

M-ary Chaotic Sequence Based SLM-OFDM System for PAPR Reduction without Side-Information

A.Goel, M. Agrawal, P. Gupta Poddar

Abstract—Selected Mapping (SLM) is a PAPR reduction technique, which converts the OFDM signal into several independent signals by multiplication with the phase sequence set and transmits one of the signals with lowest PAPR. But it requires the index of the selected signal i.e. side information (SI) to be transmitted with each OFDM symbol. The PAPR reduction capability of the SLM scheme depends on the selection of phase sequence set. In this paper, we have proposed a new phase sequence set generation scheme based on M-ary chaotic sequence and a mapping scheme to map quaternary data to concentric circle constellation (CCC) is used. It is shown that this method does not require SI and provides better SER performance with good PAPR reduction capability as compared to existing SLM-OFDM methods.

Keywords—Orthogonal frequency division multiplexing (OFDM), Peak-to-average power ratio (PAPR), Selected mapping (SLM), Side information (SI)

I. INTRODUCTION

OFDM, a multicarrier modulation technique, has high spectral efficiency, good immunity to the interference and multipath effects in wireless channels[1]. It is widely used for high speed wireless communication systems. The main difficulty of using OFDM signal, is its large envelope fluctuation, which results in high PAPR. Various PAPR reduction techniques have been proposed in the literature [2]. Selected Mapping (SLM) provides a good PAPR reduction capability without distorting the shape of the OFDM signal. A large number of phase sequence sets have been proposed by many researchers for SLM-OFDM systems, these include Newman[3], Hadamard[4], Shapario-Rudin[5], chaotic sequences[5] and Riemann sequence[6] etc. But all of these techniques[3]-[6] require side information (SI), i.e. the index of the selected signal required to be transmitted with each OFDM symbol, which decreases the data rate of SLM-OFDM system. In this type of OFDM system, SI is required at the receiver to retrieve the original OFDM signal. Therefore, SI has the prime importance and if it gets corrupted then the symbol error rate (SER) performance of the SLM-OFDM system degrades drastically. To eliminate the requirement of SI at the receiver, various SI embedding schemes[7]-[10] have been proposed by researchers. In many of these techniques [7]-[9], the accuracy of SI detection depends on the signal-to-noise ratio (SNR). Therefore, when a SLM-OFDM system is operated at low SNR, then SER performance of the system becomes very poor.

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In this paper, a new phase sequence set generation based on the M-ary chaotic sequence has been proposed for achieving better PAPR reduction capability without SER performance degradation. Further, to eliminate the requirement of SI at the receiver, concentric circle constellation mapping [10] has been suggested. It has been shown that the proposed method provides good PAPR reduction capability and has better symbol error rate (SER) performance than its counterpart[3]-[6], other SLM systems.

The remainder of the paper is organized as follows. Section II describes the OFDM system and PAPR. In section III, we discuss the existing SLM based techniques and also present the SER analysis of Riemann matrix based phase sequence set[6]. Section IV deals with the proposed approach for PAPR reduction without side information. The results obtained from computer simulations and their discussions are carried out in section V. Finally, we conclude in section VI.

II. OFDM SYSTEM AND PAPR

In an OFDM system, a serial data stream is first converted into N parallel data streams, then binary data available on these parallel streams are mapped to desired constellation points (M-ary Phase Shift Keying or M-ary Quadrature Amplitude Modulation). Let $X = \{X_r\}_{r=0}^{N-1}$ be the N complex symbols to be transmitted over N narrow band subcarriers of an OFDM system. The time domain baseband OFDM signal at the transmitting end is given by a single IDFT operation as follows

$$x[n] = \frac{1}{\sqrt{N}} \sum_{r=0}^{N-1} X_r \exp\left(\frac{j2\pi rn}{N}\right) \quad 0 \leq n \leq N-1 \quad (1)$$

where N is the total number of subcarriers and X_r is the transmitted data symbol on r^{th} subcarrier.

PAPR of OFDM signal: The PAPR is the ratio of peak power to the average power of OFDM signal. For discrete-time OFDM signal ($x[n]$), it is defined as

$$PAPR \text{ (dB)} = 10 \log_{10} \left(\frac{\max \{ |x[n]|^2 \}}{E \{ |x[n]|^2 \}} \right) \quad (2)$$

where $E\{\cdot\}$ is the expectation operator, $\max\{\cdot\}$ gives the maximum value and $|\cdot|$ gives the absolute value.

