c} \dot{x}(t) \\ \dot{x}(t - \tau_2) \end{array} \right), \]
\[ = 2 \left( \begin{array}{c} x(t) \\ x(t - \tau_2) \end{array} \right)^T \left( \begin{array}{c} Ax(t) + Bz(t - \tau_1(t)) + C\dot{x}(t - \tau_2(t)) + f_1(t) + f_2(t) + f_3(t) \\ x(t) - x(t - \tau_2(t)) \end{array} \right), \]
\[ V_2(t) = \left( \begin{array}{c} x(t) \\ \dot{x}(t) \end{array} \right)^T \left( \begin{array}{c} R_{11} \quad R_{12} \\ R_{12} \quad R_{22} \end{array} \right) \left( \begin{array}{c} x(t) \\ \dot{x}(t) \end{array} \right) - \left( \begin{array}{c} x(t - \tau_2) \\ \dot{x}(t - \tau_2) \end{array} \right)^T \left( \begin{array}{c} R_{11} \quad R_{12} \\ R_{12} \quad R_{22} \end{array} \right) \left( \begin{array}{c} x(t - \tau_2) \\ \dot{x}(t - \tau_2) \end{array} \right) \]
\[ + x^T(t) R_{11}(x(t) + \dot{x}(t)) R_{22}\dot{x}(t) - x(t - \tau_2) x(t - \tau_2) - \dot{x}(t - \tau_2) \dot{x}(t - \tau_2) \]
\[ + 2x^T(t) R_{12}[Ax(t) + Bz(t - \tau_1(t))] + C\dot{x}(t - \tau_2(t)) + f_1 + f_2 + f_3, \]
\[ V_3(t) = x^T(t) Q_1 x(t) - (1 - \tau_1(t)) x^T(t - \tau_1(t)) \]
\[ \leq x^T(t) Q_1 x(t) - (1 - \tau_1(t)) x^T(t - \tau_1(t)) \]
\[ \times Q_1 x(t - \tau_1(t)), \]

It’s from the Lemmal and Lemma2 that we have
\[ \dot{V}_4(t) = x^T(t) \tau_2 Q_2 x(t) - \int_{t-\tau_2}^{t} x^T(s) Q_2 x(s) ds \]
\[ \leq x^T(t) \tau_2 Q_2 x(t) - \int_{t-\tau_2}^{t} \left( \frac{1}{\tau_2} \int_{t-\tau_2}^{t} x(s) ds \right)^T \]
\[ \times \tau_2 Q_2 \left( \frac{1}{\tau_2} \int_{t-\tau_2}^{t} x(s) ds \right) \]
\[ \dot{V}_5(t) = \dot{x}^T(t) \tau_1 Q_3 \dot{x}(t) - \int_{t-\tau_1}^{t} \dot{x}^T(s) Q_3 \dot{x}(s) ds \]
\[ \leq \dot{x}^T(t) \tau_1 Q_3 \dot{x}(t) + \xi(t)^T \left( \begin{array}{c} M_{11} \\ M_{12} \end{array} \right) \xi(t), \]
with
\[ M_{11} = M_1 + M_1^T + \tau Z_{11}, \]
\[ M_{12} = -M_1^T + M_2 + \tau Z_{12}, \]
\[ M_{22} = -M_2 - M_2^T + \tau Z_{22}, \]
\[ \xi(t) = \cos \left( \theta(t) - \theta(t - \tau(t)) \right) \]
\[ \dot{V}_6(t) = x^T(t) \tau_1 N_{99} x(t) - \int_{t-\tau_1}^{t} x^T(s) N_{99} x(s) ds \]
\[ \dot{V}_6(t) = \dot{x}^T(t) \tau_1 N_{99} x(t) - \int_{t-\tau_1}^{t} \dot{x}^T(s) N_{99} x(s) ds. \]
From the Leibniz-Newton formula, the following equation is true for any appropriate dimensional matrices $N_{ij}$ ($i = 1, \ldots, 8$)

$$2 \left\{ x^T (t) N_{19} + x^T (t - \tau_2) N_{29} + \dot{x}^T (t - \tau_2) N_{39} + x^T (t - \tau_1) N_{49} + f_{11}^T N_{59} + f_{21}^T N_{69} + f_{31}^T N_{79} + \left( \frac{1}{R_{t}} \right)^T (t - \tau_2) f (s) ds \right\}^{T} N_{89}, \quad (14)$$

