Abstract—High redundancy and strong uncertainty are two main characteristics for underwater robotic manipulators with unlimited workspace and mobility, but they also make the motion planning and control difficult and complex. In order to setup the groundwork for the research on control schemes, the mathematical representation is built by using the Denavit-Hartenberg (D-H) method [9]&[12]; in addition to the geometry of the manipulator which was studied for establishing the direct and inverse kinematics. Then, the dynamic model is developed and used by employing the Lagrange theorem. Furthermore, derivation and computer simulation is accomplished using the MATLAB environment. The result obtained is compared with mechanical system dynamics analysis software, ADAMS. In addition, the creation of intelligent artificial skin using Interlink Force Sensing ResistorTM technology is presented as groundwork for future work.

Keywords—Manipulator System, Robot, AUV, Denavit-Hartenberg method Lagrange theorem, MALTAB, ADAMS, Direct and Inverse Kinematics, Dynamics, PD Control-law, Interlink Force Sensing ResistorTM, intelligent artificial skin system.

I. INTRODUCTION

The significance of the underwater robot lays in the potential and promising future of its application as an effective and intelligent solution for ocean-based industrial applications. A properly designed Autonomous Underwater Vehicle (AUV) should be able to carry out operations in an unstructured oceanic underwater environment while providing a real-time interface to the operator for handling the tasks. The tasks may include: diving down to search for a certain object, picking the object, manipulating the object such as opening a jar, turning a wellhead, and bring the collected items back to the surface (see Figs. 1, 2 & 3 (a), (b) & (c)).

Although the ability to operate in the subsea environment presents many opportunities and benefits in research, commercial and military endeavors, the added challenge of working underwater often results in technologies that are either very expensive or have limited functionality. The design work started from analyzing the current available various components and operating control technologies of the AUV. Then the mechanical structure is determined, including the body frame, sealing methods and gripper mechanism.

In this paper, we are going to address the kinematics model of the manipulator, as well as investigate its inverse kinematics and present, as groundwork, the creation of intelligent artificial skin using Interlink Force Sensing ResistorTM technology.

Computer simulation is conducted to verify the movement control process of the arm, through which the parameters of joint angles, locations and torques are obtained.
Before calculations and analysis of manipulator kinematics, it is necessary to setup mathematical models for representing the positions, orientations and frames of each manipulator link. The general tool used for the space parameters is called the homogeneous transformation matrix that can interrelate kinematical conditions within different coordinate systems. The transformation matrix in three-dimensional spaces with a size of 4 by 4, and consists of a position vector and a rotation matrix.

\[
\begin{bmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

Then the joint velocity and acceleration along this path are:

\[
\begin{align*}
\dot{\theta}(0) &= \dot{\theta}_0 \\
\dot{\theta}(t_f) &= \dot{\theta}_f \\
\ddot{\theta}(0) &= 0 \\
\ddot{\theta}(t_f) &= 0 \\
\end{align*}
\]

where \( t_f \) is the time when the manipulator reaches its goal position, \( \theta_0 \) is the initial angle position and \( \theta_f \) is the goal angle position. The cubic polynomial has the form:

\[
\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3
\]

Then the joint velocity and acceleration along this path are:

\[
\begin{align*}
\dot{\theta}(t) &= a_1 + 2a_2 t + 3a_3 t^2 \\
\ddot{\theta} &= 2a_2 + 6a_3 t
\end{align*}
\]

Combining the two equations above with the four desired constraints yields four equations in four unknowns:

\[
\begin{cases}
\theta_0 = a_0 \\
\theta_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 \\
0 = a_1 \\
0 = a_1 + 2a_2 t_f + 3a_3 t_f^2
\end{cases}
\]

Solving these equations, we obtain:

II. D-H REPRESENTATION OF THE MANIPULATOR

III. TRAJECTORY GENERATION

Rough, jerky motions tend to cause increased wear on the mechanism and vibrations by exciting resonances in the manipulator. Therefore, it is desirable for the motion of the manipulator to be smooth. The path of the manipulator is governed by a smooth and continuous function. Furthermore, the first and second derivatives of that function are used to regulate velocity and acceleration respectively [5]. For making a signal smooth motion, a cubic polynomial and four constraints are evident. The four constraints are:

\[
\begin{align*}
\theta(0) &= \theta_0 \\
\theta(t_f) &= \theta_f \\
\dot{\theta}(0) &= 0 \\
\dot{\theta}(t_f) &= 0 \\
\end{align*}
\]
The obtained coefficient of the polynomial can be calculated, when the joint angle position, time and desired final angle position is known (see Table I).

