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Abstract—In this paper, a Dynamic Economic Dispatch (DED) model is developed for the system consisting of both thermal generators and wind turbines. The inclusion of a significant amount of wind energy into power systems has resulted in additional constraints on DED to accommodate the intermittent nature of the output. The probability of stochastic wind power based on the Weibull probability density function is included in the model as a constraint; A Here-and-Now Approach. The Environmental Protection Agency’s hourly emission target, which gives the maximum emission during the day, is used as a constraint to reduce the atmospheric pollution. A 69-bus test system with non-smooth cost function is used to illustrate the effectiveness of the proposed model compared with static economic dispatch model with including the wind power.

Keywords—Dynamic Economic Dispatch, Stochastic Optimization, Weibull Distribution, Wind Power

I. INTRODUCTION

Wind power (WP) has attracted much attention as a promising renewable energy resource. It has potential benefits in curbing emissions and reducing the consumption of irreplaceable fuel reserves. Conventional economic dispatch problem uses deterministic models, which can not reflect situations considering the WP injection. Since the wind farms connected to power systems have characteristics of dynamic and stochastic performance, stochastic models are more suitable. There are several studies intended to investigate the injection of WP into conventional power networks and its impact on the generation resource management due to its stochastic and non-dispatchable characteristics.

The paper [1] used two approaches to deal with wind generators on the load dispatching calculation. The first approach is negative load, where the wind forecast is treated as a ‘negative load’. Therefore, load demand is reduced by the forecast WP producing a new load demand. This new load demand is then used in the ED process. The second one is inclusive approach; wind turbines are included in the calculation. In order to maximize the wind output for the purpose of reducing emission, wind output should be used as much as possible.

The other important effect of WP on the power system is reserve requirement. Based on the case of the power system in Ireland, Doherty [2] asserts that a high installed capacity of WP causes an increase of reserve requirements due to wind forecasting error. Dany [3] also investigates the impact of WP on the increasing need for reserve requirements. Doherty [4] shows that the increase of the forecast time horizon will also increase reserve requirement due to the increase of the standard deviation of the total WP forecast.

In power market, minimizing the operational cost of differing generators and the risk level are two vital objectives. Because integrating the unpredictable and uncertainty characteristics of WP into the traditional thermal generation systems will bring the problem of system security, which the operator concerns. Reference [5] defined a fuzzy membership function \( \mu \) as the system security level, which can be described as two ways. In one side, the relationship between system security level and WP penetration in ED can be linearization when the available WP penetration is in the limit. In another side, a quadratic membership function is defined to reflect dispatcher’s different attitude, which is a corporate tactical or strategic plan that views WP penetration with a pessimistic or optimistic attitude. That is to say that the security level \( \mu \) will alter with WP penetration.

In [6-7], a bi-objective economic dispatch problem related with WP penetration is described. In this model, it considers operational cost and security factors as opposite objectives which should be minimized simultaneously. A multi-objective mimetic particle swarm optimization (MOMPSO) algorithm is developed to derive the non-dominated Pareto-optimal solutions in terms of the specified multiple design objectives. But the probabilistic methods are not adopted to handle the uncertainties in power systems due to including WP penetration. It only limits the used WP to minimize the risk.

The Optimal Power Flow (OPF) has generally been addressed as a deterministic optimization problem. However, it is becoming increasingly important that solution methods to the OPF problem be developed to address probabilistic quantities, transform into the probabilistic optimal power flow (P-OPF) problem. The randomness introduced tends to have some structure to it, and this structure is generally represented with a probability density function (PDF). The goal of the P-OPF problem is to determine the PDFs for all variables in the problem. These PDFs are the distributions of the optimal solutions [8-12].

Finally, one of the challenges is how to appropriately characterize WP in the load dispatch model. A conventional way was to use the average WP similar to all approaches in [1]-[12]. The probabilistic conventional approaches tried to find probabilistic characteristics of solutions of the problem.
under investigation [8]-[12]. This kind of approaches is called the wait-and-see (WS) strategy in the context of stochastic programming. Although these approaches can be easily implemented, it has a less-known pitfall, called the probabilistic infeasibility. The probabilistic feasibility of the average WP is 0.25, or equivalently, the probabilistic infeasibility is as large as 0.75 [12], [13].

For this reason, one of the more appropriate strategies in contrast, the here-and-now (HN) strategy introduces the probabilistic characteristics to the model of optimization problem itself. The probability of stochastic WP is included in the model as a constraint [13], [14]. This strategy, referred to as the here-and-now approach, avoids the probabilistic infeasibility appearing in conventional models. The Reference [13] developed a new generic ED model to minimize the fuel cost and take the stochastic probability distribution function of WP as constraint. In particular, Liu introduced a threshold parameter \( p_a \) into the constraint to characterize the tolerance that the total load demand cannot be satisfied [13].