From central limit theorem, it follows that, for large values of N , the real and imaginary part of $x[n]$ become Gaussian distributed, each with zero mean and variance $E[|x[n]|^2]/2$. Thus the magnitude of OFDM signal has Rayleigh distribution,

while its power has chi-square distribution [11]. The cumulative distribution function (CDF) of an OFDM signal [11] with N subcarriers is given by

$$CDF(\gamma_0) = \Pr(PAPR(x[n]) \leq \gamma_0) \quad (3)$$

Therefore, the complementary cumulative distribution function (CCDF) of an OFDM signal becomes

$$CCDF(\gamma_0) = \Pr(PAPR(x[n]) > \gamma_0) \quad (4)$$

$$= 1 - (1 - \exp(-\gamma_0))^N \quad (5)$$

where γ_0 is the given threshold value of PAPR. In order to obtain the more precise CCDF of PAPR, an oversampling by a factor $\alpha > 1$ has to be performed and the CCDF given by (5) changes to

$$CCDF(\gamma_0) = 1 - (1 - \exp(-\gamma_0))^{\alpha N} \quad (6)$$

III. SLM BASED TECHNIQUES

In conventional SLM scheme, frequency domain OFDM signal $X = \{X_r\}_{r=0}^{N-1}$, is multiplied with the phase sequence set containing U different phase sequences of length N and generate U alternative signals $\{X_r^u\}_{r=0}^{N-1} = \{X_r \cdot P_r^u\}_{r=0}^{N-1}$, $1 \leq u \leq U$, where phase factor, $P_r^u \in \left\{ \phi_l = \exp\left(\frac{j2\pi l}{M}\right) \mid 0 \leq l \leq M-1 \right\}$. The generated U alternative frequency domain OFDM signals $X^u = [X_0^u, X_1^u, X_2^u, \dots, X_r^u, \dots, X_{N-1}^u]$, $1 \leq u \leq U$ are independent in nature. The corresponding time domain OFDM signal x^u , for $1 \leq u \leq U$ is given by

$$x^u(n) = \frac{1}{\sqrt{N}} \sum_{r=0}^{N-1} X_r^u \exp\left(\frac{j2\pi nr}{N}\right), \quad 0 \leq n \leq N-1 \quad (7)$$

One out of U alternative OFDM signals x^u , $1 \leq u \leq U$, with lowest PAPR is selected for transmission, along with the index u . The block diagram of conventional SLM-OFDM system is shown in Fig. 1.

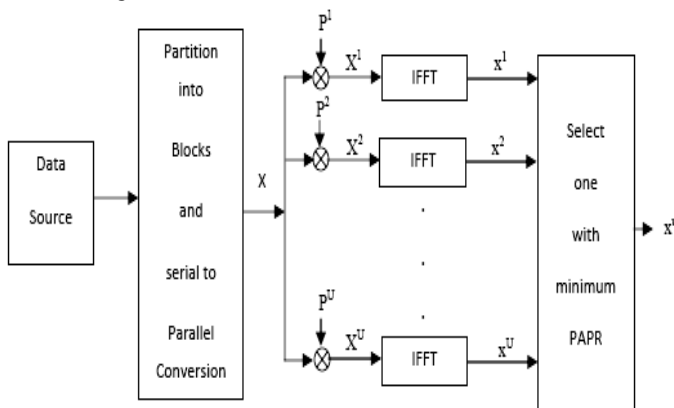


Fig. 1 Block diagram of SLM-OFDM system

The selection of a phase sequence set to achieve better PAPR reduction capability is a crucial aspect in SLM-OFDM system. In [3], for $M=4$, a phase sequence set is generated randomly from set $\{\pm 1, \pm j\}$, for achieving good PAPR reduction capability, but it requires large number bits to encode the SI, as there are M^N possible combinations of phase sequences and any one of the them can be generated randomly. Therefore, $\log_2(M^N)$ bits per OFDM symbol are required to encode the SI, which results in high data rate loss.

