Abstract—A model based fault detection and diagnosis technique for DC motor is proposed in this paper. Fault detection using Kalman filter and its different variants are compared. Only incipient faults are considered for the study. The Kalman Filter iterations and all the related computations required for fault detection and fault confirmation are presented. A second order linear state space model of DC motor is used for this work. A comparative assessment of the estimates computed from four different observers and their relative performance is evaluated.

Keywords—DC motor model, Fault detection and diagnosis, Kalman Filter, Unscented Kalman Filter

I. INTRODUCTION

Fault Detection and Diagnostic (FDD) techniques to identify faults in electric machines typically use motor current signature analysis, and vibration monitoring [1]. These methods are primarily used for induction machines as they are the main workhorse of industry.

A model based fault detection and diagnostics methodology is proposed in this paper, where a second order state variable model, with armature current \( I_a \), and rotor speed \( \omega \) as the state variables, is used [2]. Model based fault detection algorithms are based on the use of stochastic observers such as Kalman filter, whereas conventional fault detection strategy use signal based methods. The comparison of the estimated state variable and actual measurements, termed residues, provide the information required to judge the nature, time of occurrence and severity of faults. The variations in the residues thus generated, are used for detecting and identifying different type of faults. A comparative study on the use of four variants of Kalman Filters for fault detection of DC machine is presented.

II. MODEL BASED FAULT DETECTION USING OBSERVERS.

Observer based fault detection method use dynamic modeling of the physical system. Deployment of stochastic observers such as Kalman filter needs precise tuning by appropriate selection of state noise matrix (system uncertainty) and measurement noise matrix, in order to ensure faster convergence. In addition, the model needs to satisfy controllability and observability criteria and the noise terms are assumed to be Gaussian.

The state space motor model chosen for this study comprises of a linear time invariant second order system. The two states considered are the armature current and the speed of the DC motor. The input is 12 V DC and the output considered here are the same as the states.

A. Continuous time state variable DC motor model.

The continuous time state variable model of DC motor, used for this work is,

\[
X = AX + BU \\
Y = CX;
\]

\[
X = \begin{bmatrix} I_a \\ \omega \end{bmatrix}
\]

\[
A = \begin{bmatrix} -\frac{R_m}{L_m} & -\frac{K_b}{L_m} \\ \frac{K_t}{J_m} & -\frac{B_m}{J_m} \end{bmatrix}
\]

\[
B = \begin{bmatrix} 1/L_m \\ 0 \end{bmatrix}
\]

where, 
\( I_a \) is the armature current and \( \omega \) is the rotor speed,
\( R_m = 2.06 \, \Omega \), Armature resistance;
\( L_m = 0.238 \, mH \), Inductance
\( K_b = 0.02352 \), Back EMF constant (Volt-sec/rad)
\( K_t = 0.0235 \), Torque constant (Nm/A)
\( J_m = 1.07e-6 \), Rotor inertia (Kg m²)
\( B_m = 12e-7 \), Mechanical damping factor

For continuous time system, the characteristic matrices are as per equation (7). Fig. 1 shows the linear response of first state, \( I_a \). Starting current reaches up to 5.5 amps and settles to a steady state value of 0.25 amps.

![Fig. 1. DC motor current, \( I_a \) Vs Time](image-url)
B. Discrete time state variable model

The continuous time model as described by equations (1) and (2) is discretised using the Matlab® ‘c2d’ function. The system matrices are as shown below.

\[
X(k+1) = A_dX(k) + B_dU(k) \quad (8)
\]

\[
Y(k) = C_dX(k) \quad (9)
\]

The discrete time model is as described by equations (8) and (9). The sampling time is taken as 10 microseconds

\[
A_d = \begin{bmatrix}
0.917 & -0.0009 \\
0.2104 & 0.9999
\end{bmatrix} B_d = \begin{bmatrix}
0.4 & 0.25 \\
0.04 & 0.48
\end{bmatrix} C_d = \begin{bmatrix}
1 & 0
\end{bmatrix} \quad (10)
\]

Equation (10) gives the discretised system matrices. The time response for discretised model is as shown in fig. 3. The speed is scaled for ‘Y’ axis normalization to show both variables.

