Volterra Filter for Color Image Segmentation

M. B. Meenavathi, and K. Rajesh

Abstract—Color image segmentation plays an important role in computer vision and image processing areas. In this paper, the features of Volterra filter are utilized for color image segmentation. The discrete Volterra filter exhibits both linear and nonlinear characteristics. The linear part smoothes the image features in uniform gray zones and is used for getting a gross representation of objects of interest. The nonlinear term compensates for the blurring due to the linear term and preserves the edges which are mainly used to distinguish the various objects. The truncated quadratic Volterra filters are mainly used for edge preserving along with Gaussian noise cancellation. In our approach, the segmentation is based on K-means clustering algorithm in HSI space. Both the hue and the intensity components are fully utilized. For hue clustering, the special cyclic property of the hue component is taken into consideration. The experimental results show that the proposed technique segments the color image while preserving significant features and removing noise effects.

Keywords—Color image segmentation, HIS space, K–means clustering, Volterra filter.

1. INTRODUCTION

Image segmentation is an important step for many image processing and computer vision algorithms. The interest is motivated by applications over a wide spectrum of topics. For example, analyzing different regions of an aerial photo helps to better understand the vegetation cover. Scene segmentation is helpful to retrieve images from large image databases for content-based image retrieval [1] [2]. Most of the segmentation methods require image features that characterize the regions to be segmented. Particularly, texture and color have been independently and extensively used [3] [4].

The color information is a multidimensional vector and hence the segmentation techniques for gray images cannot be directly applied. The existing color image segmentation techniques can be broadly classified into five approaches based on edge detection, region growing, neural network based, fuzzy based, and histogram thresholding.

An edge detector finds the boundary of an object exploiting the fact that the pixel intensity values change rapidly at the boundary of two regions. Sobel, Prewitt and Roberts are some of the examples of edge detector operators [12] [13]. For color images the edge detection can be performed on color components (Red, Green, and Blue) separately. These edges are merged to get the final image. Jie and Fei [16] proposed an algorithm for natural color image segmentation, where edges are calculated in terms of high phase congruency in the gray level image. It uses a K-means clustering algorithm to label the long edge lines. The global color information is used to detect approximately the objects within an image, while the short edges are merged based on their positions.

Region growing techniques are used to find the homogeneous regions in an image [14]. Here, we need to assume a set of seed points initially. The homogeneous regions are formed by attaching to each seed point those neighboring pixels that have correlated properties [9] [15]. This process is repeated until all the pixels within an image are classified. However, the obscurity with region based approach is the selection of initial seed points.

Neural networks are formed by several elements that are connected by links with variable weights [6]. Artificial neural networks are widely applied for pattern recognition. Their processing potential and nonlinear characteristics are used for clustering [5]. Self organization of Kohonen Feature Map (SOFM) network is a powerful tool for clustering [19]. Ji and Park [18] proposed an algorithm for watershed segmentation based on SOM. This method finds the watershed segmentation of luminance component of color image. Li and Li [20] proposed an unsupervised algorithm for color image segmentation. The algorithm uses a neural network to extract features of the image automatically. The multiple color features are analyzed using a self organizing feature map (SOFM). Then the useful feature sequence is determined. The encoded feature vector is used for final segmentation.

Fuzzy set theory gives a mechanism to represent ambiguity within an image [8]. Each pixel of an image has a degree of belongingness to a region or a boundary. A number of fuzzy approaches for image segmentation are reported in [7] [10].

Histogram thresholding is one of the popular methods for monochrome image segmentation [21]. Considering the fact that an image consists of different regions corresponding to the gray level ranges, the histogram of an image can be separated using peaks corresponding to different regions. A threshold value corresponding to the valley between two adjacent peaks can be used to separate the object. Guteman [11] proposed a neural network based adaptive thresholding segmentation algorithm for monochrome image. The main advantage of this method is that, it does not require a priori knowledge about number of objects in the image. Navon et al [17] proposed an algorithm for color image segmentation using a local threshold values. This technique divides an
image into homogeneous regions by using a local threshold values. It calculates the threshold values automatically with the help of merging process.

In this paper, we present a method for segmenting color images in HSI space for Volterra filtered images. The Volterra filter enhances the uniform zones by preserving edges. Hence segmentation of filtered images gives more features compared with other conventional segmentation methods. Here, segmentation is done in HSI space using K-means clustering technique [22]. HSI color representation is compatible with the vision psychology of human eyes [23]. Both the hue and intensity components of HSI are utilized. The cyclic property of the hue component is also taken into consideration.