In [4], the PAPR reduction is performed by choosing Hadamard matrix as phase sequence set, but its PAPR reduction capability is very limited. The binary chaotic sequence based phase sequence set generation scheme is proposed in [5], its PAPR reduction capability is better than Hadamard sequence[4] and Shapario-Rudin [5] sequence. The PAPR reduction capability of SLM-OFDM system has been further improved by using rows of Riemann matrix as a phase sequence[6]. Before going into the details of the SER analysis of the Riemann matrix based phase sequence set, first we discuss the construction and some important facts of Riemann matrix, these are as follows:

The Riemann matrix [12] (R) of size $N \times N$ is obtained by removing the first row and first column of matrix A of size $(N+1) \times (N+1)$,

$$A(i, j) = \begin{cases} i-1 & \text{if } i \text{ divides } j \\ -1 & \text{otherwise} \end{cases} \quad (8)$$

The elements of v^{th} row in Riemann matrix (R) are either v or -1 , $1 \leq v \leq N$. In a v^{th} row of Riemann matrix (R), $C = \left\lfloor \frac{N}{v+1} \right\rfloor$ number of elements has a value v and remaining elements are -1 . Therefore, when a v^{th} , $2 \leq v \leq N$ row of this matrix R is used as a phase sequence P^v , it results in not only phase change but also the amplitude of the modulated data symbols. Therefore, the average transmitted power of alternative frequency domain OFDM signal, X^u , will not be same as original OFDM signal X . Here, it has also been shown mathematically, when a frequency domain OFDM signal is multiplied by v^{th} row, $2 \leq v \leq U$ of Riemann matrix (R), then average power of resulted frequency domain OFDM signal increases. In the case of M-PSK constellation, let the data symbols (before multiplication with the phase sequence) have average symbol energy E_s . Assume that the v^{th} row, $2 \leq v \leq U$ of Riemann matrix (R), is the best available phase sequence for a particular OFDM signal to provide the least possible PAPR. Therefore, after multiplication with v^{th} row, the symbol energy of C symbols is increased by a factor of v^2 and the symbol energy of remaining symbols remain unchanged to E_s . The average energy of alternative OFDM signal (original frequency domain

OFDM symbol after multiplication with v^{th} phase sequence) can be calculated as

$$E_{av} = \frac{E_s(N - C) + Cv^2 E_s}{N} \quad (9)$$

If each of the M-PSK modulated subcarrier is transmitted over AWGN channel, the average error probability P_e (SER) obtained with Riemann matrix phase sequence in absence of any SI detection error is given by

$$P_e = \left(\frac{N - C}{N}\right) P_{e1} + \left(\frac{C}{N}\right) P_{e2} \quad (10)$$

where P_{e1} and P_{e2} are the probability of errors over AWGN channel for subcarriers with symbol energy E_s and $v^2 E_s$ respectively. The error probabilities P_{e1} and P_{e2} are given by

$$P_{e1} = \text{erfc} \left(\sqrt{\frac{E_s}{N_o}} \sin \left(\frac{\pi}{M} \right) \right) \quad (11)$$

$$P_{e2} = \text{erfc} \left(\sqrt{\frac{v^2 E_s}{N_o}} \sin \left(\frac{\pi}{M} \right) \right) \quad (12)$$

After substituting the values of P_{e1} and P_{e2} from (11) & (12) into (10), we have

$$P_e = \left(\frac{N - C}{N}\right) \text{erfc} \left(\sqrt{\frac{E_s}{N_o}} \sin \left(\frac{\pi}{M} \right) \right) + \left(\frac{C}{N}\right) \text{erfc} \left(\sqrt{\frac{v^2 E_s}{N_o}} \sin \left(\frac{\pi}{M} \right) \right) \quad (13)$$

After substituting the value of average symbol energy (before multiplication with phase sequence) E_s from (9) into (13), we have

$$P_e = \left(\frac{N - C}{N}\right) \text{erfc} \left[\sqrt{\frac{NE_{av}}{(N - C) + Cv^2} \sin \left(\frac{\pi}{M} \right)} \right] + \left(\frac{C}{N}\right) \text{erfc} \left[\sqrt{\frac{v^2 NE_{av}}{(N - C) + Cv^2} \sin \left(\frac{\pi}{M} \right)} \right] \quad (14)$$

The results for the average probability of error (SER) obtained from (14) for $v = 4$ & 8 are shown in Fig. 2. It can be easily observed from the Fig. 2 that the average probability of error increases with v , $2 \leq v \leq U$. The main reason behind it is that the data symbols $X_0^u, X_1^u, X_2^u, \dots, X_{N-1}^u$ are not having equal powers and the symbols with low powers can easily be corrupted during the transmission over AWGN channel. Therefore, SER performance degrades by increasing the value of U .