Measurement update equations are,

\[
K_k = P^{-1}_k C_d^T \left[ C_d P_k C_d^T + R \right]^{-1} \quad (13)
\]

\[
\hat{X}(k) = \hat{X}(k-1) + K_k [Z(k) - C_d \hat{X}(k)] \quad (14)
\]

\[
P_k = (I - K_k C_d) P_k \quad (15)
\]

where \( K_k \) is the Kalman gain, \( P_k \) is the posterior co-variance and \( P^{-1}_k \) is the prior co-variance. ‘Q’ is the state noise (system uncertainty) and ‘R’ is the measurement noise.

The Kalman Filter iterations are as given by a set of time update equations (11, 12) and another set of measurement update equations (13-15). It is assumed that the dc motor current (measurement) is corrupted by noise, whose distribution is Gaussian. Typical noise signal is shown in fig. 4, which is generated through Matlab® using ‘randn’ function, for the required number of samples.

Kalman filter is a recursive computational algorithm with a set of prediction (time update) and correction equations (measurement update). Proper selections of the ‘Q’ and ‘R’ matrices are very important for optimal filter performance. It

III. APPLICATION OF KALMAN FILTER TO DC MOTOR

Kalman filter is an optimal stochastic observer which is widely used for state estimation even if the measurement data is corrupted by random noise, which is Gaussian [3-4]. Here in this case, the observation or the measurement is the armature current of DC motor. The Kalman filter equations are represented as,

\[
\hat{X}(k+1) = A_d \hat{X}(k) + B_d U(k) \quad (11)
\]

\[
P^{-1}_k = A_d P_k A_d^T + Q \quad (12)
\]

Fig. 4. Noise signal with Gaussian distribution.

Fig. 5 shows a motor armature current measurement, which is obtained by the model response and corrupted by a noise signal as already mentioned. By suitable application of Kalman filter, the actual current can be estimated.
can lead to improper results and may show convergence issues if the same is not carefully initialised. Fig. 6 shows the estimated armature current of DC motor by Kalman Filter algorithm even with corrupted measurements.

Fig. 6. KF estimation of armature current

Fig. 7 shows the estimated current and scaled speed. From this, it is evident that in spite of measurement disturbances, KF can estimate the actual states very accurately.

Fig. 7. KF estimation of DC motor current and speed

Along with the Kalman filter iterations, the residue generated during computations can give an insight into the healthiness of the motor parameters. The residue generated is a white noise under healthy conditions. In this case, an open loop system is considered for fault detection. The same can be extended to other parametric variation also.

Fig. 8 shows the current residue signal in this case, when there is no fault. It can be observed that the residue has a zero mean Gaussian distribution.

Fig. 8. Current residue computed during KF iterations.

A case is considered, where there is a bias fault in measurements. For certain period during the measurements, typically for 1000 samples, the current measurement is biased by a step change. Fig. 9 shows a bias fault in the measurements.

Fig. 9. Shift in current residue due to a bias fault.

As can be seen from fig. 10, in spite of the fault, the Kalman filter estimates both armature current and speed.

Fig. 10. KF estimates of current and speed after the fault.

IV. FAULT DETECTION AND CONFIRMATION IN DC MACHINES

Fault detection in DC motor is based on the innovations generated during Kalman filter iterations. The test statistic
based on innovations at each time sample used for fault detection is given by [5-6],

\[ \varepsilon(k) = \gamma^T(k) V^{-1}(k) \gamma(k) \]  

(12)

where \( V(k) \) represents error covariance matrix and is computed as,

\[ CP_{k/k-1} C^T + R \]  

(13)

The test statistic \( \varepsilon(k) \) follows a central Chi-square distribution with \( n \) degrees of freedom [5]. A fault flag is set at any time, 't' when the test statistic \( \varepsilon(k) \) exceeds the test criterion, with an assumed level of significance. The rejection of the null hypothesis by Fault Detection Test (FDT) at time 't' indicates that a fault might have occurred. On detection of a fault, in order to avoid a false alarm, a fault confirmation test can be initiated. After N sampling instances, a confirmatory statistical test is done by making use of all innovations in the time \( [t, t+N] \), as shown below.

\[ \varepsilon(N,t) = \sum_{k=t}^{t+N} \gamma(k)^T V^{-1}(k) \gamma(k) \]  

(14)

The occurrence of a fault at time 't' is confirmed if the test statistic \( \varepsilon(N,t) \) exceeds the test criteria. During the Kalman filter computations, the test statistic also is computed, which can be checked at each sampling instances for a threshold. The value of threshold is decided by the level of significance, for a particular confidence level, which is taken as 95 in this case. This means, if the hypothesis test reject a test statistic at a specific sampling instance, it can be statistically interpreted that the sample is well within the dataset in a window. The shift in residue due to the bias fault in current sensor is shown in fig. 11.