And consider the fact that, for any $m > 0$ and any function $f(t)$,

$$m f(t) - \int_{t-m}^{t} f(t) dt = 0, \quad (15)$$

the following inequality is also true for any appropriate dimensional matrices $N_{ij}$ ($i, j = 1, \ldots, 8$)

$$\tau_1 \xi^T (t) \begin{pmatrix} N_{11} & N_{12} & \ldots & N_{18} \\
* & N_{22} & \ldots & N_{28} \\
* & * & \ldots & N_{38} \\
* & * & \ldots & N_{88} \end{pmatrix} \xi (t) - \int_{t-\tau_1 (t)}^{t} \xi^T (t) \begin{pmatrix} N_{11} & N_{12} & \ldots & N_{18} \\
* & N_{22} & \ldots & N_{28} \\
* & * & \ldots & N_{38} \\
* & * & \ldots & N_{88} \end{pmatrix} \xi (t) \, dt \geq 0, \quad (16)$$

Choosing $M = \tau_1 N_{99} + R_{22} + \tau_1 Q_3$, use systems (1) to obtain

$$\dot{x}^T (t) (\tau_1 N_{99} + R_{22} + \tau_1 Q_3) \dot{x} = [A x (t) + B x (t - \tau_1 (t)) + C (x (t - \tau_2)) + f_1 + f_2 + f_3^T] \cdot x (t) + [A x (t) + B x (t - \tau_1 (t)) + C (x (t - \tau_2)) + f_1 + f_2 + f_3] \cdot x (t - \tau_2) \quad (17)$$

Then, we add the terms on the left sides of (14) and (15) to $V(t)$, and use the Schur’s complement in [15] on the term of (16), we obtain

$$\dot{V} (t) \leq \xi^T (t) \varphi \xi (t) - \int_{t-\tau_1 (t)}^{t} \zeta^T (t) N \zeta (t) dt,$$

where

$$\xi^T (t) \varphi \xi (t) = \begin{pmatrix} x^T (t) & x^T (t - \tau_2) & \dot{x}^T (t - \tau_2) \end{pmatrix} \begin{pmatrix} x^T (t - \tau_1) & f_1^T & f_2^T & f_3^T \end{pmatrix} \left( \frac{1}{R_{t}} \right) (t - \tau_2) f (s) ds \right\}^{T} x (t),$$

and all elements of $\varphi$ are the same as the elements of $\phi$, except that the following:

$$\varphi_{11} = (L_{11} + R_{12}) A + A^T (L_{11} + R_{12})^T + L_{13} + L_{13} \quad (18)$$

By the theorem 9.8.1 in [13], the system (1) with $A_1$ is asymptotically stable if there exist $L > 0, R \geq 0, Q_1 > 0, Q_2 > 0, Q_3 > 0, N_{99} > 0$ and $N > 0$ which were defined in Theorem 1 such that:

$$\dot{V} (t) \leq \sum_{\xi (t) \neq 0, \zeta (t, s) \neq 0} \dot{\xi}^T (t) \varphi \xi (t) - \int_{t-\tau_1 (t)}^{t} \zeta^T (t, s) N \zeta (t, s) dt < 0$$

for all $\xi (t) = 0, \zeta (t, s) = 0$ satisfying (3). Using the S-procedure [19], we see that this condition is implied by (6) such that:

$$\sum_{\xi (t) \neq 0, \zeta (t, s) \neq 0} \dot{\xi}^T (t) \varphi \xi (t) - \int_{t-\tau_1 (t)}^{t} \zeta^T (t, s) N \zeta (t, s) dt$$

for all $\xi (t) = 0, \zeta (t, s) = 0$. Therefore, there exist $L > 0, R \geq 0, Q_1 > 0, Q_2 > 0, Q_3 > 0, N_{99} > 0$ and $N > 0$ which were defined in Theorem 1, and scalars $\varepsilon_1 > 0, \varepsilon_2 > 0, \varepsilon_3 > 0$, such that the LMIs (4), (5) and (6) are satisfied, then systems (1), with uncertainty (2), is asymptotically stable. This completes the proof.

Remark 1: Many existing delay-derivative-dependent stability criteria of system with severely time-varying delay generally require a constraint $\tau_d < 1$. In this paper, we omit this assumption and obtained a less conservative stability condition. As a matter of fact, the chosen Lyapunov-Krasovskii functional in this theorem is the same as our latest article [20], however, in the process of the derivative of the functional, the lemma 1 is very important to our less conservative results, which will be shown subsequently in the examples.

If we set $\beta = 0$, similar to the proof of Theorem 1, we can obtain the following Corollary.