Choosing small \( p_a \) will mitigate the risk of insufficient WP, while increasing the demand for thermal power. There are several remarks for the last two works. First, the transmission losses are omitted in analysis. The second remark is about the objective function adopted in this paper, which is based on the quadratic curve that describes thermoelectric power production costs. The most important remark, third, it is remarked that, static economic dispatch model is used by assuming constant load during the dispatch period.

The problem of allocating the customers' load demands among the available thermal power generating units in an economic, secure and reliable way has received considerable attention since 1920 or even earlier. The problem has been formulated as a minimization problem of the fuel cost under load demand constraint and various other constraints at a certain time of interest. It has been frequently known as the static economic dispatch (SED) problem. SED can handle only a single load level. However, SED may fail to deal with the large variations of the load demand due to the ramp rate limits of the generators; moreover, it does not have the look-ahead capability. Owing to the large variation of the customers load demand and the dynamic nature of the power system, it is necessary the investigation of DED problem [15].

In this paper, a DED model is developed for the system consisting of both thermal generators and wind turbines. It determines the optimal settings of generator units with predicted load demand over a certain period of time. The probability of stochastic wind power based on the Weibull probability density function is included in the model as a constraint, as the here-and-now strategy, to avoid the probabilistic infeasibility.

The losses in terms of B-coefficients will be added to our model in the power balance constraint. The proposed model is extendible to more general cost functions. The inclusion of a significant amount of wind energy into power systems has resulted in additional constraints on DED to accommodate the intermittent nature of the output. With increasing concern over global climate change, policy makers are promoting renewable energy sources, predominantly wind generation, as a means of meeting emissions reduction targets. Although wind generation does not itself produce any harmful emissions, its effect on power system operation can actually cause an increase in the emissions of conventional plants [16]. So that Environmental Protection Agency’s (EPA) hourly emission target [17], which gives maximum emission during the day, is used as a constraint, to reduce the atmospheric pollution.

II.ECONOMIC DISPATCH MODEL

The new generic ED problem to minimize the fuel cost and take the stochastic WP as constraint takes the following form:

\[
\begin{align*}
\text{Minimize : } & \quad TOC = \sum_{i}^{N} \sum_{m}^{W} C_{in}\left(P_{im}\right) \\
\text{Subject to : } & \quad P_{i,min} \leq P_{im} \leq P_{i,max} \\
& \quad \Pr\left(\sum_{i}^{N} P_{im} + \Omega(W) \right) \leq P_{d} + P_{L,min,m} \\
\end{align*}
\]

Where:
- \( TOC \): Total Operating Cost
- \( C_{in} \): Conventional Thermal Units Cost
- \( W \): Real power generated by WPG unit \( i \)
- \( P_{i,min} \): Min power generated by generator \( i \) during the interval \( m \)
- \( P_{i,max} \): Max power generated by generator \( i \)
- \( N \): Number of Thermal generators
- \( W \): Number of Dispatch intervals
- \( Pr\left(E\right) \): Probability of event \( E \)
- \( P_{d} \): Total Load Demand
- \( \Omega(W) \): A PDF functional of random variable \( W \)
- \( p_a \): specified threshold representing the tolerance that the total demand cannot be satisfied.

A.Objective Function

In the past, to solve economic dispatch problem effectively, most algorithms require the incremental cost curves to be of monotonically smooth increasing nature. The generating units with the multi-valve steam turbines exhibit a greater variation by the fuel-cost functions, where the valve point results in the ripple form of the heat-rate curve and the cost function contains higher order nonlinearity due to the valve-point effects [18], as shown in Fig.(1). The more general fuel cost function of each thermal generator, considering the valve-point effect, is expressed as the sum of a quadratic and a sinusoidal function. The total non-smooth fuel cost function in terms of real power output can be expressed as [19]:

\[
C_{in}\left(P_{im}\right) = a_{i} + b_{i}P_{im} + c_{i}P_{im}^{2} + d_{i}\sin\left(e_{i}\left(P_{i,min} - P_{im}\right)\right)
\]

Where \( a_{i}, b_{i}, c_{i}, d_{i}, \text{and } e_{i} \) are cost coefficients of \( i \) th unit.
B. Real Power Operating Limits:

\[ P_{i,\text{min}} \leq P_i \leq P_{i,\text{max}} \]  

(4)

