GenCos’ Optimal Bidding Strategy Considering Market Power and Transmission Constraints: A Cournot-based Model

A. Badri

Abstract—Restructured electricity markets may provide opportunities for producers to exercise market power maintaining prices in excess of competitive levels. In this paper an oligopolistic market is presented that all Generation Companies (GenCos) bid in a Cournot model. Genetic algorithm (GA) is applied to obtain generation scheduling of each GenCo as well as hourly market clearing prices (MCP). In order to consider network constraints a multiperiod framework is presented to simulate market clearing mechanism in which the behaviors of market participants are modeled through piecewise block curves. A mixed integer linear programming (MILP) is employed to solve the problem. Impacts of market clearing process on participants’ characteristic and final market prices are presented. Consequently, a novel multi-objective model is addressed for security constrained optimal bidding strategy of GenCos. The capability of price-maker GenCos to alter MCP is evaluated through introducing an effective-supply curve. In addition, the impact of exercising market power on the variation of market characteristics as well as GenCos scheduling is studied.

Keywords—Optimal bidding strategy, Cournot equilibrium, market power, network constraints, market auction mechanism

I. INTRODUCTION

Electric power markets move from traditional monopolistic systems toward restructured power markets in which producers are able to compete with each other through optimal bidding strategies and generation scheduling. In vertically integrated environments Unit Commitment (UC) program is applied in order to minimize total production costs while satisfying some constraints such as power demand, spinning reserve and unit constraints [1]. Some of researches deal with the impact of system security constraints on UC entitled Security- Constraint UC (SCUC) [2,3]. On the other hand in deregulated environments GenCos run UC not for minimizing total system cost but for maximizing their own profit using Profit-Based UC (PBUC) algorithm [4,5]. Accordingly Independent System Operator (ISO) will try to alleviate possible line violations by minimizing total generation costs [2]. This paper provides a tool for simulation of a pool-based electricity market. In the first stage the problem of GenCos’ scheduling in order to obtain the maximum profit is investigated. The electricity market is modeled on the basis of Cournot model for analysis of an oligopolistic market. Among different approaches proposed for UC [6-10], genetic algorithm is applied to obtain generation scheduling of generators located in the GenCos.

The model is developed for evaluation of market equilibrium condition in terms of MCPs and generation output powers. In the second stage impact of market clearing auction on GenCos’ behaviours is investigated. A network—constrained multiperiod auction to maximize social welfare is used to clear the market. According to the piecewise characteristics of supply and demand curves, a Mixed Integer Linear Programming (MILP) is employed to solve the problem. Finally, in the third stage a novel multi-objective model is presented for GenCos’ security constrained optimal bidding strategy, called security constrained price-based unit commitment (SCPBUC) problem. It is shown that SCPBUC can be used as a preventive action to reach the most profitable state in presence of transmission constraints. Following, at first GenCos’ profit-based unit commitment is presented and then using market clearing model, the proposed SCPBUC is developed.

II. PROFIT-BASED UC

A mathematical model is proposed for GenCos’ profit maximization which takes into account the oligopolistic aspect of the market on the basis of the Cournot model. Considering an oligopolistic market with \( n \) GenCos the objective function of each GenCo is maximizing its profit as shown in Eqs. (1) and (2):

\[
\text{Max } R_{Gi} = \sum_{i=1}^{T} \sum_{t=1}^{T} \left( P_{gi,t} - C_{gi,t} \right)
\]

Subject to:

\[
\sum_{i=1}^{ng} P_{gi,t} = P_{Gli} \quad t = 1,..., T
\]

\[
P_{gi,t} \leq P_{gi,t} \quad i = 1,..., ng , \quad t = 1,..., T
\]

\[
P_{gi,t} \geq P_{gi,t} \quad i = 1,..., ng , \quad t = 1,..., T
\]

\[
X_{i,t-1}^{on} - T_{i}^{on} \geq 0 \quad i = 1,..., ng
\]
\[ t = 1, ..., T \]

\[ [X^\text{off}_{i,t-1} - T^\text{off}_{i,t}]\{ | I_{i,t} - I_{i,t-1} | \} \geq 0 \quad i = 1, ..., ng , \]

\[ t = 1, ..., T \]

In which:

