Verified Experiment: Intelligent Fuzzy Weighted Input Estimation Method to Inverse Heat Conduction Problem

Chen-Yu Wang, Tsung-Chien Chen, Ming-Hui Lee and Jen-Feng Huang

Abstract—In this paper, the innovative intelligent fuzzy weighted input estimation method (FWIEM) can be applied to the inverse heat transfer conduction problem (IHCP) to estimate the unknown time-varying heat flux efficiently as presented. The feasibility of this method can be verified by adopting the temperature measurement experiment. We would like to focus attention on the heat flux estimation to three kinds of samples (Copper, Iron and Steel/AISI 304) with the same 3mm thickness. The temperature measurements are then regarded as the inputs into the FWIEM to estimate the heat flux. The experiment results show that the proposed algorithm can estimate the unknown time-varying heat flux on-line.

Keywords—Fuzzy Weighted Input Estimation Method, IHCP and Heat Flux.

NOMENCLATURE

<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$M$</td>
<td>Global conductance matrix</td>
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<tr>
<td>$N$</td>
<td>Total number of nodes</td>
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<tr>
<td>$n$</td>
<td>Total number of time steps</td>
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<td>$P$</td>
<td>Filter’s error covariance matrix</td>
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<td>$\sigma$</td>
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I. INTRODUCTION

Given the known initial conditions, boundary conditions, and the thermal properties of materials in the heat conduction problem, to investigate the temperature distribution in the solid by utilizing the heat-conducting equations is called the direct heat conduction problem (DHCP). On the other hand, the estimation of unknown heat flux, heat contact coefficient, heat conduction coefficient, and heat source by utilizing the temperature measurements inside the heat-conducting solid is called the inverse heat conduction problem (IHCP). Inverse problems in heat conduction have been of interest for many researchers in recent years [1]–[3]. It is sometimes necessary to calculate the transient surface heat flux and the surface temperature from a temperature measured at some location inside a body. Typical examples include the skin surface heat flux estimation for a reentry vehicle, new rocket heat-shield materials and heat-dissipating control of electronic devices [4].

The related researches about the inverse heat transfer experiment with modern estimation theory based on the Kalman filtering technique and the recursive least square algorithm are discussed as follows. Tuan [5] presented an adaptive weighted input estimation method, which combines the Kalman filter (KF) [6] with a recursive least square estimator (RLSE) [7]. The residual innovation sequence is...
generated by the Kalman filter and applied to the real-time recursive least square algorithm to estimate the unknown heat flux. The constant weighting factor is applied to the RLSE to emphasize the weights of the latest data. In order to improve the adaptation and estimation capability of the estimator, the adaptive weighting function is used to replace the constant weighting factor in 1998 [8]. Although the input estimates converge slowly in the initial time when the adaptive weighting function is used in the RLSE, the estimator has relatively better overall tracking performance when the unknown input is time-varying regardless of the influence of the measurement noise interference. A non-destruction ballistic experimental method was established in 2006 to measure the temperature by using the thermocouple equipped on the outer wall of gun barrel during the firing process [9]. A feasibility investigation to estimate the heat flux is produced with regard to the estimation of the high temperature due to the rapidly burning propellant in the inner wall of the gun tube [10]. The input is time-varying and may not be predicted easily. As the result, it is difficult to choose an adaptive and efficient weighting function. To resolve this situation, Chen et al. [11], [12] in 2008 presented an intelligent fuzzy weighting function to replace the weighting factor, of the RLSE.

Improving the weighting efficiency of the RLSE is essential, because the unknown input is time-varying and changes continuously. The adaptive weighting function takes any input variation into consideration. Therefore, the inverse method is developed to rapidly track the target and effectively reduce the effect due to the noise. In this paper, the feasibility of this method can be verified by adopting the temperature measurement experiment. Three different kinds of samples (Copper, Iron and Steel/AISI 304) with the same 3mm thickness are used. The bottoms of samples are heated by applying the standard heat source. The thermocouples are used to measure the temperatures on the top of samples. The temperature measurements are then regarded as the inputs into the presented method, which can estimate the heat flux in the bottoms of samples. The influence on the estimation will be investigated by utilizing the experiment verification. The results show that this method is efficient and robust to estimate the unknown time-vary heat flux.

