Abstract—Suspended cable structures are most preferable for large spans covering due to rational use of structural materials, but the problem of suspended cable structures is initial shape change under the action of non-symmetrical load. The problem can be solved by increasing of relation of dead weight and imposed load, but this methods cause increasing of materials consumption. Prestressed cable truss usage is another way how the problem of shape change under the action of non-symmetrical load can be fixed. The better results can be achieved if we replace top chord with cable truss with cross web.

Rational structure of the cable truss for prestressed cable truss top chord was developed using optimization realized in FEM program ANSYS 12 environment. Single cable and cable truss model work was discovered. Analytical and model testing results indicate, that usage of cable truss with the cross web as a top chord of prestressed cable truss instead of single cable allows to reduce total displacements by 13-16% in the case of non-symmetrical load.

In case of uniformly distributed load single cable is preferable.

Keywords—Cable trusses, Non-symmetrical load, Cable truss models, Vertical displacements

I. INTRODUCTION

The main advantage of suspended cable structures (Fig.1) is possibility to use in full scale high strength materials, for example steel ropes with tensile strength of wires 1960 MPa, because tensile elements can’t lose its stability and stresses are uniformly distributed by cross-section [5], [7] – [10], [14], [25]. But the problem of suspended cable structures is increased deformability or initial shape change, especially in case of non-symmetrical load (Fig.2). The problem is especially important for suspension bridges, where this case of loading is possible [11], [18], [21], [28] and [29]. The problem can be solved by increasing of relation of dead weight and imposed load, which is realized by adding of cantledge (Fig.3) [15], [22]. But this methods cause increasing of material consumption. The prestressed cable truss usage is another way how the problem of shape change under the action of unsymmetrical load can be fixed (Fig.4) [13], [16], [26], [27], [30]. The better results can be achieved if we replace single cable top chord with cable truss with cross web (Fig.5) [6].

The aim of this study is to develop rational structure of cable truss for top chord of prestressed cable truss and to
compare single cable and cable truss as top chord of prestressed cable truss. Achieved results should be checked on the physical models.

II. INPUT PARAMETERS FOR CABLE TRUSS OPTIMIZATION

Design scheme of investigation object is shown on the Fig.6. The structural material is pretensioned steel rope [2], [3]. The dead load, applied to the structure is 51.1kN/m, imposed load is 82.2 kN/m [1], and can be applied to full span or to a half of span. The load is applied to the deck, which is placed between chords of the prestressed cable truss.

![Image](Fig. 6 Design scheme of cable truss)

Fig. 6 Design scheme of cable truss q – imposed load; g – dead load; \( f_b \) – bottom chord camber; \( f_t \) – top chord camber; \( x_2 \) and \( x_4 \) – distance from the pylon to the connection of web element and top cord; \( x_1 \) and \( x_3 \) – distance from the pylon to the connection of web element and bottom cord

Constants for optimization are bottom chord camber, material consumption of cable truss \( q \), material consumption of stabilization cable, level of prestressing, prestressed cable truss height, span and suspension step.

Variable factors for optimization are relation of top and bottom chord camber, position of web elements, which is expressed by distance from the pylon to the connection of web element with the top chord for each web element depending on the distance from the pylon to the connection of the same element with bottom chord, the number of web elements inclined to the center of cable truss, distribution of material consumption among bottom cord, top chord and web elements. Responses for optimization are vertical displacements \( w \), and objective of optimization is to minimize vertical displacements for the non-symmetrical load.

III. CALCULATION OF RATION PARAMETERS OF CABLE TRUSS

Optimization was realized by enumeration of possible variants [4], [12], using cycles in the FEM program ANSYS 12 environment [20]. Enumeration was realized in three steps, at each step finding optimal field, then increasing precision by 10 times and finding new optimal field. The main principles of preprocessor imputing are described.

![Image](Fig. 7 Design scheme of cable truss)

Fig. 7 Design scheme of cable truss

A. Definition of finite element type and material

Cable truss is modeled using finite element (FE) type LINK10 with options of small stiffness assigned to slack cable for both longitudinal and perpendicular motions and tension-only (cable) option. The material of elements is linear isotropic with modulus of elasticity 167 Gpa and Poisson's ratio 0.3.

The analysis type is static with including large-deflection effects, because suspension cable structures are characterized with large deflections before stabilization.