\( T \): Market time horizon

\( ng \): Number of total units

\( ng_i \): Number of units in GenCo \( i \)

\( \lambda_i \): Market clearing price at \( t \)

\( P_{gi,t} \): Output of unit \( i \) at \( t \)

\( C_{gi,t} \): Operation cost of unit \( i \) at \( t \)

\( F(P_{gi,t}) \): Fuel consumption function of unit \( i \) at \( t \)

\( I_{i,t} \): Commitment state of unit \( i \) at \( t \)

\( SC_{i,t} \): Start up cost of unit \( i \) at \( t \)

\( P_{gi,\text{min/max}} \): Minimum (maximum) output limits of unit \( i \)

\( UR_i \): Ramp-up rate limit of unit \( i \)

\( DR_i \): Ramp-down rate limit of unit \( i \)

\( X^\text{on/off}_{i,t} \): On (off) time duration of unit \( i \) at \( t \)

\( T^\text{on/off}_{i,t} \): Maximum up (down) time of unit \( i \) at \( t \)

To obtain GenCos’ aggregated generation outputs \( P_{Gi} \) it is assumed that all the participants bid in a Cournot model. Accordingly, the objective function of each GenCo is as illustrated in Eqs. (9) and (10).

\[ \text{Max} \quad R_{gi} = P_{gi} \lambda(P_{Gi}, ..., P_{Gn}) - C_i(P_{Gi}) \quad i = 1, ..., n_c \]  

\[ C_i(P_{Gi}) = a_i P_{Gi}^2 + b_i P_{Gi} + c_i \]  

Where \( P_{Gi} \) indicates the output supplied by \( i \)th GenCo and \( C_i(P_{Gi}) \) is GenCos’ equivalent cost function. Assuming, uniform price market, \( \lambda \) denotes market price (MCP) that is highly dependent on power supplied by the participants. In Cournot model each firm chooses its output independently trying to maximize its profit. Taking the deriviate of Eq. (9) with respect to output power we have:

\[ \frac{\partial R_{gi}}{\partial P_{Gi}} = \lambda R_{gi} + \lambda \frac{\partial (P_{Gi} P_{Gj})}{\partial P_{Gi}} - \frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} = 0 \]  

\[ i = 1, ..., n_c \]

In which \( P_{Gi} \) indicates the outputs of all competitors of \( i \)th GenCo. On the other hand similar to generators’ cost function behavior of each elastic consumer is quantified by its utility function as illustrated in Eq. (12).

\[ U_i(P_b) = -\alpha_i P_b^2 + \beta_i P_b + \gamma_i \quad i = 1, ..., n_t \]  

Where \( P_b \) is power consumed by the \( i \)th consumer and \( n_t \) is the number of consumers. Similarly the \( i \)th consumer’s benefit is given by Eq. (13).

\[ R_i = U_i(P_b) - \lambda \cdot P_b \]  

Consequently, optimal consumption level is specified by maximizing the consumers’ profit as shown in Eq. (14).

\[ \frac{\partial R_i}{\partial P_b} = -2\alpha_i P_b + \beta_i - \lambda = 0 \]  

\[ i = 1, ..., n_t \]

Assuming a lossless network and uniform price system and using Eq. (14) the total load demand of system at each hour will be as follows:

\[ \sum_{i=1}^{nc} P_{Gi} + \sum_{i=1}^{nl} P_{bi} - \lambda \cdot \left( \sum_{i=1}^{nl} \sum_{i=1}^{2\beta_i} \right) = \frac{\beta}{2\alpha} - \lambda \]  

Where \( \alpha \) and \( \beta \) are equivalent coefficients of total elastic load demand of the system. Obtaining system market price with respect to \( P_{Gi} \) and substituting in Eq. (11) the equilibrium condition of the market \((P_{Gi} , i = 1, ..., n_c)\) is derived by solving \( n_c \) equations. Note that Cournot equilibrium results in GenCos’ optimal aggregated outputs as well as market clearing prices. Genetic algorithm is used to solve the proposed unit commitment problem. To save CPU time the proposed genetic programming is based on real coding.