For a particular case, we define the three rotary joint positions of the initial manipulator configuration. These are:

\[ \theta_{10} = 0, \theta_{20} = -\frac{\pi}{2}, \theta_{30} = 0 \]

The desired final positions are:

\[ \theta_{1f} = \frac{\pi}{3}, \theta_{2f} = \frac{\pi}{2}, \theta_{3f} = -\frac{\pi}{6} \]

The time interval \( t_f = 10\) s. By substituting the obtained coefficients in (5) into the assumed polynomial, we can define the position, velocity and acceleration of the each joint as a function of time, which have been plotted in Figs. 4, 5 and 6.

**IV. DYNAMICS MODEL OF MANIPULATOR**

The underwater manipulator’s dynamics model is established using the Lagrange method. The first step to establish the dynamics model is to choose a generalized coordinate system, using which we can determine the manipulator’s geometry and the forces applied. We choose the Homogeneous Coordinate System, since it is often simpler and

![Fig. 4 Joint Displacement (radian)](image_url)

![Fig. 5 Joint Angular Velocity (radian/s)](image_url)

![Fig. 6 Joint Angular Acceleration (radian/s^2)](image_url)
more symmetric than Cartesian Coordinate Counterparts [5]. We assume that the manipulator has \( n \) degrees of freedom and consists of \( n \) mass point system. Then each center point vector can be described in generalized coordinates by \( \vec{r} = \vec{r}(q_1, q_2, \ldots q_n), \) \( i = 1, 2 \ldots n \). Where \( q_1, q_2, \ldots q_n \) are the coordinate variables. The position of a point in Homogeneous Coordinate is \( \vec{r} = \begin{bmatrix} x & y & z & 1 \end{bmatrix}^T \). The change of frame from \( o \) to base zero is \( \vec{r} = \vec{A}_i \vec{r}^o \). \( A \) is the transformation matrix. The homogeneous speed of this point mass is defined as:

\[
\vec{v} = \frac{d}{dt} \vec{r} = \vec{A}_i \vec{v}^o
\]

Lagrangian dynamic formulation is based on the energy-balance approach [11], so we need to get started by analyzing the potential and kinetic energies. The expression for the kinetic energy of an element \( i \) is:

\[
dT_i = \frac{1}{2} \vec{v}_i \cdot d\vec{m} = \frac{1}{2} d\text{ag}( \vec{r}(\vec{v}_i \vec{r}^o))
\]

\[
= \frac{1}{2} d\text{ag} \left[ \sum_{j=1}^{i} \left( \frac{\partial \vec{A}_i}{\partial q_j} \vec{q}_j \right) \left( \sum_{k=1}^{i} \frac{\partial \vec{A}_i}{\partial q_k} \vec{q}_k \right) \right] d\vec{m}
\]

\[
= \frac{1}{2} d\text{ag} \left[ \sum_{j=1}^{i} \sum_{k=1}^{i} \frac{\partial \vec{A}_i}{\partial q_j} \vec{q}_j \frac{\partial \vec{A}_i}{\partial q_k} \vec{q}_k \right] d\vec{m}
\]

\[
= \frac{1}{2} \text{ag} \left[ \sum_{i=1}^{n} \sum_{k=1}^{n} \frac{\partial \vec{A}_i}{\partial q_k} \vec{q}_k \frac{\partial \vec{A}_i}{\partial q_k} \vec{q}_k \right] d\vec{m}
\]