C. Generating unit ramp rate limits:

ODD problem is an extension of SED to determine the generation schedule of the committed units so as to meet the predicted load demand over a certain period of time, economic dispatch period, at minimum operating cost under ramp rate constraints and other constraints. The ramp rate constraint is a dynamic constraint which used to maintain the life of the generators. If the demand of the system was divided into \( M \) intervals, the generating unit ramp rate limit, up and down rates, must be considered as following [20]:

\[
\begin{align*}
P_{\text{in}} - P_{\text{r-(n-1)}} & \leq UR_{\text{in}} \\
P_{\text{r-(n-1)}} - P_{\text{in}} & \leq DR_{\text{in}}
\end{align*}
\]

(5)

D. Stochastic WP Constraint:

Using Weibull PDF of wind power [13]:

\[
\Pr \left( \sum_{i=1}^{M} P_W + \Omega(W) \leq P_d + P_{\text{Losses}} \right) = \Pr \left( \Omega(W) \leq P_d + P_{\text{Losses}} - \sum_{i=1}^{M} P_i \right) = 1 + \exp \left\{- \frac{v_{\text{in}}}{c} \right\}
\]

\[
\exp \left[ - \frac{1}{w_c} \left( v_{\text{in}} - v_{\text{in}} \right) \left( P_d - \sum_{i=1}^{M} P_i \right)^k \right]
\]

Where:
- \( c \): Scale factor of the Weibull distribution
- \( k \): Shape factor of the Weibull distribution
- \( v_{\text{in}} \), \( v_{\text{in}} \): Rated, cut-in, and cut-out wind speeds
- \( w_c \): Rated power generated by WPG

E. EPA Constraint:

The atmospheric pollutants such as sulphur oxides (SOx) and nitrogen oxides (NOx) caused by conventional thermal units can be modeled separately. However, the total emission of these pollutants which is the sum of a quadratic and an exponential function can be expressed as [21]:

\[
E_{\text{in}}(p_{\text{in}}) = \alpha_i + \beta_i P_{\text{in}} + \gamma_i P_{\text{in}}^2 + \eta_i \exp(\delta_i P_{\text{in}}) \leq ME
\]

(7)

Where: \( ME \) is the maximum allowable amount of pollutant during the dispatch period which is the EPA’s hourly emission target [17].

III. RESULTS AND DISCUSSION

The practical DED problems have non-smooth cost functions with equality and inequality constraints in addition to the stochastic wind power constraint that make the problem of finding the global optimum difficult using any mathematical approaches. In this paper, Genetic Algorithm Toolbox in MATLAB is used to deal with our problem. A 69-bus ten-unit test system with non-smooth fuel cost function is used in this paper to demonstrate the performance and the effectiveness of the proposed model compared with static economic dispatch model which neglects the generating unit ramp rate limits. The demand of the system was divided into 10 intervals. Unit data was taken from [20] can be found in the Appendix.

A. Table 1 shows intervals generation schedule, wind power generation, and optimum cost obtained from static economic dispatch and Table 2 shows intervals generation schedule, wind power generation, and optimum cost obtained from dynamic economic dispatch. Comparing Table 1 and Table 2, we can see that optimal total cost when considering DED is 804538.6 $ / hr, while it is 792400.2 $ / hr when considering SED. This difference (12138.4 $ / hr) is due to considering the ramp rate limits of the generators. But the SED is not reliable and not applicable due to neglecting the ramp rate limit of the generating units. Hence the SED may fail to deal with the large variations of the load demand due to the ramp rate limits of the generators.

Owing to the large variation of the customers load demand and the dynamic nature of the power system, it is necessary the investigation of DED problem. Also it can be concluded that for the large load variation, for example fourth interval (at \( PD = 1776 \) MW), the optimal cost in DED (97209.8 $ / hr) is much higher than that for SED (94755.7 $ / hr). While for the small load variation, for example the eighth interval (at \( PD = 1776 \) MW), the optimal costs on DED and SED are the same (94755.7 $ / hr).
### TABLE I