III. MARKET AUCTION MECHANISM

In order to study the impact of market operator, a network constrained multiperiod auction is introduced. It is assumed that suppliers submit their bids through piecewise price-generation curves that are built from their profit maximization algorithm (section II). Behaviors of elastic loads are modeled through piecewise constant price-demand curves. Accordingly, a MILP is used to solve the problem. From system operator point of view the objective function is maximizing system social welfare with corresponding constraints as follows:

\[ \text{Max} \quad \sum_{i=1}^{T} \sum_{d=1}^{N_d} \sum_{h=1}^{N_d} p_{ah} \lambda_{dh} - \sum_{g=1}^{N_g} \sum_{h=1}^{N_h} p_{gh} \lambda_{gh} \]  

Subject to:

\[ 0 \leq P_{Gi}^G \leq P_{Gi}^G \quad t = 1, ..., T, \quad g = 1, ..., N_g \]

\[ b = 1, ..., N_b \]
subject to equation (21), which imposes limits on the output of each generator at time $t$ and bus $j$:

$$p_{ij}^{min} \leq \frac{1}{|X_{ij}|} (\theta_d - \theta_p) \leq p_{ij}^{max}$$

where:

$N_l$: Number of load consumers

$N_{d}$: Number of demand blocks of each load

$N_g$: Number of generators

$N_b$: Number of generation blocks of each generator

$N_B$: Number of buses

$p_{D_{th}}^{D}$: Power consumed by $h$th block of consumer $d$ at time $t$

$p_{gb}^{G}$: Power produced by $b$th block of generator $g$ at time $t$

$\lambda_{D_{th}}^{D}$: Price of $h$th block of consumer $d$ at time $t$

$\lambda_{gb}^{G}$: Price of $b$th block of generator $g$ at time $t$

$p_{D_{bid}}^{D}$: Size of $b$th block offered by generator $g$ at time $t$

$p_{D_{bid}}^{D}$: Size of $h$th block offered by consumer $d$ at time $t$

$p_{gb}^{G}$, $p_{gb}^{G}$: Minimum and maximum output powers of generator $g$

$\psi_n$: set of generators connected to bus $n$

$\Delta_n$: set of consumers connected to bus $n$

$X_{ij}$: Reactance of line between buses $i$ and $j$

$\theta_p$: Phase angle of bus $i$ at time $t$

$p_{ij}^{min}$, $p_{ij}^{max}$: Minimum and maximum capacities of line between buses $i$ and $j$

In the above problem, Eq. (16) states social welfare maximization as the objective function. Eqs. (17) and (18) express limits of blocks offered by market participants. Eq. (19) defines generators’ total outputs considering bidding blocks and corresponding operation limits. Eq. (20) provides power balance in each bus, stating that total generation minus consumption in each bus must be net injection through the connected lines and finally Eq. (21) shows the constraints on transmission capacity limits based on DC power flow assumptions.

### IV. Security Constrained Profit-Based UC

In order to achieve the maximum payoff each GenCo may consider transmission constraints while bidding to the market. In the other words, it can use the network as an instrument to exert market power. For this purpose a novel multi-objective SCPBUC model is presented that enables each (price-maker) GenCo to take into account the network to reach the highest profit.

$$\text{Max } R_{Gi} = \sum_{t=1}^{T} \sum_{i=1}^{N_g} \left( \lambda_{i} p_{gi}, t \right) - \sum_{t=1}^{T} \sum_{i=1}^{N_g} C_{gi}, t$$

subject to:

$$P_{ij}^T, P_{ij}^R \in \Omega_j$$

Eq. (22) is the proposed multi-objective problem (from ith GenCo’s point of view) in which upper and lower levels deal with profit maximization and transmission constraints, respectively. $\Pi$ and $\Omega$ are set of corresponding constraints, as illustrated in sections II and III. From ith GenCo’s point of view, the main objective of proposed framework is to find the optimum power quantities that provide the maximum payoff over a period of time, considering transmission security constraints. This is accomplished by means of effective supply curves that are obtained by each GenCo for each hour. The relation between market clearing prices and the contribution of each GenCo at each hour is represented by a heuristic curve, so called effective-supply curve. These curves are released by simulating market auction mechanism by each GenCo and deriving market clearing prices as the function of corresponding contributions for each hour. These are stepwise monotonically decreasing curves (See Fig. 3 in section V). The discontinuous nature of these curves is the result of using multiple bid blocks. The decreasing behavior is a consequence of different costs of GenCos’ generation units. Since these curves embody the effects of all interactions with competitors and transmission constraints, the 24 hourly day-ahead effective-supply curves provide all market information that a given price-maker needs to perform security constrained optimal generation scheduling. The flowchart of solving proposed security constrained optimal generation scheduling problem is illustrated in Fig. 1.
According to Fig. 1 a detailed description for solving security constrained profit-based unit commitment problem, Eq (22) is as follows:

1- Each GenCo will provide 24 effective supply curves.
2- Accordingly, the GenCo will perform a profit-based self-scheduling for all possible price blocks ($b_{HN}$) for $H$th hour and the contributions corresponding to maximum profit ($p^{*}_{G,H}$) is extracted. Here $GR$ is GenCo’s profit and $b^{*}_{H}$ stands for the selected block of the $H$th hour. Note that only this block is considered for the $H$th hour in next iterations.
3- Step 2 is repeated for 24 effective supply curves and GenCos’ maximum profit is determined.

According to the proposed solution it is concluded that for solving Eq. (22) the lower level that considers competitors’ behaviors and transmission security constraints is modeled with effective supply curves. This will really simplifies the solution and causes that each price maker GenCo do its self scheduling independently of the problems of other producers.

In order to do the market clearing mechanism each GenCo should access to some information about transmission network and other participants’ bid blocks. Here, it is assumed that all necessary information is made available either by the market or by estimating other rivals’ behaviors. A flowchart describing the proposed algorithm is shown in Fig. 2:

The manner in which the GenCo should produce to maximize its profit is a complex dynamic decision problem. Considering the structure of the proposed model the solution is an efficient coordinate-descent technique coupled with MILP techniques.

V. CASE STUDY

A 9 bus IEEE test system [11] is employed for demonstrating simulation results in which 7 generators located in buses 1 to 7 supply elastic loads located in buses 1 to 9. Table I shows characteristics of the generators in three main GenCos. Transmission line data are provided in [11]. Using individual utility coefficients ($\alpha_i, \beta_i$) offered by each load at each hour, Table II illustrates hourly equivalent coefficients of total elastic loads ($\alpha, \beta$). Considering behaviors of total elastic loads at each hour and using Cournot equilibrium model, GenCos’ initial self-scheduling and corresponding market clearing prices are illustrated in Table III.
TABLE I
GENERATORS’ DATA

<table>
<thead>
<tr>
<th>No</th>
<th>GenCo1</th>
<th>GenCo2</th>
<th>GenCo3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G1</td>
<td>G2</td>
<td>G3</td>
</tr>
<tr>
<td>$P_{min}$</td>
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<tr>
<td>$P_{max}$</td>
<td>240</td>
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<td>$a$</td>
<td>0.05</td>
<td>0.08</td>
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<tr>
<td>$b$</td>
<td>14</td>
<td>15.4</td>
<td>19.4</td>
</tr>
<tr>
<td>$c$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$SC($)</td>
<td>110</td>
<td>350</td>
<td>170</td>
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<td>$T^{con}$</td>
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<tr>
<td>$T^{off}$</td>
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<tr>
<td>$UR/DR(Mw/h)$</td>
<td>150</td>
<td>100</td>
<td>50</td>
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TABLE II
HOURLY COEFFICIENTS OF TOTAL ELASTIC LOAD DEMANDS

<table>
<thead>
<tr>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<tr>
<td>Hour</td>
<td>1-6</td>
<td>7</td>
<td>8,24</td>
<td>9,15,</td>
<td>23</td>
<td>10,16</td>
<td>11-14,</td>
<td>17,18,22</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.01</td>
<td>0.009</td>
<td>0.0078</td>
<td>0.0065</td>
<td>0.0064</td>
<td>0.00628</td>
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<td>$\beta$</td>
<td>45.8</td>
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<td>45.8</td>
<td>45.8</td>
<td>45.8</td>
<td>45.8</td>
<td>45.8</td>
</tr>
</tbody>
</table>