### II. HSI COLOR REPRESENTATION

In the HSI color representation, I component represents the intensity, H component represents the hue and S component represents the saturation. RGB representation is transformed to HSI representation by:

\[
\begin{bmatrix}
Y \\
C_1 \\
C_2
\end{bmatrix} = \begin{bmatrix}
1/3 & 1/3 & 1/3 \\
1 & -1/2 & -1/2 \\
0 & -\sqrt{3}/2 & \sqrt{3}/2
\end{bmatrix} \begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

Then HSI values can be given as:

\[
H = \left\{ \begin{array}{ll}
\text{Arc cos}(C_1/S) & C_1 \geq 0 \\
2\pi - \text{Arc cos}(C_1/S) & C_1 < 0
\end{array} \right.
\]

(2)

The H component is an angular value and it displays a special cyclic property. The cyclic property of the hue component is the most challenging aspect of color image segmentation in the HSI space. In hue space, we should redefine the distance and the center, which are the basis of clustering algorithms. Considering the cyclic property of hue values, the following definitions are given.

**Definition 1:** The distance between two Hue values \(H_1\) and \(H_2\) is:

\[
d(H_1, H_2) = \left\{ \begin{array}{ll}
|H_1 - H_2| & |H_1 - H_2| \leq \pi \\
2\pi - |H_1 - H_2| & |H_1 - H_2| > \pi
\end{array} \right.
\]

(3)

**Definition 2:** The directed distance between two Hue values \(H_1\) and \(H_2\) is:

\[
\overline{d}(H_1, H_2) = \left\{ \begin{array}{ll}
H_2 - H_1 & |H_2 - H_1| \leq \pi \\
H_2 - H_1 - 2\pi & |H_2 - H_1| > \pi
\end{array} \right.
\]

(4)

**Definition 3:** The interval \([H_1, H_2]\) determined by two Hue values \(H_1\) and \(H_2\) in the hue space is a set of points in the hue space:

\[
\text{if } |H_1 - H_2| \leq \pi \quad \{H_1, H_2\} = \{H/|H_1 \leq H \leq H_2]\}
\]

\[
\text{if } |H_1 - H_2| > \pi \quad \{H_1, H_2\} = \{H/[\max(H_1, H_2) \leq H \leq 2\pi + \max(H_1, H_2)]\}
\]

and the midpoint \(H_m\) of the interval is defined as:

\[
H_m = \left\{ \begin{array}{ll}
(H_1 + H_2)/2 & |H_1 - H_2| \leq \pi \\
(H_1 + H_2)/2 - \pi, & |H_1 - H_2| > \pi
\end{array} \right.
\]

(5)

**Definition 4:** Let \(x_1, x_2, x_3, ..., x_n\) are \(n\) points in the hue space in the interval \([H_1, H_2]\). The center point of \(x_1, x_2, x_3, ..., x_n\) is the point in the hue space that satisfies:

\[
\sum_{i=1}^{n} \overline{d}(X_i, X) = 0 \quad \text{and} \quad X \in [H_1, H_2]
\]

(6)

The second condition in (6) is to prevent the ambiguity of the center point, or to ensure the center point falls within the interval \([H_1, H_2]\).

**Euclidean theory in hue space:** \(x_1, x_2, x_3, ..., x_n\) are \(n\) points in the hue space. All the points fall within the interval \([H_1, H_2]\). The mid point of \([H_1, H_2]\) is \(H_m\). The center point of \(x_1, x_2, x_3, ..., x_n\) can be given by the following equation:

\[
X = H_m + \frac{1}{n} \sum_{i=1}^{n} \overline{d}(H_m, X_i)
\]

(7)

### III. HSI COLOR SEGMENTATION

Good color segmentation algorithms should consider both hue and intensity for segmentation [2]. In some cases, because of the occlusion and the variation of the projected light intensity, the brightness of the same object surface will not be uniform. However, the hue values determined by the reflective property of the object surface are relatively stable.

**Proposed Methodology:**

The proposed method for segmentation of color images consists of the following four steps.