Corollary 1: Under $A_1$, the systems (1) is asymptotically stable, if there exist matrices $L = \begin{pmatrix} L_{11} & L_{12} & L_{13} \\
L_{21} & L_{22} & L_{23} \\
L_{31} & L_{32} & L_{33} \end{pmatrix} \geq 0$ with $L_{11} > 0,$

$$R = \begin{pmatrix} R_{11} & R_{12} \\
R_{21} & R_{22} \end{pmatrix} \geq 0, \quad Q_1 > 0, \quad Q_2 > 0, \quad Q_3 > 0, \quad N_{99} > 0,$$ and any appropriate dimensional matrices $N_{ij}$ ($i, j = 1, \ldots, 8$), $M_1 \in R^{n}, M_2 \in R^{n}, Y = \begin{pmatrix} M_1 & M_2 \end{pmatrix} \in R^{n \times 2n}$, $Z = \begin{pmatrix} Z_{11} & Z_{12} \\
* & Z_{22} \end{pmatrix}$ and scalars $\varepsilon_1 > 0, \varepsilon_2 > 0, \varepsilon_3 > 0$, such that the following LMIs holds:

$$\begin{pmatrix} Q_3 & Y \\
* & Z \end{pmatrix} > 0$$
\[
N = \begin{pmatrix}
N_{11} & N_{12} & N_{13} & \cdots & N_{18} \\
N_{21} & N_{22} & N_{23} & \cdots & N_{28} \\
N_{31} & * & N_{33} & \cdots & N_{38} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
* & * & * & \cdots & N_{88}
\end{pmatrix}
\]

If \( C \equiv 0 \) and \( f_3(t, \dot{x} - t_2) \equiv 0 \), then system (1) reduces to the following system:

\[
\begin{cases}
\dot{x}(t) = A x(t) + B x(t - t_1(t)) + f_1(t, x(t) + f_2(t, x(t) - t_1(t))) \\
x(t_0 + \theta) = \varphi(\theta), \quad \forall \theta \in [-\tau_1, 0]
\end{cases}
\]

According to Theorem 1, we have the following corollary for the delay-dependent stability of system (22).

**Corollary 2**: Under \( A_1 \), the systems (1) is asymptotically stability, if there exist matrices \( L > 0 \), \( Q_1 > 0 \), \( Q_2 > 0 \), \( N_{55} > 0 \) and any appropriate dimensional matrices \( N_{ij} (i, j = 1, 2, \cdots, 5) \), \( M_1 \in \mathbb{R}^n \), \( M_2 \in \mathbb{R}^m \), \( Y = [M_1 \quad M_2] \in \mathbb{R}^{m \times 2n} \), \( Z = [Z_{11} \quad Z_{12} \quad \cdots \quad Z_{22}] \) and scalars \( \epsilon_1 > 0 \), \( \epsilon_2 > 0 \), such that the following LMIs holds:

\[
\begin{pmatrix}
Q_2 & Y \\
* & Z
\end{pmatrix} < 0
\]

\[
\begin{pmatrix}
N_{11} & * & * & * & * \\
* & N_{22} & * & * & * \\
* & * & N_{33} & * & * \\
* & * & * & N_{44} & * \\
* & * & * & * & N_{55}
\end{pmatrix} > 0
\]

\[
\phi = \begin{pmatrix}
\phi_{11} & \phi_{12} & \phi_{13} & \cdots & \phi_{18} \\
\phi_{21} & \phi_{22} & \phi_{23} & \cdots & \phi_{28} \\
\phi_{31} & * & \phi_{33} & \cdots & \phi_{38} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
* & * & * & \cdots & \phi_{88}
\end{pmatrix}
\]

where

\[
\phi_{11} = (L_{11} + R_{12}) A + A^T (L_{11} + R_{12})^T + L_{13} + L_{13}^T + N_{18} + N_{18}^T + R_{11} + Q_{1} + \tau_2 Q_2 + M_1 + M_1^T + \tau_1 Z_{11} + \epsilon_1 \beta_1^2 I + \tau_1 N_{11},
\]

\[
\phi_{12} = A^T L_{12} + N_{18}^T + L_{13} + L_{13}^T + \tau_1 N_{12},
\]

\[
\phi_{13} = (L_{11} + R_{12}) C + C + N_{38}^T + \tau_1 N_{13},
\]

\[
\phi_{14} = (L_{11} + R_{12}) B + N_{18}^T + \tau_1 N_{14} - M_1^T + M_2 + \tau_1 Z_{12},
\]

\[
\phi_{15} = L_{11} + R_{12} + N_{38}^T + \tau_1 N_{15},
\]