B. Input parameters

The main input parameters of cable truss are span, step of suspensions (load application points), bottom chord camber, top chord camber, stabilization cable camber, material consumption of cable truss, material consumption of bottom chord of truss, material consumption of top chord of truss, material consumption of the web of truss, material consumption of stabilization cable, material consumption of suspensions.

C. Geometry construction

As the model of cable truss is not simple, and it takes a lot of time separately input point, they are inputted by formula using cycles. Coordinates for deck points can be calculated by (1).

\[
\begin{align*}
x &= a \cdot i \\
y &= 0
\end{align*}
\]

where \( x \) – horizontal coordinate; \( y \) – vertical coordinate; \( i \) – point number, changes from 0 to \( l/la \); \( a \) – step of suspensions; \( l \) – span of cable truss.

The program should know to what part of structure belongs each point. So points are counted and first and last point number for each structure is stored. Coordinates for cable truss bottom chord points can be calculated by (2).

\[
\begin{align*}
x &= a \cdot i \\
y &= -4 \cdot f_b \cdot x \cdot (l-x)/l + h_p
\end{align*}
\]

where \( f_b \) – bottom chord camber; \( h_p \) – pylon height.

Coordinates of point for stabilization cable can be found by the same principle, as for bottom chord. The points of top chord are defined by the position of web element. The position of each web element is expressed by the distance from pylon to the connection of the web element with top chord depending on the distance from the pylon to the connection of the same web element with bottom chord (Fig.8) and is expressed by polynomial equation.
Fig. 8 Position of web elements

The position of web elements, inclined to the edges of cable truss can be expressed by (3), the position of web elements, inclined to the edges of cable truss can be expressed by (4).

\[ x_2 = x_1 - (\text{root}1 \cdot x_1^2 + \text{root}2 \cdot x_1 + \text{root}3), \quad (3) \]
\[ x_4 = x_3 + (\text{root}4 \cdot x_3^2 + \text{root}5 \cdot x_3 + \text{root}6), \quad (4) \]

where \( x_2 \) and \( x_4 \) – distance from the pylon to the connection of web element and top chord;

\( x_1 \) and \( x_3 \) – distance from the pylon to the connection of web element and bottom chord;

\( \text{root}1 \ldots \text{root}6 \) – roots of the system of equation.

The roots of the polynomial equation for web elements inclined to the center of cable truss were found solving system (5).

\[
\begin{align*}
  s_1 &= \text{root}1 \cdot a_1^2 + \text{root}2 \cdot a_1 + \text{root}3 \\
  s_2 &= \text{root}1 \cdot a_2^2 + \text{root}2 \cdot a_2 + \text{root}3 \\
  s_3 &= \text{root}1 \cdot a_3^2 + \text{root}2 \cdot a_3 + \text{root}3
\end{align*}
\]

where \( s_1 \) – distance \( x_2 \) for \( x_1 = a_1 \);

\( s_2 \) – distance \( x_2 \) for \( x_1 = a_2 \);

\( s_3 \) – distance \( x_2 \) for \( x_1 = a_3 \);

\( a_1 \) – distance from the pylon to the connection of first web element with bottom chord;

\( a_2 \) – distance from the pylon to the connection of middle web element with bottom chord;

\( a_3 \) – distance from the pylon to the connection of last web element with bottom chord, counting for the middle of span.

Distances \( s_1, s_2 \) and \( s_3 \) were found by optimization.

The system of equations is solved by Ansys, using matrix method (6) - (10).

\[
\Delta = a_1^2 \cdot a_2 - a_1^2 \cdot a_3 - a_1 \cdot a_2^2 + a_1 \cdot a_3^2 + a_2^2 \cdot a_3 - a_2 \cdot a_3^2. \quad (6)
\]
\[
\Delta_1 = s_1 \cdot a_2 - s_1 \cdot a_3 - s_1 \cdot a_4 + a_1 \cdot s_1 + s_2 \cdot a_1 - a_2 \cdot s_2. \quad (7)
\]
\[
\Delta_2 = a_1^2 \cdot s_2 - a_1^2 \cdot s_3 - s_1 \cdot a_2^2 + s_1 \cdot a_3^2 + a_2^2 \cdot s_3 - s_2 \cdot a_3^2. \quad (8)
\]
\[
\Delta_3 = a_1^2 \cdot s_2 - a_1^2 \cdot s_3 - s_1 \cdot a_2^2 + s_1 \cdot a_3^2 + a_2^2 \cdot s_3 - s_2 \cdot a_3^2.
\]

where \( \Delta \) – determinant.