In the below analysis we assumed that SI is known to the receiver or detected without error. But in general, exact SI is required at the receiver to recover the original OFDM signal.

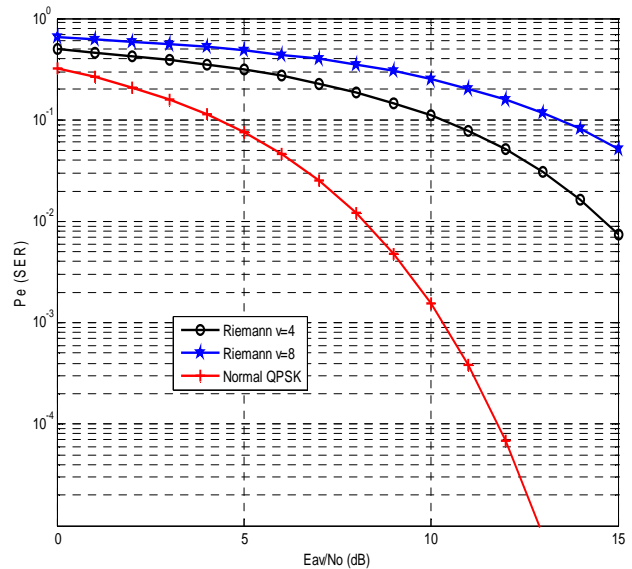


Fig. 2 SER performance of Riemann matrix phase sequence $v = 4$ & 8

Because, the received signal has to be multiplied with the reciprocal of the phase sequence P^u (used by the transmitter to provide the least PAPR). The SER performance of scheme [6] can be further degraded if SI gets corrupted. Therefore, the main drawback of existing SLM-OFDM techniques [3]-[6] is the requirement of side information (SI) and if SI detection is incorrect, it tremendously degrades the SER performance of the system.

In order to avoid this problem, various SI embedding techniques [7]-[9] have been proposed in the literature. A scrambling based method [7] proposed by Breiling et al. and maximum-likelihood decoding [8] proposed by Jayalath et al., are used to eliminate the requirement of SI for SLM-OFDM system. In [9], SI embedding is performed by increasing the amplitude level of certain numbers (locations) of subcarriers. The index of the subcarriers with higher amplitude have one to one mapping with the index u , which is the desired SI. At the receiver of a SLM-OFDM system, a technique based on maximum-likelihood detection is used to retrieve the SI. However, in this method [9] SI detection error varies with signal-to-noise ratio. Therefore, we have proposed a scheme, which completely eliminates the requirement of SI.

IV. PROPOSED METHOD

In the proposed scheme, a new phase sequence set based on M -ary chaotic sequence is generated to reduce the PAPR, and concentric circle constellation (CCC) [10] based mapping is used to eliminate the requirement of SI. In the proposed approach, first binary data is converted into the quaternary data (0,1,2 & 3) stream and then concentric circle constellation (CCC) [10] based mapping as given in Table 1 is used to obtain the data symbols, after that conventional SLM scheme along with subcarrier modulation is applied for PAPR reduction. At the receiver, after performing the subcarrier demodulation, CCC decoding and a de-mapping operation as given in the Table II, has also to be performed to retrieve the

original data without any SI. Detailed description of proposed phase sequence set generation and CCC mapping is given in the following sub section.

A. Generation of phase sequence set

Chaotic sequence[5] is a non-converging and non-periodic sequence that exhibits random behavior. The *M*-ary chaotic sequence $C_n \in \{0, 1, 2, \dots, M-1\}$, $0 \leq n \leq N-1$ of length *N* is generated by

$$C_n = \left\lfloor \frac{My_{n+1}}{2} \right\rfloor + \frac{M}{2}, \quad y_{n+1} = f(y_n) = 1 - \alpha y_n^2, \quad \alpha \in [1.4015, 1.99], \quad y_n \in (-1, 1) \quad (15)$$

where, $\lfloor z \rfloor$ is greatest integer not exceeding to *z* and from C_r , the *r*th element of first phase sequence becomes $P_r^1 = \exp(j2\pi C_r / M) = \phi_{C_r}$, e.g., if $C_2 = 3$, then 2nd element of the first phase sequence $P_2^1 = \phi_3$. Therefore, the first phase sequence is constituted as $P^1 = [\phi_{C_0}, \phi_{C_1}, \phi_{C_2}, \dots, \phi_{C_{N-1}}]$ and *u*th phase sequence P^u , $2 < u \leq U$ can be easily obtained from P^1 after *u* - 1 right circular shifts.