**OPTIMUM SOLUTION OBTAINED FROM STATIC ECONOMIC DISPATCH**

<table>
<thead>
<tr>
<th>( P_0 ) (MW)</th>
<th>1036</th>
<th>1258</th>
<th>1480</th>
<th>1776</th>
<th>1850</th>
<th>1776</th>
<th>1480</th>
<th>1776</th>
<th>1332</th>
<th>1184</th>
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<tr>
<td>( P_{1, \text{opt}} )</td>
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<td>150.1643</td>
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<td>150.0000</td>
<td>240.4172</td>
<td>150.0000</td>
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<td>150.0000</td>
<td>150.0000</td>
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</tr>
<tr>
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<td>135.1057</td>
<td>135.0000</td>
<td>197.8265</td>
<td>257.2515</td>
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<td>135.0000</td>
<td>197.8265</td>
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<td>135.0000</td>
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<tr>
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<td>173.9701</td>
<td>185.1997</td>
<td>294.8253</td>
<td>228.3312</td>
<td>294.8253</td>
<td>185.1997</td>
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<td>241.2457</td>
<td>133.7098</td>
<td>241.2457</td>
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<td>120.4152</td>
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<td>243.0000</td>
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<td>243.0000</td>
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<td>160.0000</td>
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<td>130.0000</td>
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<td>120.0000</td>
<td>120.0000</td>
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<td>( P_{9, \text{opt}} )</td>
<td>52.0571</td>
<td>44.8768</td>
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<td>80.0000</td>
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</tr>
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<td>10.2003</td>
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</table>

\( \Sigma (P_{\text{opt}}) \) = 898.481

WP (%)\(^a\) = 14.6928

Losses (MW) = 22.2457

Min Cost = 792400.2

Total Cost = 792400.2

\( ^a \) Wind Power Percentage of the total load

### TABLE II

**OPTIMUM SOLUTION OBTAINED FROM DYNAMIC ECONOMIC DISPATCH**

<table>
<thead>
<tr>
<th>( P_0 ) (MW)</th>
<th>1036</th>
<th>1258</th>
<th>1480</th>
<th>1776</th>
<th>1850</th>
<th>1776</th>
<th>1480</th>
<th>1776</th>
<th>1332</th>
<th>1184</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{1, \text{opt}} )</td>
<td>150.0014</td>
<td>192.0971</td>
<td>150.0000</td>
<td>189.2877</td>
<td>240.4172</td>
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<td>( P_{2, \text{opt}} )</td>
<td>135.0022</td>
<td>135.0000</td>
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<td>197.8265</td>
<td>135.0000</td>
<td>135.0000</td>
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<td>123.7257</td>
<td>203.7257</td>
<td>283.7257</td>
<td>228.3312</td>
<td>296.1897</td>
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<td>10.0000</td>
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</table>

\( \Sigma (P_{\text{opt}}) \) = 898.322

WP (%)\(^a\) = 14.6928

Losses (MW) = 10.2849

Min Cost = 55749.4

Total Cost = 804538.6

\( ^a \) Wind Power Percentage of the total load
The total transmission losses is 343.4027 MW during the whole dispatching period with using SED, while it is 398.7680 MW with using the DED. This increase in transmission losses is also due to limiting the generating units output with the ramp rate limit. Although the DED increases the optimal total operating cost and increases the transmission losses, it is necessary to obtain realistic, practical, and applicable economic dispatch model. The used strategy, referred to as the here-and-now approach, avoids the probabilistic infeasibility caused by using the average of random variables appearing in conventional models.

**IV. CONCLUSION**

This paper presents a new DED model including generator ramp rate limitation for the system consisting of both thermal generators and wind turbines. This model is more realistic, practical, and accurate economic dispatch model. The proposed model will minimize the risk due to uncertainty, and hence will minimize the required spinning reserve. We can conclude that the proposed DED model will provide valuable information and suggestions for safe, reliable, and economic operation of power systems. So the obtained results would provide direct guidelines for system operators to make correct decisions to schedule the system with WP.

Another important area of future research is that the total WP is characterized by a single random variable, this assumes that all wind turbines are located in a coherent geographic area. This remains a challenge for future work. To analytically remove this assumption, the correlated Weibull distribution is needed. Hence, from the Weibull distribution model of each WP cluster, the correlated Weibull distribution (Multivariate Distributions according to Probability Theorems) of the sum of WP will be derived in the next publication to be used in our models.

**APPENDIX**

**TABLE**

**CONVENTIONAL GENERATORS CHARACTERISTICS**

<table>
<thead>
<tr>
<th>Generator i</th>
<th>$P_i$</th>
<th>$P_{max}$</th>
<th>$a_i$</th>
<th>$b_i$</th>
<th>$c_i$</th>
<th>$d_i$</th>
<th>$e_i$</th>
<th>$f_i$</th>
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<th>$h_i$</th>
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<td>MW</td>
<td>$$/h$</td>
<td>$$/h$</td>
<td>$$/kW$</td>
<td>$$/h$</td>
<td>$$/h$</td>
<td>$$/h$</td>
<td>$$/h$</td>
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</table>

The wind parameters were: $c = 15 \text{ m/s}$, $k = 1.7$, $w_r = 150 \text{ MW}$.

The Transmission Losses Coefficients: $B = 10^{18}$.

**REFERENCES**


