Abstract—A new method, based on the normal shrink and modified version of Katssagelous and Lay, is proposed for multiscale blind image restoration. The method deals with the noise and blur in the images. It is shown that the normal shrink gives the highest S/N (signal to noise ratio) for image denoising process. The multiscale blind image restoration is divided in two sections. The first part of this paper proposes normal shrink for image denoising and the second part of paper proposes modified version of katssagelous and Lay for blur estimation and the combination of both methods to reach a multiscale blind image restoration.

Keywords—Multiscale blind image restoration, image denoising, blur estimation.

I. INTRODUCTION

IMAGE are the main sources of information in different field such as medical imaging, astronomy, public security, and satellite imaging. But, due to some reasons, observed images are degraded which are mainly caused by blur and noise. Therefore, image restoration is necessary. Many conventional approaches have been developed to restore the original image require the prior knowledge of blur and some features of noise [7]. Unfortunately in most cases such knowledge is not available and the blind image restoration should be used to restore original image.

Blind restoration is a far more complicated problem than simple image restoration. One of the most important tools for blind image restoration is wavelets. Wavelets are mathematical functions that cut up data into different frequency components and then each component is studied with different resolution matched to its scale [2] since Gaussian noise is one of the factor that was caused the image was degraded. Then, removing noise is necessary. For removing noise we use from Normal shrink in the scale of wavelet. In the second part of paper, we use from modified method of Katssagelous and Lay for blur estimation. Finally we use both methods for Multiscale blind image restoration.

Several blind restoration algorithms have been proposed in the past. Multiscale Blind image Restoration using a wavelet decomposition was proposed by Sze-Ho in 1996[3]. Spatially adaptive wavelet-based Multiscale image restoration was proposed by Mark.R.banham in 1996[4].

Blind image restoration for MMW radiometer based on wavelet techniques was proposed by Hyuk Park in 2005[1]. Adaptive Wavelet thresholding for image denoising and compression was proposed in 2000 [5] and some surveys can be found in [6].

This paper is structured as follows: in section II the image restoration formulation is reviewed. Section III deals with the wavelet threshold and section IV describes blur estimation. In section V The proposed new method for multiscaled blind image restoration is described. Sections VI and VII describe the experimental results and conclusions respectively.

II. FORMULATION

The process of image degradation is shown in Fig. 1.

\[ Y(x, y) = f(x, y) \ast h(x, y) + V(x, y) \] (1)

Where \( f(x, y) \) represents an original image and \( Y(x, y) \) is the degraded image. \( V(x, y) \) represents an additive noise (usually, noise is Gaussian noise with zero mean). \( h(x, y) \) is the point spread function of blur. For more details, the reader is referred to [7]:

\[ Y(x, y) = f(x, y) \ast h(x, y) + V(x, y) \] (1)

The expression of the equation in the frequency domain by the Fourier transform is

\[ Y(u, v) = F(u, v)H(u, v) + V(u, v) \] (2)

In the blind image restoration we should identify PSF and noise as estimated image get similar original image approximately.

III. WAVELET THRESHOLD

A. Description of Wavelet Threshold

Let the original image be \( \{f(x, y)\}_{x, y = 1, \ldots, N} \) where \( N \) is some integer power of 2. The original image is corrupted by additive noise and one observes

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Where $V(x, y)$ are independent and identically distributed (iid) as normal $N(0, \sigma^2)$ and independent of $f(x, y)$. The goal is to remove the noise from $Y(x, y)$ and to obtain an estimate $(\hat{f}(x, y))$ which minimize the mean square error (MSE)\[8\].

Let $W$ and $W^{-1}$ denote the two dimensional orthogonal discrete wavelet transform (DWT) matrix and its inverse respectively. Then $R=Wy$ represents the matrix of wavelet coefficients of $Y$ having four subbands (LL, LH, HL, and HH)\[9\]. The sub-bands $LH_k$, $HL_k$, $HH_k$ are called the details, where $k$ is the scale varying from $J, ..., 2, 1$ and $J$ is the total number of decompositions. The size of the subbands at scale $k$ is $N/2^k \times N/2^k$. The subbands LLJ is the low-resolution residue.

The wavelet thresholding procedure removes noise by thresholding only the wavelet coefficients of the detail subbands whereas keeping the low resolution coefficients fixed.

![Subbands of the 2-D orthogonal wavelet transform](image)

**IV. BLUR ESTIMATION**

**A. Formulation of Blur Estimation**

In this section we give blur estimation by Katssagelous and Lay\[11\].

\[
E_{j/y}^{(p)}(m, n) = \frac{H^{(p)}(m, n)S_{j/y}^{(p)}(m, n)Y(m, n)}{|H^{(p)}(m, n)|^2 S_{j/y}^{(p)}(m, n) + \sigma_v^{2(p)}}
\]

\[
S_{j/y}^{(p)}(m, n) = \frac{S_f^{(p)}(m, n)\sigma_v^{2(p)}}{|H^{(p)}(m, n)|^2 S_f^{(p)}(m, n) + \sigma_v^{2(p)}}
\]

\[
S_{f}^{(p+1)}(m, n) = S_{j/y}^{(p)}(m, n) + \frac{1}{N^2}E_{j/y}^{(p)}(m, n)
\]

\[
H^{(p+1)}(m, n) = \frac{1}{N^2} Y(m, n)E_{j/y}^{(p)}(m, n)
\]

\[
\sigma_v^{2(p+1)} = \frac{1}{N^2} \sum_{m=1}^{M} \sum_{n=1}^{M} \left| E_{j/y}^{(p)}(m, n) \right|^2 \left| S_{j/y}^{(p)}(m, n) - \frac{1}{N^2}E_{j/y}^{(p)}(m, n) \right|^2 \\
+ \frac{1}{N^2} |Y(m, n)|^2 - 2 \text{Re} \left[ Y(m, n)H^{(p+1)}(m, n)E_{j/y}^{(p)}(m, n) \right]
\]

In (8) $Y(m, n)$ and $E_{j/y}^{(p)}(m, n)$ are respectively the 2D DFT’S (Discrete Fourier Transform) of the observed image...
$Y(x, y)$ and restored image. Re[$\eta$], $\eta^*$ and $|\eta|$ denote real part, complex conjugation and the magnitude of the complex number $\eta$ respectively.