B. Proposed SLM-OFDM System & Mapping Scheme

In the proposed scheme, first the original bit stream is transformed into quaternary data points 0, 1, 2 & 3. After that these quaternary data points {0, 1, 2 & 3} has to be mapped to four different points of concentric circle constellation according the mapping scheme presented in Table I and shown in Fig.3. In SLM-OFDM scheme a block of *N* modulated data symbols is multiplied with *M*-ary chaotic sequence based phase sequence set P^u , $1 < u \leq U$ and generate *U* alternative signals $X^u = [X_0^u, X_1^u, X_2^u, \dots, X_r^u, \dots, X_{N-1}^u]$, $1 \leq u \leq U$ as per (1). One of the alternative OFDM signal (x^u), with lowest PAPR is selected for transmission. With this mapping scheme, data symbols before multiplication with the phase sequence $X_r \in \{0 + j0, 0 + j2, -4 + j0 \& 2\sqrt{2}(1 - j)\}$. For *M*=4 the elements of chaotic phase sequence set takes the values from set $\{1, j, -1, -j\}$, therefore corresponding alternative frequency domain OFDM signal X^u will take the values as per the last column of Table I and generate 13 distinct points of concentric circle constellation (CCC).

The decoding of received data symbols is performed by minimum distance decoding rule. All 13 points are distinct; therefore they can be decoded uniquely. To recover the original quaternary data signal, de-mapping of the decoded signal is performed according to the scheme presented in Table II.

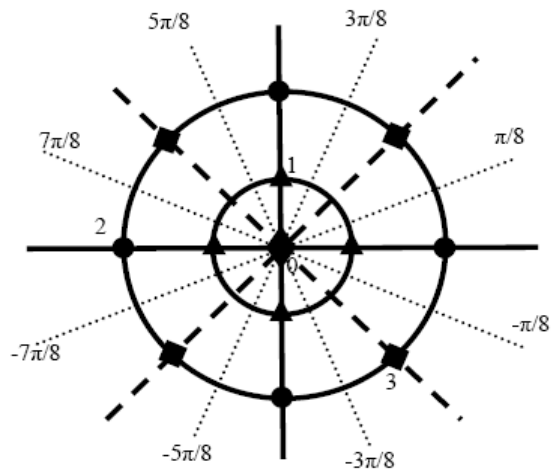


Fig. 3 Concentric circle constellation and mapping of quaternary data symbols

TABLE I
 MAPPING OF QUATERNARY DATA POINTS TO CONCENTRIC CIRCLE CONSTELLATION

Quater-nary data points	Mapped Constellation Points (X_r)	Constellation symbols (X_r) after multiplication with phase factors $\{1, j, -1, -j\}$
0	$0 + j0$	$\{0 + j0\}$
1	$0 + j2$	$\{j2, -2, -j2, 2\}$
2	$-4 + j0$	$\{-4, -j4, 4, j4\}$
3	$2\sqrt{2}(1 - j)$	$\{2\sqrt{2}(1 - j), 2\sqrt{2}(1 + j), 2\sqrt{2}(-1 + j), 2\sqrt{2}(-1 - j)\}$

TABLE II
 DE- MAPPING OF CONCENTRIC CIRCLE CONSTELLATION SYMBOLS TO QUATERNARY DATA POINTS

Demodulated Constellation symbols	De-mapped Constellation Point	Recovered Quaternary data point
$\{0 + j0\}$	$0 + j0$	0
$\{j2, -2, -j2, 2\}$	$0 + j2$	1
$\{-4, -j4, 4, j4\}$	$-4 + j0$	2

$$\{2\sqrt{2}(1-j), 2\sqrt{2}(1+j), 2\sqrt{2}(1-j), 2\sqrt{2}(-1+j), 2\sqrt{2}(-1-j)\}$$

Hence, original data can be easily recovered without requiring any side information. Moreover, in the proposed scheme phase set sequence $P^u, 1 < u \leq U$ does not produce amplitude change in any modulated subcarrier of OFDM signal $X = [X_1, X_2, \dots, X_r, \dots, X_N]$. Therefore the power of $X_1^u, X_2^u, X_3^u, \dots, X_N^u$ will remain same and hence the SER performance remains unchanged with U .