**Step 1:** The input noisy image is filtered using Volterra filter.

**Step 2:** Compute the histogram of both the Hue and the intensity components.

**Step 3:** Apply K-means clustering technique to obtain segmented images.

**Step 4:** Finally both segmented H and I images are combined to obtain desired segmented output image.

The block diagram of the proposed method is as shown in Fig. (1). The Volterra filtered color image is transformed to...
HSI color space and is used for K-means clustering. K-means is one of the simplest unsupervised learning algorithms that solve the well known clustering problem. The procedure follows a simple and easy way to classify a given data set into a number of clusters. The main idea is to define K-centroids, one for each cluster. Take each point belonging to a given data set and associate it to the nearest centroid.

### IV. EXPERIMENTAL RESULTS

The proposed method is a general purpose tool for the segmentation of synthetic and real world color images. It has been tested using MATLAB 7X environment on P-IV, 2.8 GHz machine. After filtering the noisy images using Volterra filter [24], segmentation is performed based on K-means clustering in HSI space. Several experiments have been conducted on synthetic and real world images. The performance of the segmentation based on the proposed Volterra filter is analyzed using the quantitative parameters such as cluster validity factor $S_J$ and squared error function $J$. These parameters are computed as:

$$S_J = \frac{\min \sum_{i=1}^{d} \sum_{j=1}^{k} (m_{ij} - m_j)^2}{\frac{1}{n_k} \sum_{i=1}^{n_k} \sum_{j=1}^{d} (x_{ij} - m_j)^2}$$  \hspace{1cm} (10)$$

and

$$J = \frac{\sum_{i=1}^{d} \sum_{j=1}^{k} \left\| x_i - c_j \right\|^2}{n_k}$$  \hspace{1cm} (11)$$

where

- $n_k \rightarrow$ Number of patterns in cluster $k$
- $m_j \rightarrow$ Cluster center for cluster $k$ along feature $j$
- $d \rightarrow$ The number of features
- $x_{ij} \rightarrow$ Value of the $j^{th}$ feature for the $i^{th}$ pattern belonging to cluster $k$
- $c_j \rightarrow$ Chosen data point
- $\rightarrow$ Cluster center

To test the robustness and efficacy of our method different noise is added to the original image. In our work Gaussian and mixed Gaussian-impulse noise is considered. Segmentation is performed for noisy images using conventional K-means clustering and our proposed technique. Table I lists the cluster validity factor and squared error for the segmentation of different images.

Fig. 2 (a) shows a synthetic color image representing different geometrical shapes. The use of synthetic images for which ground truth is known enables us to give some quantitative estimate of the segmentation. For testing the proposed approach under noise condition an additive impulse noise with density 0.02 and zero mean Gaussian noise with variance of 0.01 are added and is shown in Fig. 2(b). The segmentation of the noisy image using k-means clustering is shown in Fig. 2(c). The noisy image is filtered by Volterra filter with 3X3 linear and 9X9 nonlinear masks. The Filter coefficients are calculated using FIR-Hamming window technique. The coefficients are arranged in the form of block lexicographic matrix and are reduced based on symmetry conditions [24]. The filtered image is segmented based on K-means clustering with known centroid values as is shown in Fig. 2(d). From Fig. 2(d) it is observed that by using Volterra filter output, the segmentation accuracy could be increased. Since we know the number of clusters priori in the synthetic images, the squared error function is computed. The squared mean error is minimized using our method and is as shown in Table I.

To test the performance of our approach, we conducted experiments on several real world images. Fig. 3(a) shows a real world color image of radish. Gaussian noise of zero mean and variance of 0.01 is added to original image and is shown in Fig. 3(b). The Hue and Intensity images of the noisy image are shown in Fig. 3(c) and 3(d) respectively. The hue attribute gives the pure color components of the image. From Fig. 3(c) we observe that the original image has only two prominent color components.
The intensity attribute is a subjective descriptor and is a key factor in describing the color sensation of various objects of the image and is as shown in Fig. 3(d). The brightness image shown in Fig. 3(d) embodies the achromatic notation of intensity attribute and is definitely measurable and interpretable. The hue and intensity component images are filtered using Volterra filter. For the Hue image the threshold value $t$ is $62 < t < 154$ and for the intensity image $t$ is $130 < t < 230$. The filtered images are given in Fig. 3(e) and 3(f) respectively. The K-means clustered segmentation of the filtered hue component is as shown in Fig. 3(g). Here, the centroid of the hue components is 66 and 160 when the number of clusters, $k = 2$. The objects of the hue components are assigned to 66 and 160 and recalculated to obtain the new centroid. The new centroid values are 62 and 154 and they do not change further. Similarly the segmented image of the intensity component with centroid values 130 and 230 is shown in Fig. 3(h). To test the correctness of our approach, conventional K-means clustering is performed for original image with centroids 66 and 207 and is shown in Fig. 3(i). The segmented hue and intensity image after the Volterra filtering are combined and is shown in Fig. 3(j). From Fig. 3(j) we observe that the Volterra filtered image in HSI space gives relatively good segmentation results. The cluster validity factor and squared mean error of the same are shown in Table I.