$S_f^{(p+1)}(m, n)$ and $H_f^{(p+1)}(m, n)$ represent respectively the power spectral density of the input signal and the Fourier transform of the PSF at the $(p+1)$ iteration. The iterative algorithm developed in [11]. (Equations 8, 9, 10, 11, and 12) compute the restored image and PSF.

The Blackman-Tukey algorithm was used to compute an estimate of the power spectral density of $Y$, which in turn was used as $S_f^{(0)}$, initial estimate of the power spectral density of $x$. The 2D impulse $h(x, y) = 1\forall x = y = 0$

\[ h(x, y) = 0\forall\text{elsewhere} \]
as $(h^{(0)})$, the initial estimate of PSF.

B. Modified Blur Estimation

In the pervious section we introduce the method of Katssagelous and lay for estimation blur. In this section, we introduced a small modification on pervious method. We suppose the degraded image be

\[ Y(x, y) = f(x, y) \otimes h(x, y) \]

We want to identify PSF. Therefore we modify algorithm as follows

\[ E_{f/Y}^{(p)}(m, n) = \frac{Y(m, n)}{H_f^{(p)}}(m, n) \]

\[ S_f^{(p+1)}(m, n) = \frac{1}{N^2}E_{f/Y}^{(p)}(m, n)^2 \]

\[ H_f^{(p+1)}(m, n) = \frac{Y(m, n)E_{f/Y}^{(p)}(m, n)}{E_{f/Y}^{(p)}(m, n)^2} \]

We can use the equation (15) to identify PSF and then, by using the equation (13) restore image.

VI. NEW METHOD FOR MULTISCALE BLIND IMAGE RESTORATION

In the pervious sections, we identify noise and blur. Let the degraded image be:

$\tilde{Y}(x, y) = f(x, y) \otimes h(x, y) + V(x, y)$

In this section we want to use two pervious methods for blind image restoration.

At first we apply Normal shrink for removing noise($x_1$) then we apply modified method for removing blur (the input of modified blur estimation is output Normal shrink). Finally image restoration($x_2$) was obtained.

VI. EXPERIMENTAL RESULTS

Test 1

Our first test consists of an image of cameraman of size $(256 \times 256)$. The kind of noise is Gaussian with variance 0.05. In the test, Gaussian noise is added to original image. We try to restore image with different methods. The results show median filter and wiener filter denoise weak as well as they remove a lot of details of original image during denoising. Global thresholding use wavelet Transform. In this method, all of subband has one threshold therefore because of lack of exact threshold, the denoised image get blurred. The result of soft and hard thresholding also is not satisfactory. The result shows normal shrink is the best method. Because this method define threshold for every subband besides the threshold is proportional to distribution coefficient of wavelet in every scale.

We can use from MSE and SNR as global measure of objective improvement. The value of MSE represents mean square error and SNR shows the value of removing noise. The equations are as follows:

\[ MSE = 10\log_{10} \left\{ \frac{1}{N^2} \sum_{x,y} \left[ f(x, y) - \hat{f}(x, y) \right]^2 \right\} \]

\[ SNR = 10\log_{10} \left\{ \frac{\sum_{x,y} [f(x, y)]^2}{\sum_{x,y} [f(x, y) - \hat{f}(x, y)]^2} \right\} \]

For an $N \times N$ image, where $f(x, y)$ and $\hat{f}(x, y)$ are the original image and restored image. The results bring in the Table I.

<table>
<thead>
<tr>
<th>Method</th>
<th>MSE</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without filtering</td>
<td>58.3180</td>
<td>8.3127</td>
</tr>
<tr>
<td>Median filter</td>
<td>52.4689</td>
<td>14.1618</td>
</tr>
<tr>
<td>Wiener filter</td>
<td>52.1183</td>
<td>14.5125</td>
</tr>
<tr>
<td>Soft threshold</td>
<td>51.7479</td>
<td>14.8828</td>
</tr>
<tr>
<td>Hard threshold</td>
<td>50.8965</td>
<td>15.7343</td>
</tr>
<tr>
<td>Global threshold</td>
<td>51.5608</td>
<td>15.0699</td>
</tr>
<tr>
<td>Normal shrink</td>
<td>50.5881</td>
<td>16.0426</td>
</tr>
</tbody>
</table>
Test 2

In the pervious test no blur was added to the original image, our second test consists of an image cameraman of size (256×256). Suppose PSF is a Gaussian blur with variance 0.02 to a support of size (9×9). The blurred image was obtained by convoluting the original image and the PSF. Gaussian, zero mean noise is added to the blurred noise. The proposed algorithm derived in section 4 is used to this test and the results are shown in Fig. 5.

VII. CONCLUSION

A new method based on the combination of normal shrink and modified method of katsaggelous and Lay is introduced. In this method normal shrink is used for removing noise and it gives the highest S/N, then modified blur estimation for removing blur of image was carried out to obtain the best possible image.

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REFERENCES