\[
\phi_{16} = L_{11} + R_{12} + N_{38}^T + \tau_1 N_{16},
\]

\[
\phi_{17} = \tau_2 L_{13} + N_{38}^{12} + N_{18}^T + \tau_1 N_{17},
\]

\[
\phi_{18} = A^T (\tau_1 N_{48} + R_{22} + \tau_1 Q_3),
\]

\[
\phi_{22} = -L_{23} - L_{23}^T - R_{11} + \tau_1 N_{22},
\]

\[
\phi_{23} = -L_{23} + L_{12} C - R_{12} + \tau_1 N_{23},
\]

\[
\phi_{24} = L_{12} B - N_{28}^T + \tau_1 N_{24},
\]

\[
\phi_{25} = L_{12} C + \tau_1 N_{25},
\]

\[
\phi_{26} = -L_{12} C + \tau_1 N_{26},
\]

\[
\phi_{27} = -\tau_2 L_{23} + \tau_1 N_{27},
\]

\[
\phi_{28} = 0
\]

\[
\phi_{33} = -R_{22} + \tau_1 N_{33},
\]

\[
\phi_{34} = -N_{38} + \tau_1 N_{34},
\]

\[
\phi_{35} = \tau_2 C T L_{13} + \tau_2 L_{23} + \tau_1 N_{37},
\]

\[
\phi_{38} = C T (\tau_1 N_{48} + R_{22} + \tau_1 Q_3),
\]

\[
\phi_{44} = -\left(1 - \tau_1 \epsilon_1\right) Q_1 - \tau_2 Q_2 - N_{48}^T - M_2 - M_2^T + \tau_1 Z_{22} + \epsilon_2 \beta_1^2 I + \tau_1 N_{44},
\]

\[
\phi_{45} = -N_{58} + \tau_1 N_{45},
\]

\[
\phi_{46} = -N_{68} + \tau_1 N_{46},
\]

\[
\phi_{47} = -N_{78} + \tau_1 N_{47},
\]

\[
\phi_{48} = B^T (\tau_1 N_{48} + R_{22} + \tau_1 Q_3),
\]

\[
\phi_{55} = -\epsilon_1 I + \tau_1 N_{55},
\]

\[
\phi_{56} = \tau_1 N_{56},
\]

\[
\phi_{57} = \tau_2 L_{13} + \tau_1 N_{57},
\]

\[
\phi_{58} = \tau_1 N_{58} + R_{22} + \tau_1 Q_3,
\]

\[
\phi_{66} = -\epsilon_2 I + \tau_1 N_{66},
\]

\[
\phi_{67} = \tau_2 L_{13} + \tau_1 N_{67},
\]

\[
\phi_{68} = \tau_1 N_{68} + R_{22} + \tau_1 Q_3,
\]

\[
\phi_{77} = \tau_2 Q_2 + \tau_1 N_{77},
\]

\[
\phi_{88} = -\tau_1 N_{88} - R_{22} - \tau_1 Q_3.
\]
IV. NUMERICAL EXAMPLES

In order to show the effectiveness of the approaches presented in Section 3, in this section, two numerical examples are provided.

Example1. Consider the neutral systems (1) which was considered in [21] with

\[
A = \begin{pmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{pmatrix}, \quad C = \begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix},
\]

\[
\|f_1(t, x(t))\| \leq \alpha_1\|x(t)\|, \quad \|f_2(t, x(t - \tau_1(t)))\| \leq \alpha_2\|x(t - \tau_1(t))\|, \quad \|f_3(t, \dot{x}(t - \tau_2))\| \leq \alpha_3\|\dot{x}(t - \tau_2)\|
\]

where \(\alpha_1 \geq 0, \alpha_2 \geq 0, \alpha_3 \geq 0\) and \(0 < c < 1\).

We also consider the effect of the bound \(c\) on the maximal allowable value \(\tau_{1m}\). For \(c = 0.1, \tau_2 = 1, \tau_{1d} = 0.5, \alpha_2 = 0.1\), and different values of \(\alpha_3\), we apply theorem 1 and Corollary to calculate the maximal allowable value \(\tau_{1m}\) that guarantees the asymptotical stability of the system.

Table I gives the comparison of our results with those in [6] and [21]. We can see from Table I that the upper bound of \(\alpha_3\) has a remarkable effect on \(\tau_{1m}\). \(\tau_{1m}\) decreases as \(\alpha_3\) increases. In conclusion, the results obtained in this paper are less conservative than that presented in [6] and [21].