The roots of the polynomial equation for web elements inclined to the center of cable truss can be founded by the same method.

Top chord point coordinates for one half of span for web elements inclined to the edges of cable truss is expressed by (11).

\[
\begin{align*}
  x_2 &= x_1 - (\text{root}1 \cdot x_1^2 + \text{root}2 \cdot x_1 + \text{root}3) \\
  y &= \frac{4 \cdot f_t \cdot x_2 \cdot (1 - x_2) + h_p}{l^2}
\end{align*}
\]

where \( x_1 = a \cdot i \);

\( i \) changes from 1 to \((l/a)/2\);

\( f_t \) – top chord camber;

Top chord point coordinates for one half of span for web elements inclined to the center of cable truss is expressed by (12).

\[
\begin{align*}
  x_4 &= x_3 + (\text{root}4 \cdot x_3^2 + \text{root}5 \cdot x_3 + \text{root}6) \\
  y &= \frac{4 \cdot f_t \cdot x_4 \cdot (1 - x_4) + h_p}{l^2}
\end{align*}
\]

where \( x_3 = a \cdot i \);

\( i \) changes from 1 to \((l/a)/2\).

Top chord point coordinates for other half of span can be founded by the same method.

As the points are not sorted, it is not possible to connect them. Grouping of points of top chord, so the point with first number has the smallest horizontal coordinate, but the point with last point has the largest horizontal coordinate, should be done. To order array on point numbers and coordinates by coordinate, bubble sort is used (Fig.9). Bubble sort works by repeatedly stepping through the list to be sorted, comparing each pair of adjacent items and swapping them if they are in the wrong order. The pass through the list is repeated until no swaps are needed, which indicates that the list is sorted [31].
When the sorting is done, points can be connected with lines.

D. Definition of cross-section parameters

For LINK10 elements we need to define cross-section area and initial strain.

In this work the whole mass of structure is given, the distribution of material consumption for each element of the truss was found by optimization. The material consumption for all elements is expressed, as a part of whole truss material consumption.

The prestressing of structure is organized in stabilization cable by applying initial strain.

When the preprocessor operations are done, calculations can be launched [17], [19], [23].

IV. RATIONAL PARAMETERS OF CABLE TRUSS

Rational parameters of cable truss were founded.

Rational relation of top chord camber and bottom chord camber: \( \frac{f_{fL}}{g} = 0.71 \).

Rational relation of bottom chord material consumption and material consumption of whole truss: \( \frac{g_d}{g} = 0.6 \)

Rational relation of web elements material consumption and material consumption of whole truss: \( \frac{g_w}{g} = 0.05 \)

Rational number of web elements inclined to the center of cable truss is achieved removing element from 5 to 11 from both sides.

Rational position of web elements inclined to the edges of cable truss is expressed by rational value of distance \( x_2 \) of each web element on distance \( x_1 \) from the pylon in the form of polynomial equation (13).

\[
x_2 = x_1 - \left( 6.783 \cdot 10^{-4} \cdot x_1^2 + 0.1817 \cdot x_1 + 2.108 \right) \tag{13}
\]

where \( x_1 \) – distance from the pylon to the bottom chord’s node,

\( x_2 \) – distance from the pylon to the top chord’s node

Position of web elements inclined to the center of cable truss was founded by mirroring elements inclined to the edges of cable truss.

V. COMPARISON OF SINGLE CABLE AND CABLE TRUSS FOR TOP CHORD OF PRESTRESSED CABLE TRUSS

Cable truss top chord of prestressed truss was compared with single cable top chord of cable truss. Shape of displacements in non-symmetrical loading case is shown on the Fig.10. Loaded part of structure displace to the downwards direction, but unloaded part – to the upper direction. Total displacements are calculated as sum of upwards and downwards displacements. The results of displacements in symmetrical and non-symmetrical loading case are shown in Table I. The difference between single cable and cable truss displacements upwards is 34%, the difference between single cable and cable truss displacements downwards is 8% and difference between single cable and cable truss total displacements is 16%.