II. EXPERIMENT EQUIPMENT

The entire experiment modular includes the signal source, the test samples, the sensors, the data acquisition device, and the computer module. The purpose of experiment is to inversely estimate the temperature and heat flux of the sample by using the temperature measurements of the sample surface. Therefore, the samples are heated by adopting the standard heat source, and the thermocouples (K type) are equipped on the sample surface. The structure chart of experiment are shown in Figure 1. The experiment devices and the samples used are illustrated as follows:

A. The signal source:

The standard heat source generator with the maximum output power of 200W is compatible with an alternating/direct current power source of 110 volt. The 60 VDC power is series connected and can supply stable power to the heater. The standard heat source generator provides heat from its bottom layer. The inner wall and top of this device is insulated. It is a critical technique to perform the experiment in the insulated condition for the direct heat conduction problem. In addition, 5 holes with the diameter of 2mm have been punched on the top of the generator for the thermocouples to measure the surface temperatures of the test samples.

B. The test samples:

The pure copper, iron and stainless steel (AISI 304) with the same thicknesses of 3mm are used. The following thermal properties are used in the calculation.

- Copper: \( \rho = 801 \), \( \alpha = 8933 \), and \( g = 385 \).
- Iron: \( \rho = 80.2 \), \( \alpha = 7870 \), and \( g = 447 \).
- Steel: \( \rho = 14.9 \), \( \alpha = 7900 \), and \( g = 477 \).

C. Sensors:

The thermocouples (K type) are used in this experiment.

D. The temperature data acquisition device:

The device with the type of NI-9211 is manufactured by the National Instruments Company and can be used to implement the signal acquisition, procedure and transformation. It is composed of the high performance measurement and control card, the signal process modular, the filter amplifier, and the electric charge amplifier.

E. The computer module (including the software programs):

1. Intel processor 2.1Ghz computer, the signal express software, and the Matlab programming language can be used to process the signal data.
2. The SIGNAL EXPRESS acquisition software: The software in coordination with the data acquisition system developed by the National Instruments Company can collect data from the subject system in real time. The sampling rate, the temperature range, the sampling time, the sensor type, the compensation of the cold junction, the frequency channel, and the record style to record the real-time signal of the system can be configured.
3. The FWIEM algorithm can be programmed by using the Matlab programming language. The temperature measurements are then regarded as the inputs into the method, which is to estimate the heat flux in the bottoms of samples.
III. MATHEMATICAL FORMULATION

The boundary condition at the position, \( x = x_s \), is assumed to be heat-insulated. By equipping the thermocouple sensor at the position, \( x = x_s \), and measuring the surface temperature of the test sample, the heat-conducting model is formed as shown in Figure 2.

The heat-conducting governing equations are as follows:

\[
\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \quad 0 < x < x_s, \quad 0 < t \leq t_f \tag{1}
\]
\[
T(x, 0) = T_0 \quad 0 \leq x \leq x_s, \quad t = 0 \tag{2}
\]
\[
-k \frac{\partial T(0, t)}{\partial x} = q(t) \quad x = 0, \quad t > 0 \tag{3}
\]
\[
\frac{\partial T(x_s, t)}{\partial x} = 0 \quad x = x_s, \quad t > 0 \tag{4}
\]
\[
Z(t) = T(x_s, t) + v(t) \quad x = x_s, \quad t > 0 \tag{5}
\]

where \( T(x, t) \) represents that the temperature is a function of time, \( t \), and the position, \( x \). \( T_0 \) is the initial temperature. \( k \) is the heat-conducting coefficient of the sample material. \( \rho c_p \) is the heat-melting coefficient of the sample material. \( \alpha = k/\rho c_p \) is the heat-diffusing coefficient. \( v(t) \) is the measurement noise. \( z(t) \) is the temperature measurement, which is assumed to be the Gaussian white noise with zero mean. By using the central differential method to disperse Equation (1) with respect to the space derivative, the following equation is obtained.