Then the whole domain was integrated with consideration of the kinetic energy \( T_i \) for link \( i \):

\[
T_i = \int_{\text{link } i} dT_i = \frac{1}{2} \sum_{i=1}^{n} \sum_{k=1}^{n} \frac{\partial \vec{A}_i}{\partial q_k} \vec{q}_k \frac{\partial \vec{A}_i}{\partial q_k} \vec{q}_k \int_{\text{link } i} d\vec{m}
\]

Next is the process to derive the potential energy. In frame \( i \), the mass center point position is expressed as \( \vec{r}_{ci} \), and the expression in the base zero frame is \( \vec{r}_i = \vec{A}_i \vec{r}_{ci} \), and then the potential energy expression in frame \( i \) is:

\[
V_i = -\vec{m}_i \vec{g} \cdot \vec{r}_{ci} = -\vec{m}_i \vec{g} \cdot \vec{A}_i \vec{r}_{ci}
\]

The Lagrangian dynamic formulation provides a means of deriving the equations of motion from a scalar function called the Lagrangian, which is defined as the difference between the kinetic and potential energy of a mechanical system [11].

\[
L = T - V
\]

\[
\frac{dL}{dt} \frac{\partial \vec{q}_k}{\partial \vec{q}_j} = \sum_{k=1}^{n} h_{ij} \vec{q}_k
\]

\[
\frac{dL}{\partial \vec{q}_j} = \sum_{k=1}^{n} h_{ij} \vec{q}_k + \sum_{k=1}^{n} h_{ij} \vec{q}_k
\]

\[
= \frac{1}{2} \vec{q}^T H(\vec{q}) \vec{q}
\]

where: \( m \) is the mass of a link; \( dm = m \) for any link. The \( x \), \( y \) and \( z \) are the position vectors of an integral element within the link body domain. The inertia tensor \( H \) showing the mass distribution of link \( i \) is:

\[
H_i = \begin{bmatrix}
\frac{1}{2} m_i & m_i \cdot 0 & m_i \cdot z_i \\
0 & \frac{1}{2} m_i & m_i \cdot y_i \\
m_i \cdot z_i & m_i \cdot y_i & \frac{1}{2} m_i
\end{bmatrix}
\]

In which the mass distribution matrix is \( m \).
\[
\frac{1}{2} \dot{q}^T \frac{\partial^2 L}{\partial q \partial q^T} \dot{q} - \sum_{i=1}^{n} m_i \ddot{q}^T \frac{\partial \mathbf{A}_i}{\partial \dot{q}} \frac{\partial \mathbf{q}_e}{\partial q_i} = \frac{1}{2} \dot{q}^T \frac{\partial H}{\partial q_i} \dot{q} - g_j
\]

In our notation, the Lagrangian of a manipulator is:
\[
L(q_j, \dot{q}_j) = T(q_j, \dot{q}_j) - V(q_j)
\]

The equations of motion for the manipulator are then given by:
\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = \tau
\]

Substituting all the values into this equation we get:
\[
\sum_{k=1}^{n} h_{ij} \ddot{q}_k + \sum_{k=1}^{n} h_{ij} \dot{q}_k - \frac{1}{2} \dot{q}^T \frac{\partial H}{\partial \dot{q}_i} \dot{q} + g_j = \tau_j
\]

where \( j = 1, \ldots, n \). And rewriting the equation into matrix format:
\[
\begin{bmatrix}
\dot{q}^T C_i \dot{q} \\
\frac{\partial G(q)}{\partial q_i}
\end{bmatrix} + G = \tau
\]

where:
\[
C_i = \text{Coriolis and centrifugal coefficient matrix.}
\]

\[
G(q) = \begin{bmatrix}
\dot{q}^T C_i \dot{q} \\
\frac{\partial G(q)}{\partial q_i}
\end{bmatrix}
\]

\[
\tau = \begin{bmatrix}
\tau_1 \\
\tau_2
\end{bmatrix}
\]