![Fig. 10 Shape of displacements in non-symmetrical loading case](image)

![Table I: Displacements of Prestressed Cable Truss with Single Cable and Cable Truss Top Chord](table)

<table>
<thead>
<tr>
<th>Displacement direction</th>
<th>Single cable</th>
<th>Non-symmetrical load</th>
<th>Cable truss</th>
<th>Non-symmetrical load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deck displacements downwards, mm</td>
<td>0.4965 m</td>
<td>0.6684 m</td>
<td>0.5498 m</td>
<td>0.6157 m</td>
</tr>
<tr>
<td>Deck displacements upwards, mm</td>
<td>0.3039 m</td>
<td>0.1995 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deck displacements total, mm</td>
<td>0.4965 m</td>
<td>0.9723 m</td>
<td>0.5498 m</td>
<td>0.8152 m</td>
</tr>
</tbody>
</table>

VI. EXPERIMENTAL MODEL TESTING

Two models were constructed to compare behaviors of single cable and cable truss for top chord of prestressed cable truss. (Fig.11 and Fig.12).

![Fig. 11 Model of single cable top chord](image)
The span of the models of cable truss is equal to 2.1 m. Top chord camber is equal to 22 centimeters. The deck is connected to main load caring structure by suspensions in 15 points (Fig. 13).

The prestressing is organized in stabilization cable and is developed by rotating a screw and moving a bar. To allow cable to move, it is supported by the block (Fig. 15).

The deck of the models is made from wood oriented strand board (OSB). Due to OSB good deformability, it does not take load, but only distribute it among suspensions. The deck is not connected to suspensions in horizontal plane.

To connect a deck to load bearing cable, adjustable suspensions are used, that allow leveling the deck. Suspensions are connected to the cables using U-bolt clips to prevent moving them along the cable (Fig. 16).

### Elements of the Models

The elements of the cable truss and single cable top chord for prestressed cable truss models are made from steel cables. Two types of cables are used: 6x7+WSC (wire steel core) and 6x19+WSC (Fig. 14). Both cables have ordinary left hand lay (Zs). Tensile strength of wires for both cables is 1770 MPa. The modulus of elasticity was experimentally obtained and is 60000 MPa. The diameters of elements are shown in Table II. With selected diameters of elements was achieved that cable truss has the same cable material consumption as single cable, what is important for comparison of models.

### Table II

<table>
<thead>
<tr>
<th>Elements</th>
<th>Cable type</th>
<th>Diameter</th>
<th>Breaking force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main cable</td>
<td>6x19+WSC</td>
<td>10.0 mm</td>
<td>63.0 kN</td>
</tr>
<tr>
<td>Stabilization</td>
<td>6x19+WSC</td>
<td>8.0 mm</td>
<td>40.3 kN</td>
</tr>
<tr>
<td>Cable truss top</td>
<td>6x19+WSC</td>
<td>8.0 mm</td>
<td>40.3 kN</td>
</tr>
<tr>
<td>Bottom chord</td>
<td>6x19+WSC</td>
<td>8.0 mm</td>
<td>40.3 kN</td>
</tr>
<tr>
<td>Top chord</td>
<td>6x19+WSC</td>
<td>5.5 mm</td>
<td>17.8 kN</td>
</tr>
<tr>
<td>Web elements</td>
<td>6x7+WSC</td>
<td>2.0 mm</td>
<td>2.7 kN</td>
</tr>
<tr>
<td>Stabilization</td>
<td>6x19+WSC</td>
<td>8.0 mm</td>
<td>40.3 kN</td>
</tr>
</tbody>
</table>
Web elements of cable truss are connected to the chords by contact connection (Fig.17), where web elements are pressed to the chord. This type of connection showed enough strength and it was possible to make small elements of the web with this type of connection.

**B. Measurement Instruments**

The tensile force in stabilization cable was measured by electronic dynamometer Scaime IPB50 (Fig.18) and self-made mechanical dynamometer (Fig.19). Electronic dynamometer work principle is based on changes of electrical bridge resistance. The precision of measurements for electronic dynamometer is 0.25 kg. Mechanical dynamometer work principle is based on measurements of spring displacements and converting them to forces using dependences. The precision of measurements for mechanical dynamometer is 0.5 kg.

Displacements were measured in 8 points by Aistov deflectometers, which allow to measure displacements in wide range (Fig.20). Deck displacements were measured.