TABLE III
GENCOS’ INITIAL SELF SCHEDULING

<table>
<thead>
<tr>
<th>Period</th>
<th>GenCo1</th>
<th>GenCo2</th>
<th>GenCo3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G1</td>
<td>G2</td>
<td>G3</td>
</tr>
<tr>
<td>1</td>
<td>138</td>
<td>78</td>
<td>106</td>
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<tr>
<td>2</td>
<td>146</td>
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<tr>
<td>8</td>
<td>184</td>
<td>107160</td>
<td>43</td>
</tr>
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</table>

In order to study the impact of market clearing mechanism on GenCos’ bidding strategies, proposed multiperiod algorithm is employed. Here, it is assumed that all participants bid with increasing and decreasing piecewise curves so that suppliers and consumers bid with the three blocks consisting 1/2, 1/3 and 1/6 of their aggregated output powers, $P_a$ and aggregated consumptions level $P_c$, respectively. Substituting GenCos’ bidding blocks to eqs. (16)-(21), Table IV shows generators’ output powers for the period of 24 hours. As illustrated some GenCos’ outputs are reduced due to satisfying network constraints and social welfare. However, since GenCo1 contains cheap units its contributions are not changed in comparison to other competitors that include more expensive units.

TABLE IV
GENCOS’ SCHEDULING AFTER MARKET CLEARING

<table>
<thead>
<tr>
<th>Period</th>
<th>GenCo1</th>
<th>GenCo2</th>
<th>GenCo3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G1</td>
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<td>184</td>
<td>107160</td>
<td>43</td>
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</tbody>
</table>

As mentioned before, using market simulation process, the (price-maker) GenCos are able to construct their effective-supply curves at each hour. For more clarification, the effective-supply curve of GenCo1, illustrating the effect of its power contribution on the market equilibrium at the first period is depicted in Fig. 3. As shown, as the amount of power contribution is decreased, the corresponding market price is increased and vice versa. This is due to the fact that GenCo 1 that may be interpreted as the price-maker GenCo contains cheap units that play a major role in determining market characteristics. Taking into account the network constraints, suppliers may use their effective-supply curves to exercise market power.

Table V shows GenCos’ optimal self-scheduling dedicated from SCPBUC problem. In comparison to Table IV, it is appear that for all periods, production of GenCo1 is reduced (in order to exert market power and increase the prices), while generation of two other suppliers are increased to benefit more from increased prices.

Fig. 3 Effective-supply curve of GenCo1 for the first period
TABLE V

<table>
<thead>
<tr>
<th>Period</th>
<th>GenCo1</th>
<th>GenCo2</th>
<th>GenCo3</th>
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<tbody>
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<td></td>
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<tr>
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</table>

Fig. 4 illustrates the variation of hourly market clearing prices in three cases: Cournot equilibrium output, after market clearing (without market power) and security-constrained optimal bidding strategy (with market power), respectively. It is apparent that in all hours, applying market clearing mechanism results in reduction in market prices. However, exercising market power (by price maker GenCos) causes some increases in corresponding prices. Note that, in peak hours, in order to meet the demand all expensive units are working, therefore the prices are unchanged.

VI. CONCLUSION

In this paper the problem of optimal generation scheduling of GenCos is investigated in which market power and network constraints are taken into account. A Cournot-based model is presented for GenCos’ bidding strategies to obtain participant outputs and market clearing prices, regardless of transmission constraints. In order to consider network security constraints a multiperiod framework is proposed to provide GenCos’ power accepted in the market and actual market clearing prices. Based on binary variables included in the framework a mixed integer linear programming is employed to solve the problem. Applying a novel introduced effective-supply curve the capability of price-maker GenCo to exercise market power is presented. Utilizing these curves each price-maker GenCo is able to use transmission constraints as the preventive action to predict market characteristics, while bidding to the market and maintain the prices in excess of competitive levels, to benefit more from the market.

REFERENCES

Ali Badri was born in Iran, on 1973. He received the B.Sc degree in electrical engineering from Isfahan University of Technology, Isfahan, Iran in 1995 and the M.Sc and Ph.D degrees from Iran University of Science and Technology, Tehran, Iran in 2000 and 2008, respectively. He is currently an assistant professor in the Department of Electrical Engineering, Shahid Rajaee University, Tehran, Iran. His research interests are restructred power systems, power system operation, energy management and FACTS. Dr. Badri is member of Iranian association of electrical and electronic engineers.