V. PERFORMANCE EVALUATION

Here, we considered an OFDM system with $N = 64$ orthogonal subcarriers and 10,000 OFDM symbols to evaluate the SER and PAPR performance of the proposed scheme. The PAPR reduction capability of the chaotic sequence remains same for $\forall y_0 \in (-1, 1)$ and $\forall \alpha \in [1.4015, 1.99]$ [5]. In this paper, we have used $y_0 = 0.1, \alpha = 1.99$ to generate M -ary chaotic sequence. Fig. 4 shows the PAPR reduction capabilities of Hadamard sequence[4], Riemann matrix phase sequence set[6] and proposed scheme called ' M -ary chaotic phase sequence' for $U = 8 \& 32$. The complementary cumulative distribution function (CCDF) of original OFDM signal (without PAPR reduction) is also shown for comparison. Fig. 5 shows the SER performance comparison for above systems i.e. Riemann matrix phase sequence and M -ary chaotic phase sequence based scheme for $U = 8 \& 32$. The phase sequence set which provides low PAPR and low SER is considered to be the best. Riemann matrix phase sequence set for $U = 32$ provides best PAPR reduction capability among all phase sequence sets under consideration but its SER performance even at 20dB signal-to-noise ratio (SNR) is above 10^{-1} , which is unacceptable in any practical communication systems, therefore it can not be a good choice. As seen from Fig. 5 the SER performance of the M -ary chaotic phase sequence with $U = 8 \& 32$ are coinciding and provide significantly low SER = 2×10^{-4} , whereas it degrades in case of the Riemann matrix phase set sequence by increasing the value of U . The PAPR reduction capabilities of Riemann matrix phase sequence and M -ary chaotic phase sequence for $U = 8$ are almost equal but M -ary chaotic phase sequence requires 2dB less SNR to achieve a SER of 2×10^{-4} and hence the suggested sequence provides a better SER performance than Riemann matrix phase set sequence. As seen from Fig.4, the M -ary chaotic phase sequence with $U = 8$ provides a PAPR reduction of 2.8dB where only 1.2dB reduction is provided by Hadamard sequence with $U = 8$ (CCDF of PAPR 0.002). Further, the PAPR reduction capability of the M -ary chaotic phase sequence can be improved by increasing the value of U without affecting the SER performance. Hence the proposed M -ary chaotic phase

sequence based scheme provides low PAPR and SER in comparison to all schemes under consideration.