V. KINEMATICS EQUATION DERIVATION

In this section, we use MATLAB programming to assist us in deriving the kinematics equation of the three link underwater manipulator. The advantage of MATLAB is that it can deal with complex matrix manipulation, which will make the derivation process easier and quicker. As revealed before, the velocity of a point on the manipulator can be obtained by taking derivatives on the position functions. From the previous results, the position in terms of the homogeneous coordinates can be obtained using transformation matrix. From the value of link parameters, the individual link-transformation matrices can be computed. Then the link transformations can be multiplied together to obtain the single transformation that relates frame \([N]\) to frame\([0]\):

\[
0 \mathbf{T}_n = 0 \mathbf{T}_1 \ldots 0 \mathbf{T}_n = A_1 A_2 A_3 \ldots A_4
\]

For the transformation matrix of rotational joints, the partial derivative of \( A_i \) with respect to the variable \( \theta_i \) is determined. Therefore, we can see:

\[
\frac{\partial A_i}{\partial \theta_i} = Q_i A_i
\]

In the equation:
\[
Q_i = \begin{bmatrix}
0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

Then we take the derivative of equation (17) and denote it as \( U_{ij} \):

\[
U_{ij} = \frac{\partial \mathbf{T}_i}{\partial \theta_j} = \frac{\partial (A_1 A_2 \ldots A_i \ldots A_4)}{\partial \theta_j} = A_i A_2 \ldots Q_j A_j \ldots A_4 \leq i
\]

Using the dynamics equation from the Lagrangian in the previous section, we rewrite equation (19):

\[
\tau_i = \sum_{j=1}^{n} D_{ij} \ddot{q}_j + \sum_{k=1}^{n} \sum_{l=1}^{n} D_{ijk} \dot{q}_j \ddot{q}_k + D_i
\]

where:
\[
D_{ij} = \sum_{p=\max(i,j)}^{n} \text{Trace}(U_{ip}J_p U_{pi}^T)
\]

\[
D_{ijk} = \sum_{p=\max(i,j,k)}^{n} \text{Trace}(U_{ip}J_p U_{pi}^T)
\]

\[
D_i = \sum_{p=1}^{n} -m_j \ddot{q}_p U_{pi}^T \ddot{q}_p
\]

In (21), the first section is the angular acceleration inertia, the second part is the Coriolis and centrifugal force matrix and the last part is the gravity matrix. Then, we obtain the torques \( i \) for driving the relevant joint \( i \), when a given specified motion is required.

\[
\tau_i = D_{1i} \ddot{q}_1 + D_{12} \ddot{q}_2 + D_{13} \ddot{q}_3 + D_{14} \ddot{q}_4 + D_{122} \ddot{q}_2^2 + D_{133} \ddot{q}_3^2 + D_{122} \ddot{q}_2 \ddot{q}_2 + D_{133} \ddot{q}_3 \ddot{q}_3 + D_{123} \ddot{q}_2 \ddot{q}_3 + D_{134} \ddot{q}_3 \ddot{q}_4 + D_{145} \ddot{q}_4 \ddot{q}_5 + D_{1234} \ddot{q}_2 \ddot{q}_3 \ddot{q}_4 + D_{12345} \ddot{q}_2 \ddot{q}_3 \ddot{q}_4 \ddot{q}_5
\]

Because \( D_{iab} = D_{ba} \), we further simplify (23) and define the control torque for joint 1:

\[
\tau_1 = D_{11} \ddot{q}_1 + D_{12} \ddot{q}_2 + D_{13} \ddot{q}_3 + D_{14} \ddot{q}_4 + D_{122} \ddot{q}_2^2 + D_{133} \ddot{q}_3^2 + 2D_{122} \ddot{q}_2 \ddot{q}_2 + 2D_{133} \ddot{q}_3 \ddot{q}_3 + 2D_{123} \ddot{q}_2 \ddot{q}_3 + 2D_{1234} \ddot{q}_2 \ddot{q}_3 \ddot{q}_4 + 2D_{12345} \ddot{q}_2 \ddot{q}_3 \ddot{q}_4 \ddot{q}_5
\]

