Frequency-Energy Characteristics of Local Earthquakes using Discrete Wavelet Transform (DWT)

O. H. Colak, T. C. Destici, S. Ozen, H. Arman, and O. Cerezci

Abstract—The wavelet transform is one of the most important method used in signal processing. In this study, we have introduced frequency-energy characteristics of local earthquakes using discrete wavelet transform. Frequency-energy characteristic was analyzed depend on difference between P and S wave arrival time and noise within records. We have found that local earthquakes have similar characteristics. If frequency-energy characteristics can be found accurately, this gives us a hint to calculate P and S wave arrival time. It can be seen that wavelet transform provides successful approximation for this. In this study, 100 earthquakes with 500 records were analyzed approximately.

Keywords—Discrete Wavelet Transform, Frequency-Energy Characteristics, P and S waves arrival time.

I. INTRODUCTION

If the epicentral distances is 0-10°, then these are called as local earthquakes. Seismic waves from local and regional earthquakes of low or moderate magnitude are of the short period and therefore almost exclusively recorded by short period seismograms. The seismogram length depends upon the magnitude but generally does not exceed 5 minutes or so. Strong events recorded at local or regional distances will produce seismograms with duration of several hours. For local earthquakes a dominant portion of recorded seismic waves have propagated through the crust and/or the Moho discontinuity and are commonly called crustal waves [1].

Seismic signal analysis is one of the most important topics for signal analysis. There has been lots of research in the literature today [2-5]. The wavelet transform is very useful tool in the analysis of nonstationary signals such as seismic signals.

This is due to the ability of wavelet transform to resolve features. Wavelet analysis is a windowing technique with variable sized region. For many signals, fourier analysis has a serious drawback. In transforming to the frequency domain, time information was lost.

If signal properties do not change much over time so if it is what is called a stationary signal. However, most interesting signals such as seismic signals contain nonstatinoary characteristics.

In this study, find frequency-energy characteristics based on wavelet transform in local earthquakes has been aimed, since this solutions are containing hints to find different seismic waves. Analyzed all records were received from Turkey region.

II. DISCRETE WAVELET TRANSFORM

Discrete wavelet transform (DWT) deals with s scaling parameter and k translation parameters. The integral discrete wavelet transform of a function becomes:

\[ W_{\psi} f(k2^{-s},2^{-s}) = 2^{s/2} \int f(t) \psi(2^s t - k) dt \quad (1) \]

\[ \psi_{k,s} (t) = 2^{s/2} \psi(2^s t - k) dt \quad (2) \]

where, \( \psi \) is called wavelet function for wavelet transform [6].

For many signals, the low frequency content is the most important part. The high frequency content imparts flavor or nuance. In wavelet analysis, signal is decomposed approximation and detail components [7]. The original signal passes through the complementary filters and emerges as two signals. DWT decomposition scheme is shown in Fig. 1. Circles represent downsampling 2 factor.

Fig. 1 DWT Decomposition
The decomposition process can be iterated, with successive approximations being decomposed in turn, so that one signal is broken down into many lower resolution components. This is called the wavelet decomposition tree [7]. Approximation and detail components can be assembled back into the original signal without loss of information which is called as reconstruction shown in Fig. 2.

Fig. 2 DWT reconstruction

III. FREQUENCY-ENERGY CHARACTERISTICS

A signal can be decomposed into n level. There is given by \( N=2^n \), where \( N \) is the total number of data points. Each of these wavelet levels represent a frequency band given by the equation [8].

\[
f = 2^{N} \left( f_s \right) \left( \frac{1}{N} \right)
\]

where \( f \) is the higher frequency limit of the frequency band represented by the level \( z \), \( f_s \) is the sampling frequency and \( N \) is the number of data points in the input signal. RMS value and energy of the individual wavelet level \( L \) is given by:

\[
rms(\text{level } L) = \sqrt{\frac{1}{N} \sum A(L)^2}
\]

Where \( A(L) \) is the coefficients of wavelet decomposition level \( L \) signal. Total energy of signal can be mathematically expressed as [9,10]:

\[
\sum_{n=1}^{N} |s(n)|^2 = \sum_{n=1}^{N} |a_j(n)|^2 + \sum_{j=1}^{l} \sum_{n=1}^{N} |d_j(n)|^2
\]

Where \( N \) is the number of data points, \( l \) is the total number of wavelet decomposition levels. The expression shows that the total signal energy is the sum of the \( j^\text{th} \) level approximation signal and sum of all detail level signals \( l^\text{th} \) to \( j^\text{th} \) detail.

IV. ANALYSIS OF EARTHQUAKE

All data used in this study have been supported by Boğaziçi University, Kandilli Observatory and Earthquake Research Center (KOERI). Earthquake analysis had been carried out for different stations in KOERI network. Those records have 3 different sampling time as 0.01 s, 0.02 s and 0.05 s. Two samples representing all results are the most effective ones and can be seen following.