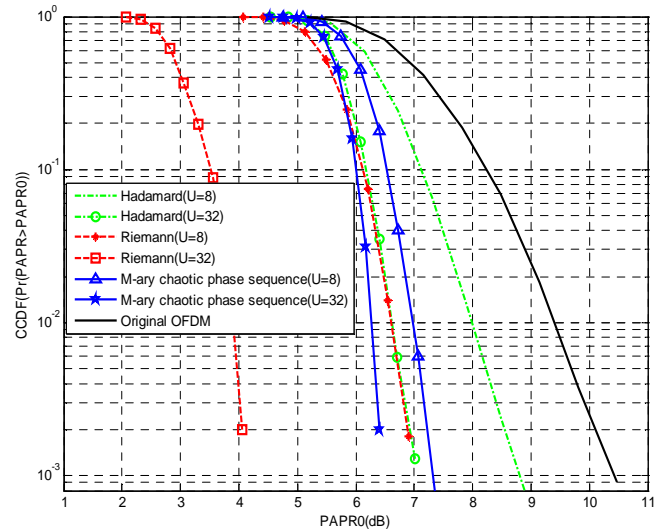


Fig. 4 CCDF of PAPR for different phase sequence sets in SLM-OFDM system

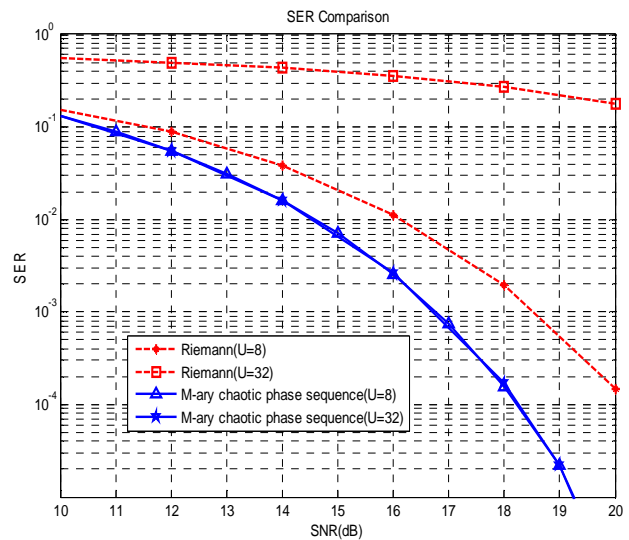


Fig. 5 SER comparison

The SER performance of the proposed technique is also evaluated over fading channel, here we have considered a three tap frequency selective fading channel (SUI-5) and a cyclic prefix of 1/8 of OFDM symbol duration. The results of SER performance over SUI-5 channel for original OFDM signal, Riemann matrix phase sequence and the proposed scheme are shown in Fig. 6. In proposed scheme with $U = 8 \& 32$, the required E_b / N_o , to achieve a SER of 10^{-4} is only 3dB worse than original OFDM signal, but at the same time it provides a PAPR reduction of 2.8dB (CCDF of PAPR 0.002). The required E_b / N_o , in the proposed scheme with $U = 8$, to achieve a SER of 10^{-4} is about 4.5dB better than Riemann matrix based phase sequence with $U = 8$. But, the PAPR reduction capability is merely 0.3 dB worse. The SER performance of the Riemann matrix based phase sequence degrades rapidly by increasing the value of U but it remains

unchanged for the proposed scheme. Hence proposed scheme has better SER performance in comparison to Riemann matrix based phase sequence. The SER performance of the proposed scheme is about 3dB worse than the original QPSK system. But the performance remains unchanged with U .

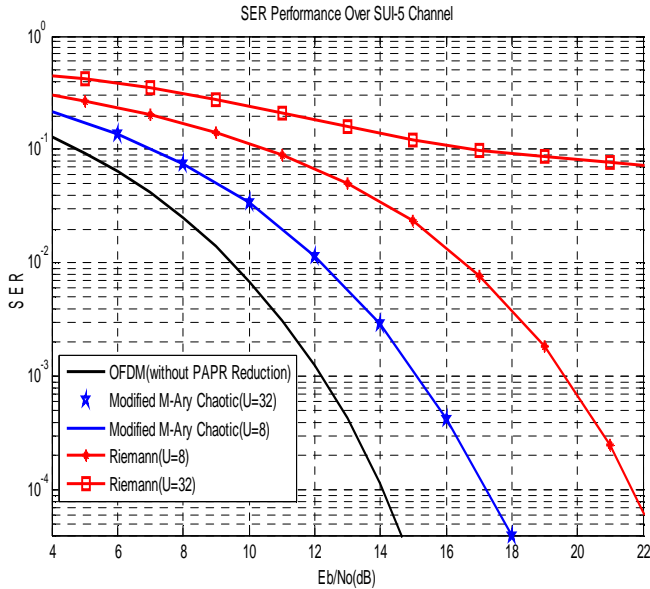


Fig. 6 SER performance comparisons over fading channel (SUI-5)

VI. CONCLUSION

This paper proposes a new scheme for generating phase sequence sets for SLM-OFDM combined with concentric circle mapping, which completely eliminates the need of conveying the side information to the receiver. The method is based on M -ary chaotic sequence, and it is shown that the performance of the SLM-OFDM system does not degrade by increasing the number of sequences generated for PAPR reduction. The overall performance of this method (in terms of SER and PAPR) is best among original OFDM signal, Hadamard sequence and Riemann matrix based phase sequence based schemes. The SER performance of proposed scheme over SUI-5 fading channel is also better in comparison to Riemann matrix based phase sequence.

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