**C. Process of model testing**

To remove free displacements and to stabilize modulus of elasticity, initial extension should be done for cable structures. The models were initially extensioned by the load, which is by 20% larger than designed load.

The model was at first prestressed by load 1000 kg for each side. Two types of loading were applied to model: symmetrical and non-symmetrical. The load was applied to the deck by placing iron weight 20 kg each. In symmetrical loading case, both models were loaded up to load 2755 kg with step 285 kg (Fig.21). In non-symmetrical loading case the models were loaded up to load 1495 kg with step 155 kg (Fig.22).
D. Models testing results

The models were tested under the action of symmetrical and non-symmetrical loads. In non-symmetrical loading case loaded part moves downwards, but unloaded part moves upwards (Fig. 23). Therefore in non-symmetrical loading case displacements upwards and downwards are decisive. Total displacements were calculated as sum of upward and downward displacements. In symmetrical loading case maximum vertical displacements downwards are decisive.

The displacements were calculated analytically by FEM program Lira 9.6 [24] and were compared with experimental results. Scheme of deformation of cable truss model is shown on the Fig. 26.

The maximum vertical displacements are generalized in Table III. Difference between displacements of model of prestressed cable truss with single cable top chord and prestressed cable truss model with cable truss top chord or cable truss advantage for experimental and analytical results are shown in Table IV.
TABLE III
EXPERIMENTAL AND ANALYTICAL (IN BRACKETS) RESULTS OF DISPLACEMENTS OF THE MODELS

<table>
<thead>
<tr>
<th>Loading scheme</th>
<th>Deck displacements downwards, mm</th>
<th>Deck displacements upwards, mm</th>
<th>Deck displacements total, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetrical load</td>
<td>14.71 (14.09)</td>
<td>23.81 (22.41)</td>
<td>37.64 (34.84)</td>
</tr>
<tr>
<td>Non-symmetrical load</td>
<td>15.06 (14.15)</td>
<td>13.83 (12.43)</td>
<td>21.04 (19.33)</td>
</tr>
</tbody>
</table>

The displacements for cable and cable truss in symmetrical loading case are equal to 15 mm, but in unsymmetrical case total displacements are 37 and 32 mm for single cable and cable truss respectively. The results of our experiment are close to analytical results. The experimental results shown, that replacing single cable with cable truss with cross web allows to decrease vertical displacements upwards by 16% and downwards by 12%, total displacements by 13%, that is a little bit smaller, that by analytical calculations. That can be described by not accurate web element placement.

VII. CONCLUSIONS

Rational structure of the cable truss for prestressed cable truss top chord was developed. It was stated, that usage of cable truss with the cross web as a top chord of prestressed cable truss instead of single cable allows to reduce vertical displacements upwards by 34%, downwards by 8% and total displacements by 16% in the case of non-symmetrical load.

Single cable and cable truss model work was discovered. Achieved results are close to analytical ones. Model testing results indicate, that usage of cable truss with the cross web as a top chord of prestressed cable truss instead of single cable allows to reduce vertical displacements upwards by 16%, downwards by 12% and total displacements by 13% in the case of non-symmetrical load.

ACKNOWLEDGMENTS

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REFERENCES

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Vadims Goremikins was born in Riga, Latvia on 6th February 1986. He is a doctoral study student. He received his professional master degree (M.Sc.Ing) in Civil Engineering from the Riga Technical University, Latvia in 2009. He graduated as a civil engineer and holds a professional bachelor degree in civil engineering (B.Sc.Ing) from the Riga Technical University on 2008. He currently works as Assistant of scientific work at the Riga Technical University, and as Assistant in the Building Structures department at the Riga Technical University. He was Internal Control Specialist, Guarantee repair Specialist, Civil engineer assistant at “MTK Construction” Ltd.M.Sc.Ing. Goremikins is a member of the Association of Latvian Young Scientists.

Karlis Rocens was born in Riga, Latvia on 3rd March 1939. He is a professor of structural engineering and director of the Institute of Structural Engineering and reconstruction at the Riga Technical University, Latvia. He is a Full member of Latvian academy of sciences and participant from Latvia in COST C25 “Sustainability of Constructions: Integrated Approach to Life Time Structural Engineering”. Author of 5 monographs and more than 250 scientific articles. His research interests include the modern structures, technological mechanics of wood and composite materials and structural material science.