Control torque for joint 2:
\[ \tau_2 = D_2 q_1 + D_2 q_2 + D_2 q_3 + D_2 q_4 + D_2 q_5 + D_2 q_6 + D_2 q_7 + D_2 q_8 \\
\quad \quad + D_2 (q_4 q_6 + q_5 q_7 + q_6 q_8 + q_7 q_9) \]

Control torque for joint 3:

\[ \tau_3 = D_3 q_1 + D_3 q_2 + D_3 q_3 + D_3 q_4 + D_3 q_5 + D_3 q_6 + D_3 q_7 + D_3 q_8 + D_3 q_9 \\
\quad \quad + D_3 (q_4 q_6 + q_5 q_7 + q_6 q_8 + q_7 q_9) \]

VI. HYDRODYNAMICS EFFECTS

In order to accurately model the underwater manipulator, we need to consider the additional influences on the manipulator during motion, which are oriented from the moving incompressible fluids in the underwater environment. The hydrodynamic forces, along with the manipulator weight and payload, determine the torques for generating a motion. Therefore, for the purpose of dynamics analysis, the integration of hydrodynamic forces into the equation of motion is needed. In the following section, we used Navier-Stokes equations to determine the hydrodynamic effects including: added mass, added Coriolis and Centripetal, drag force and buoyancy [13].

<table>
<thead>
<tr>
<th>Added mass</th>
<th>Modification of the dynamic</th>
<th>Kinetic energy of surrounded fluid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drag forces</td>
<td>Modification of the dynamic</td>
<td>Viscosity of the fluid</td>
</tr>
<tr>
<td>Buoyancy</td>
<td>No more gravity</td>
<td>Density of the fluid</td>
</tr>
<tr>
<td>Currents</td>
<td>Current loads</td>
<td>Fluid</td>
</tr>
<tr>
<td>Waves</td>
<td>Forced oscillations</td>
<td>acceleration</td>
</tr>
</tbody>
</table>

TABLE II
A LIST OF HYDRODYNAMIC EFFECTS

A. Added Mass
The added mass is generated when a rigid body moves in a fluid. The movement of the rigid body, which requires an additional force, will also accelerate the fluid. The added mass effect is normally neglected in land-based robots, since the low density of the air exerts little difference, while the water can cause much more reaction force for the same manipulator motion [13]. By approximating the manipulator as slow moving and one which has three planes of symmetry, as is common for underwater vehicles, the added mass is in the form of a 6 by 6 matrix. Based on that, the matrix form expression (27) was developed by [28] for the added mass effect:

\[ M_a = M_a^T = -\text{diag}\{X_u, Y_v, Z_w, K_p, M_q, N_r\} \]  

For simplicity, we can approximate the geometry of the designed manipulator links as cylinders. Then we can easily obtain the mass inertia matrix for each link through the equations:

\[ X_u = 0 \]

The Reynolds number \( R_n \) is expressed in the equation below:

\[ R_n = \frac{UD}{v} \]

\[ B = \rho g V \]

B. Buoyancy
Buoyancy is the force that is created due to the volume of the fluid displaced by the submerged body, and it is defined as:

\[ \tau = -\sum_{i=1}^{n} C_d \left( \frac{l_i}{D} \right) * 0.5 \rho D l_i^2 d_i |\theta| |\theta| \]

where:
\[ C_d \] is the drag coefficient, which is a function of the Reynolds numbers.
\[ D \] is the outer diameter of the cylinder,
\[ \theta \] is the joint angular displacement and
\[ l_i \] is the distance from the joints to the segmented length.

The Reynolds number \( R_n \) is expressed in the equation below:

\[ R_n = \frac{UD}{v} \]

C. Drag Force
Drag is also known as quadratic linear friction, which is the main source of effects like potential damping, wave drift damping, skin friction and so on. The drag force exerted on the links of the manipulator will be modeled based on strip theory, which is used by replacing the surface integral with a line integral along the length of the link’s cylinder. Through dividing the body surface into small elements, the force is calculated individually [13].