<table>
<thead>
<tr>
<th>Image</th>
<th>Cluster validity factor</th>
<th>Squared mean error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k-means clustering</td>
<td>Filtered k-means clustering</td>
</tr>
<tr>
<td></td>
<td></td>
<td>k-means clustering</td>
</tr>
<tr>
<td>Synthetic ($k=6$)</td>
<td>6.147</td>
<td>0.285</td>
</tr>
<tr>
<td>Radish ($k=2$)</td>
<td>3.874</td>
<td>0.415</td>
</tr>
<tr>
<td>Pepper ($k=2$)</td>
<td>5.159</td>
<td>0.314</td>
</tr>
</tbody>
</table>

![Fig. 3 Segmentation of real image](image)

(a) Original image (b) Noisy image (c) H component image (d) I component image (e),(f) Volterra filtered H&I components (g),(h) Segmentation of H&I components (i) Segmentation using k-means clustering (j) Segmentation using proposed method
To test the robustness and noise withstanding capability of our method, an additional experiment is conducted with mixed Gaussian-impulsive noise. In this experiment the noise density was increased from 0 to 67 percent and the cluster validity factor and squared mean error were evaluated. Results of this experiment show that the noise with less than 30% density has no effect on the accuracy of the segmentation. In contrast the conventional K-means clustering lose robustness against noise from 12%. After this threshold the centroid values vary randomly and hence it is difficult to get the constant values for centroid. When the noise density exceeds 24% the k-means clustering totally fails to fix the centroid values. But our method works satisfactorily up to 67% of the noise density and overall 92% of image area was correctly classified. Fig. 4 (a) shows an original pepper image. The noisy image with Gaussian white noise of variance 0.05 and impulse noise with density 0.04 is shown in Fig. 4 (b). The output of the k-means clustering with centroid values 58 and 181 is as shown in Fig. 4 (c) and 4 (d). The segmentation result of Volterra filtered image with threshold value 30 < \( t < 220 \) is as shown in Fig. 4 (e) and 4 (f). From Fig. 4 (e) and 4 (f), it is observed that by using Volterra filtered image the segmentation gives satisfactory results.

Fig. 5 draws the centroid values for red and green components versus noise density. From this, we observed that, the initial centroid value for the red component is 30 for conventional k-means segmentation and 34 for Volterra filtered segmentation. Similarly, for green component the centroid values are 130 and 185 respectively. If the noise density is increased, the centroid values of conventional k-means clustering changes randomly, whereas the changes in Volterra filtered segmentation is very less. Hence, we can fix the centroid values easily and accurately. In the experiment, the noisy density is increased from 0 to 70%. The proposed segmentation technique works satisfactorily up to 67% of noise and fixes the final centroid values 220 for green component and 62 for red component. But, in case of conventional k-means clustering the centroid values start to vary when the noise density is 12% and it will not fix the new stable centroid values.

The performance of the Volterra filter for segmentation purpose using luminance values can be shown graphically in Fig. 6. The row 128 of K-means clustered image and Volterra segmented image are shown in Fig. 6 (a) and 6 (b) respectively. From Fig. 6 (b), we observed that the Volterra segmented image has more smoothed uniform segments with preserved edges and are shown with arrows.
V. CONCLUDING REMARKS

In this paper we used the features of Volterra filter for color image segmentation. The linear characteristics of Volterra filter is utilized for getting homogeneous regions and the nonlinear characteristics are used to isolate the different segmented regions. Since Volterra filter has the ability of smoothing the image by preserving the edges, it is most suitable for segmentation purpose. Segmentation using K-means clustering does not give the proper centroid values for noisy images. Volterra filter is also effective for segmenting Gaussian and mixed Gaussian-impulse noise images. It provides closely related centroid values for noisy images. The performance of our method is evaluated using the parameters such as cluster validity factor and squared error function for different images and satisfactory results are obtained. The results indicate that the proposed approach is more robust and accurate than conventional segmentation methods.

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