\[
\dot{T}_i(t) = \frac{\alpha}{\Delta x} \left[ T_{i+1}(t) - 2T_i(t) + T_{i-1}(t) \right] \quad \text{for} \quad i = 1, 2, \ldots, N \tag{6}
\]

where \( \Delta x = x_s / (N - 1) \) is the space interval, and \( T_i(t) = T(x_i, t) \). By setting \( i = 0 \), the boundary condition, Equation (3), at the position, \( x = 0 \), can be written as follows:

\[
-k \frac{\partial T_1(t)}{\partial x} = \frac{T_2(t) - T_1(t)}{2\Delta x} = q(t) \tag{7}
\]

\[
T_0(t) = T_1(t) + \frac{2\Delta x}{k} q(t) \tag{7}
\]

By substituting Equation (7) in Equation (6), the time derivative equation when \( i = 1 \) can be obtained as the following equation.
\[ T_i(t) = \frac{\alpha}{\Delta x} \left[ -2T_i(t) + 2T_{i+1}(t) \right] + \frac{2q(t)}{k\Delta x} \]  
(8)

When \( i = 2, 3, \ldots, N-1 \) , the equation can be presented as follows:

\[ T_i(t) = \frac{\alpha}{\Delta x} \left[ T_{i+1}(t) - 2T_i(t) + T_{i-1}(t) \right] \]  
(9)

On the other hand, when \( i = N \) , and \( x = x_N = x_i \) , the boundary condition in Equation (4), can be presented as follows:

\[ T_N(t) = \frac{\alpha}{\Delta x} \left[ -2T_N(t) + T_{N-1}(t) \right] \]  
(10)

By rearranging Equations (8), (9) and (10) along with a simulated noise input, the continuous-time state equation can be obtained as the following:

\[ \dot{T}(t) = \Phi T(t) + \Omega q(t) + Gw(t) \]  
(11)

where \( w(t) \) is assumed to be the Gaussian white noise with zero mean, and it represents the modeling error. Furthermore, \( T(t) = \left[ T_1(t) T_2(t) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots T_N(t) \right]^T \)  
(12)

\[ \Omega = \left[ \begin{array}{cccc} 2 & 0 & \cdots & 0 \\ -2 & 2 & 0 & \cdots & 0 \\ 1 & -2 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & -2 & 1 \\ 0 & \cdots & 0 & \cdots & 2 \end{array} \right] \]  
(13)

\[ \Psi = \frac{\alpha}{\Delta x^2} \]  
(14)

The continuous-time state equation (Equation (11)), can be discretized with the sampling time, \( \Delta t \). The discrete-time state equation and its relative equations are shown as follows:

\[ X(k) = \Phi X(k) + \Gamma q(k) + w(k) \]  
(15)

\[ X(k) = \left[ T_1(k) T_2(k) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots T_N(k) \right]^T \]  
(16)

\[ \Phi = e^{\kappa \Delta t} \]  
(17)

\[ \Gamma = \int_{k\Delta t}^{(k+1)\Delta t} \exp \left[ \Psi \left[ \left( k+1 \right) \Delta t - \tau \right] \right] \Omega d\tau \]  
(18)

\[ w(k) = \int_{k\Delta t}^{(k+1)\Delta t} \Phi G(\tau) w(\tau) d\tau \]  
(19)

In the equations above, \( X(k) \) is the state vector. \( \Phi \) is the state transition matrix. \( \Gamma \) is the input matrix. \( q(k) \) is the definite input array. \( w(k) \) is the processing error input vector, which is assumed to be the Gaussian white noise with zero mean and with the variance, \( E\{w(k)w^T(j)\} = \sigma^2 \delta_{kj} \). \( \delta_{kj} \) is the Dirac delta function. The discrete-time measurement equation is shown below.

\[ Z(k) = HX(k) + v(k) \]  
(20)

where \( Z(k) \) is the observation vector at the kth sampling time. The measurement matrix, \( H = [0 \ 0 \ldots \ 1] \). \( v(k) \) is the measurement error vector, which is assumed to be the Gaussian white noise with zero mean and with the variance, \( E\{v(k)v^T(j)\} = \sigma^2 \delta_{kj} \). After the state equation is obtained, the inverse estimation process is carried out by using the on-line input estimation method, which is the combination of the Kalman filter mechanism and the adaptive fuzzy weighting function of the recursive least square estimation (RLSE) algorithm.