The drag coefficient is partially determined by the ratio between the internal forces and viscous forces. An expression was presented, in [27], for the drag coefficient:

\[ M(q, \dot{q}) + C(q, \dot{q}) + F(q) + G(q) + D(q, \dot{q}) = \tau \]  

where:
\[ q \] is the joint angular position,
\[ M \] is the inertia matrix,
\[ C \] denotes the Coriolis, centrifugal forces,
\[ G \] represents the gravity forces which include buoyancy effects,
\[ F \] is the friction term,
\[ D \] is the damping term.
\( D \) represents the hydraulic drag forces, which are caused by the relative velocity of the manipulator to ocean current and waves and \( \tau \) is the vector of applied joint torques, which are actually control inputs.

VII. ADAMS SIMULATION

In this investigation, computer simulation is conducted for verifying the movement control process of the arm, through which the parameters of joint angles, locations and torques are obtained.

The AUV model is simplified and values are assigned for the simulation. The initial position of the underwater vehicle (robot) is in the coordinate origin \((0, 0)\), and it is assigned an acceleration of \(0.3\text{m/s}^2\) along the X direction, and \(0.05\text{m/s}^2\) along Y direction. The joint between the main body and upper arm is assigned \(5\text{d/s}^2\) for angular acceleration and the joint between the upper and lower arm is assigned \(10\text{d/s}^2\).

In the defined motion process, the real-time position, velocity, acceleration and torques of each joint are indicated within Figs. 7 to 19.
Fig. 12 Part 2 CM Velocity X, Velocity (mm/s) versus Time (s)

Fig. 13 CM Velocity X, Velocity (mm/s) versus Time

Fig. 14 Part 3 CM Velocity X, Velocity (mm/s) versus Time (s)

Fig. 15 Part 3 CM Velocity X, Velocity (mm/s²) versus Time (s)

Fig. 16 Part 2 CM Acceleration X, Acc. (mm/s²) versus Time (s)

Fig. 17 Part 3 CM Acceleration X, Acc. (mm/s²) versus Time (s)
VIII. ARTIFICIAL SKIN

Interlink Force Sensing Resistor™ (FSRs), shown in Fig. 20, may be used as the touch sensor of artificial skin for this underwater robotic manipulator arm. They may specifically be attached to the fingers for gripping force control. This would be beneficial for applying the ideal amount of force in the underwater robotic manipulation activity illustrated in Fig. 3, to open the container without deforming it. It would also be used to repair damage in naval vessels and submarines and in many other underwater robotic manipulation applications.

The structure of this sensor can be seen in Fig. 21. It consists of two membranes and a spacer. The job of the spacer is to adhere and separate the membranes from one another such that they are combined with an air gap in between. The bottom membrane is made up of two traces on tails with an interdigitated array of fingers in between. The outer layer of the top membrane is an FSR™ carbon based-ink. When the sensor is pressed, the upper membrane makes contact with the bottom membrane and the ink connects the two traces together. The resistance of this connection is dependent upon the applied force [26].

The force sensing characteristic of this sensor can be seen in Fig. 22. As can be seen, the sensor resistance decreases with increasing applied force. For the majority of this region the governing relationship is an inverse power law [26]. This special region can be exploited by interfacing the sensor with the onboard AUV PC/104 brain via an ADC.

IX. CONCLUSION

In order to setup the groundwork for the research on control schemes, the geometry of the manipulator is studied for establishing the direct and inverse kinematics. Then, the dynamic model is built by using the Lagrange theorem and modeled in MATLAB. The challenges in designing an underwater manipulator is studied which lays a foundation for integrating the hydrodynamic effects into the normal manipulator dynamics. The outcome of this research-investigation, (mainly dynamics analysis) is successfully simulated using mechanical system simulation software ADAMS. Furthermore the groundwork for future work involving the creation of intelligent artificial skin using Interlink Force Sensing Resistor™ technology is presented.
REFERENCES