For \(c = 0\) and \(f_3(t, \dot{x}(t - \tau_2)) \equiv 0\), the system under consideration reduces to the system studied in [1]. Applying criteria in [1], [4], [6] and in this work, the maximum value of \(\tau_{1m}\) for the stability of the system is listed in Table II. It is easy to see that our proposed stability criterion gives a much less conservative result than one in [1], [4] and [6].

One should be noted that, on the one hand, from the comparison in Table I, our results are inferior to our latest results in [20], however, it is also less conservative than those conditions in [6] and [21]. On the other hand, the less conservativeness is also shown in the table II.

**TABLE I**

<table>
<thead>
<tr>
<th>(\tau_{1m}) OF EXAMPLE 1 FOR DIFFERENT VALUES OF (\alpha_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_3)</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td><a href="(%5Calpha_3=0)">6</a></td>
</tr>
<tr>
<td><a href="(%5Calpha_3=0)">21</a></td>
</tr>
<tr>
<td>This paper((\alpha_3=0))</td>
</tr>
<tr>
<td><a href="(%5Calpha_3=0.1)">6</a></td>
</tr>
<tr>
<td><a href="(%5Calpha_3=0.1)">21</a></td>
</tr>
<tr>
<td>This paper((\alpha_3=0.1))</td>
</tr>
</tbody>
</table>

Example2. Consider the neutral system

\[
\frac{dx(t) - Cx(t - \tau)}{dt} = Ax(t) + Bx(t - \tau) + f_1(t, x(t)) + f_2(t, x(t - \tau))
\]

where \(f_1, f_2, \ldots\) are nonlinear perturbations, \(A, B, C\) are matrices, \(\tau\) is the delay.

\[
A = \begin{pmatrix} -2 & 0.5 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0.4 \\ 0.4 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 0.2 & 1 \\ 1 & 0.2 \end{pmatrix}
\]

with \(\|f_1(t, x(t))\| \leq \alpha_1\|x(t)\|, \|f_2(t, x(t - \tau))\| \leq \alpha_2\|x(t - \tau)\|\) where \(\alpha_1 = 0.2, \alpha_2 = 0.1\).

This system was studied in [14], where it is found that the admissible bound of the time delay \(\tau\) for the stability of systems (25) is 0.583. Applying the criteria in this paper, the upper bound of the delay \(\tau\) has been obtained as 1.9391. This also shows that the criterion given in this paper is much less conservative than that in [14].

V. CONCLUSION

The asymptotical stability for uncertain neutral systems with nonlinear perturbations has been investigated. Based on a new class of Lyapunov-Krasovskii functionals, and combined with the use of novel integral inequalities and the Leibniz-Newton formula, some novel stability criteria have been obtained. Numerical examples have shown significant improvements over some existing results.

ACKNOWLEDGMENT

The author would like to thank the associate editor and the anonymous reviewers for their constructive comments and suggestions to improve the quality of the paper.

REFERENCES

Lianglin Xiong was born in Sichuan Province, China, in 1981. He received the B.S. degree from Neijiang teacher university, Sichuan, Neijiang, China, in 2004, obtained the M.S. and Ph.D degree from the University of Electronic Science and Technology of China (UESTC), Sichuan, in 2007 and 2009, respectively. He is a teacher with the School of Mathematics and Computer Science, Yunnan University of Nationalities. His research interests include neural systems, neutral systems, hybrid systems, fractional-order systems and so on.

Xiuyong Ding was born in Sichuan Province, China, in 1981. He received the B.S. degree from Neijiang teacher university, Sichuan, Neijiang, China, in 2004 and the M.S. degree from the University of Electronic Science and Technology of China (UESTC), Sichuan, in 2009. He is currently pursuing the Ph.D degree with UESTC. His research interests include positive systems, hybrid systems, fuzzy systems and so on.

Shouming Zhong was born in 1955 in Sichuan Province, China. He received B.S. degree in applied mathematics from UESTC, Chengdu, China, in 1982. From 1984 to 1986, he studied at the Department of Mathematics in Sun Yatsen University, Guangzhou, China. From 2005 to 2006, he was a visiting research associate with the Department of Mathematics in University of Waterloo, Waterloo, Canada. He is currently as a full professor with School of Applied Mathematics, UESTC. His current research interests include differential equations, neural networks, biomathematics and robust control. He has authored more than 80 papers in reputed journals such as the In International Journal of Systems Science, Applied Mathematics and Computation, Chaos, Solitons and Fractals, Dynamics of Continuous, Discrete and Impulsive Systems, Acta Automatica Sinica, Journal of Control Theory and Applications, Acta Electronica Sinica, Control and Decision, and Journal of Engineering Mathematics.