IV. THE INTELLIGENT FUZZY WEIGHTED RLSE INPUT ESTIMATION APPROACH

The conventional input estimation approach has two parts: one is the Kalman filter without the input term, and the other is the fuzzy weighted recursive least square estimator. The system input is the unknown time-varying heat flux. The Kalman filter is operating under the setting of the processing error variance, \( Q \), and the measurement error variance, \( R \). It is to use the difference between the measurements and the estimated values of the system temperature as the functional index. Furthermore, by using the fuzzy weighted recursive least square algorithm, the heat flux can be precisely estimated. The detailed formulation of this technique can be found in Ref.

A. The equations of the Kalman filter are shown as follows:

\[ \bar{x}(k / k-1) = \Phi \bar{x}(k-1 / k-1) \]  
(21)

\[ P(k / k-1) = \Phi P(k-1 / k-1) \Phi^T + Q \]  
(22)

\[ s(k) = HP(k / k-1)H^T + R \]  
(23)

\[ K(k) = P(k / k-1)H^T s^{-1}(k) \]  
(24)

\[ P(k / k) = [I - K(k)H]P(k / k-1) \]  
(25)

\[ \bar{x}(k / k) = \bar{x}(k / k-1) + K(k)[z(k) - \bar{x}(k / k-1)] \]  
(26)

\[ \bar{x}(k / k) = \bar{x}(k / k) + K(k)[z(k) - \bar{x}(k / k-1)] \]  
(27)

B. The recursive least square algorithm:

\[ B(k) = H[\Phi M(k-1) + I]\Gamma \]  
(28)

\[ M(k) = [I - K(k)H][\Phi M(k-1) + I] \]  
(29)

\[ K_s(k) = \gamma^{-1}P_s(k-1)B^T(k)[B(k)\gamma^{-1}P_s(k-1)B^T(k) + s(k)]^{-1} \]  
(30)

\[ P_s(k) = [I - K_s(k)B(k)]\gamma^{-1}P_s(k-1) \]  
(31)

\[ \hat{q}(k) = \hat{q}(k-1) + K_s(k)[\bar{Z}(k) - B(k)\hat{q}(k-1)] \]  
(32)

\[ \hat{x}(k / k - 1) = \Phi \hat{x}(k / k - 1) + \Gamma \hat{q}(k) \]  
(33)

\[ \hat{x}(k / k) = \hat{x}(k / k - 1) + K(k)\left[Z(k) - H \bar{x}(k / k - 1)\right] \]  
(34)

\[ \hat{q}(k) \] is the estimated input vector. \( P_s(k) \) is the error
covariance of the estimated input vector. B(k) and M(k) are the sensitivity matrices. \(K_i\) is the correction gain. \(\tilde{Z}(k)\) is the bias innovation produced by the measurement noise and the input disturbance. \(s(k)\) is the covariance of the residual. \(\gamma\) is the weighting constant or weighting factor.

\[C. The\ construction\ of\ the\ intelligent\ fuzzy\ weighting\ factor:\]

The fuzzy weighting factor is proposed based on the fuzzy logic inference system. It can be operated at each step based on the innovation from the Kalman filter. It performs as a tunable parameter which not only controls the bandwidth and magnitude of the RLSE gain, but also influences the lag in the time domain. To directly synthesize the Kalman filter with the estimator, this work presents an efficient robust forgetting method which efficiently and robustly estimates the unknown input effectively. For this reason, the weighting factor in the range between 0 and 1. The weighting factor \(\gamma(k)\) is employed to compromise between the upgrade of tracking capability and the loss of estimation precision. The relation has already been derived as follows (Tuan et al. 1998 [8]):

\[
\gamma(k) = \begin{cases} 
1 & \frac{|\tilde{Z}(k)|}{\sigma} \leq \gamma_k \\
\frac{1}{\gamma_k} & \frac{|\tilde{Z}(k)|}{\sigma} > \gamma_k
\end{cases} \tag{35}
\]