Bala-Ankara/Turkey earthquake was occurred at 30.07.2005/21.45.00. Earthquake magnitude is 5.3. Earthquake and station information was listed at Table I.

TABLE I

<table>
<thead>
<tr>
<th>Station Code and Region</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Event Time</th>
<th>Ts</th>
<th>Distance between epicenter and station (km)</th>
<th>Ts-Tp</th>
<th>(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO RM-CORUM</td>
<td>40.10</td>
<td>34.37</td>
<td>39.44</td>
<td>33.08</td>
<td>21:45</td>
<td></td>
<td>168</td>
<td>21.2</td>
<td></td>
</tr>
</tbody>
</table>

East-West component of this earthquake was shown in Fig. 3.

Fig. 3 East-West Component of Bala/Turkey earthquake

Sampling time is 0.05 s for this station. Each investigated data includes 256 data packages, and the db10 wavelet [11] was applied. Wavelet level-frequency band was listed in Table II.

TABLE II

<table>
<thead>
<tr>
<th>Levels</th>
<th>Components</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>a7</td>
<td>0.0-0.07812</td>
</tr>
<tr>
<td>1</td>
<td>d7</td>
<td>0.07812-0.15625</td>
</tr>
<tr>
<td>2</td>
<td>d6</td>
<td>0.15625-0.3125</td>
</tr>
<tr>
<td>3</td>
<td>d5</td>
<td>0.3125-0.625</td>
</tr>
<tr>
<td>4</td>
<td>d4</td>
<td>0.625-1.25</td>
</tr>
<tr>
<td>5</td>
<td>d3</td>
<td>1.25-2.5</td>
</tr>
<tr>
<td>6</td>
<td>d2</td>
<td>2.5-5</td>
</tr>
<tr>
<td>7</td>
<td>d1</td>
<td>5-10</td>
</tr>
</tbody>
</table>

Fig. 4 depicts the maximum energy values for detail components.
Two break points were marked in Fig. 4. In these points, energy-frequency changes can be seen in Fig. 5 and Fig. 6.

Maksimum energy has been moved higher frequency band in 4th window. Level 6 has 2.5-5 Hz frequency band.

d6 component predominates in 5th window. Total energy was assembled on d2, d3 and d4 components. This earthquake was shown as first type local earthquake according to energy distribution. Energy increases two times without decreasing.

Sapanca – Sakarya/Turkey Earthquake was occurred at 08.02.2006/04:07:01. Earthquake magnitude is 4.5. Sampling time is 0.01 s. Data was received by GBZX station. Distance between the epicenter and record station is 75 km. For this earthquake, z component was shown in Fig. 8, and wavelet level-frequency band was listed in Table III.

z component of this earthquake was shown in Fig. 3.

<table>
<thead>
<tr>
<th>Level</th>
<th>Components</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>a7</td>
<td>0.3906-0.7812</td>
</tr>
<tr>
<td>1</td>
<td>d7</td>
<td>0.7812-1.5625</td>
</tr>
<tr>
<td>2</td>
<td>d6</td>
<td>1.5625-3.125</td>
</tr>
<tr>
<td>3</td>
<td>d5</td>
<td>3.125-6.25</td>
</tr>
<tr>
<td>4</td>
<td>d4</td>
<td>6.25-12.5</td>
</tr>
<tr>
<td>5</td>
<td>d3</td>
<td>12.5-25</td>
</tr>
<tr>
<td>6</td>
<td>d2</td>
<td>25-50</td>
</tr>
<tr>
<td>7</td>
<td>d1</td>
<td>50-100</td>
</tr>
</tbody>
</table>
Maximum energy values for detail components were shown in Fig. 9.

This is second type local earthquake according to energy distribution. There are two peaks depicted on figure. First peak demonstrates energy of P wave and second peak represents the energy of S wave. Frequency changes in break point were shown in Figs. 10, 11.

![Fig. 9 Maximum energy values for detail components (First 55 window)](image)

![Fig. 10 Energy Change of 23rd window](image)

![Fig. 11 Energy Change of 27th window](image)

V. CONCLUSION AND DISCUSSION

In this study, we have determined two types of energy changes in local earthquakes. First type has increasing non-decreasing energy characteristics. Second type has decreasing energy characteristics after energy have increased in two break points. Frequency band changes at these points. Energy variation can be appeared at very low frequencies depending on sampling time. Discrete wavelet transform provides detection of these low frequencies. Therefore DWT gives very successful results in frequency-energy analyzing. Obtained results are so important to detect of P and S waves arrival time.

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REFERENCES