Fig. 11. Observed current by KF and innovations

The bias fault is simulated by adding a fixed value to the measurements corresponding to a set of 1000 samples. From the figure, it is evident that for the period of no fault, the residue is a white noise. The shift in residue and hence presence of a fault need to be detected with the help of a test statistic. The test statistic as per (12) need to be evaluated at every sample and gives a positive value. Fig. 12 shows the test statistic as computed by (12), where the 3001\(^{th}\) sample is not rejected by hypothesis test, indicating a fault, which has occurred after 3000\(^{th}\) sampling instant.

Fig. 12. Test statistic for fault detection

Subsequently, a fault confirmation test can be launched to check for any transient faults and not to trigger a false alarm. The fault confirmation test is used as a deterrent for false alarm. Fig. 13 shows the flowchart for fault detection and confirmation algorithm.

Fig. 13. Flow chart for fault detection and confirmation

A comparative assessment of four observers and their performance for DC motor state estimation and fault detection is given here. The observers are KF and its three variants namely Extended Kalman filter, Unscented Kalman filter and Central difference Kalman filter. In the second part of this paper, an induction motor case study also is included, where the results are different for all these filters. The computations
were performed using Matlab® and ReBEL®, for comparative evaluation of the performance of different variants of Kalman filter. The bias fault was generated by intentionally corrupting the measured data for a set of 1000 samples. Fig. 14 and 15 shows state estimates, both current and speed by four different variants of Kalman filter. Fig. 16 shows the response of the same bank of filters to a bias fault in current sensor.
V. CONCLUDING REMARKS

An observer based study on fault detection and diagnosis in DC motors is presented in this paper, where a second order self excited DC motor model is used. Kalman filter algorithm along with residue estimates and hypothesis testing is used for fault detection where shift in residues for current signal is observed for detection. Incipient soft fault is detected and confirmed by statistical hypothesis test performed on this signal. This method has the advantage that it takes care of model and measurement uncertainties, and is independent of input variations. The model is machine specific and Kalman filter and its variants require extensive tuning to account for model uncertainties. This comparative study brings out the relative advantages of observers in fault detection and diagnosis domain and enables to select the right observer for this application. In the case of DC motor, the Kalman filter has shown good performance and the other variants have not offered any significant improvement even in faulty conditions.

REFERENCES


Padmakumar. S is presently working as Engineer in charge of electrical systems refurbishment cell, research reactor maintenance division, BARC. He is associated with electrical systems maintenance of nuclear research reactors. He graduated from Kerala University in 1985 and completed his post gra graduation from IIT Bombay in 1987. He is Presently undergoing his Ph.D program at IIT Bombay.

Vivek Agarwal received the bachelor’s degree in Physics from St. Stephen’s college, Delhi University, India. He then obtained an integrated master’s degree in Electrical Engineering from Indian Institute of Science, Bangalore. Subsequently he pursued a Ph.D. degree in the dept. of Electrical and Computer Engineering, University of Victoria, Canada. After obtaining the Ph.D. degree, he briefly worked for Statpower Technologies, Burnaby, Canada as a research engineer. In 1995 he joined the department of Electrical Engineering, Indian Institute of Technology-Bombay, where he is currently a Professor.

Kallol Roy completed his B.Tech (Electrical) from REC (NIT), Calicut in 1984. He obtained his M.Tech. (Electronics Design) from IISc., Bangalore in 1990 and PhD (System and Control) from IIT, Powai, Mumbai in 2000. He has been Post-Doctoral Fellow at University of Alberta, Canada, from April 2005 to March 2006. His expertise is in the Maintenance of Research Reactors, especially Planning, Retrofitting and Development of Predictive Maintenance tasks.