The weighting factor, \(\gamma(k)\), as shown in Equation (34) is adjusted according to the measurement noise and input bias. In the industrial applications, the standard deviation \(\sigma\) is set as a constant value. The magnitude of weighting factor is determined according to the modulus of bias innovation, \(\frac{|\tilde{Z}(k)|}{\sigma}\). The unknown input prompt variation will cause the large modulus of bias innovation. In the meantime, the smaller weighting factor is obtained when the module of bias innovation is larger. Therefore, the estimator accelerates the tracking speed and produces larger vibration in the estimation process. On the contrary, the smaller variation of unknown input causes the smaller modulus of bias innovation. In the meantime, the larger weighting factor is obtained according to the small modulus of bias innovation. The estimator is unable to estimate the unknown input effectively. For this reason, the intelligent fuzzy weighting factor for the inverse estimation method which efficiently and robustly estimates the time-varying unknown input will be constructed in this research.

The intelligent fuzzy weighted input estimation method is derived following as:

The range of fuzzy logic system input, \(\theta(k)\), may be chosen in the interval, \([0,1]\). The input variable is defined as:

\[
\theta(k) = \frac{\sqrt{2\Delta \tilde{Z}(k)}}{\sqrt{2\Delta \tilde{Z}(k) + \Delta t^2}} \tag{36}
\]

where \(\Delta \tilde{Z}(k) = \tilde{Z}(k) - \tilde{Z}(k-1)\). \(\Delta t\) is the sampling interval. The proposed intelligent fuzzy weighting factor uses the input variable \(\theta(k)\) to self-adjust the factor \(\gamma(k)\) of the recursive least squares estimator. Therefore, the fuzzy logic system consists of one input and one output variables. The range of input, \(\theta(k)\), may be chosen in the interval, \([0,1]\), and the range of output, \(\gamma(k)\), may also be in the interval, \([0,1]\).

The fuzzy sets for \(\theta(k)\) and \(\gamma(k)\) are labeled in the linguistic terms of EP (extremely large positive), VP (very large positive), LP (large positive), MP (medium positive), SP (small positive), VS (very small positive), and ZE (zero). The specific membership is defined by using the Gaussian functions.

A fuzzy rule base is a collection of fuzzy IF-THEN rules:

- IF \(\theta(k)\) is zero (ZE), THEN \(\gamma(k)\) is an extremely large positive (EP);
- IF \(\theta(k)\) is a very small positive (VS), THEN \(\gamma(k)\) is a very large positive (VP);
- IF \(\theta(k)\) is a small positive (SP), THEN \(\gamma(k)\) is a large positive (LP);
- IF \(\theta(k)\) is a medium positive (MP), THEN \(\gamma(k)\) is a medium positive (MP);
- IF \(\theta(k)\) is a large positive (LP), THEN \(\gamma(k)\) is a small positive (SP);
- IF \(\theta(k)\) is a very large positive (VP) THEN \(\gamma(k)\) is a very small positive (VS);
- IF \(\theta(k)\) is an extremely large positive (EP) THEN \(\gamma(k)\) is an extremely small positive (ES),

where \(\theta(k) \in U\) and \(\gamma(k) \in V \subset R\) are the input and output of the fuzzy logic system, respectively. Therefore, the nonsingleton fuzzier can be expressed as the following equation:

\[
\mu_{a}\left(\theta(k)\right) = \exp \left(-\frac{(\theta(k) - \bar{z}_a)^2}{2\sigma_a^2}\right) \tag{37}
\]

\(\mu_{a}\left(\theta(k)\right)\) decreases from 1 as \(\theta(k)\) moves away from \(\bar{z}_a\). \(\sigma_a^2\) is a parameter characterizing the shape of \(\mu_{a}\left(\theta(k)\right)\).

The Mamdani maximum-minimum inference engine was used in this paper. The max-min-operation rule of fuzzy implication is shown below:

\[
\mu_{b}\left(\gamma(k)\right) = \max_{a=1}^{c} \left\{ \min_{i=1}^{d} \left[ \mu_{a}\left(\theta(k)\right), \mu_{a-b'}\left(\theta(k), \gamma(k)\right) \right] \right\} \tag{38}
\]

where \(c\) is the fuzzy rule, and \(d\) is the dimension of input variables.

The defuzzier maps a fuzzy set \(B\) in \(V\) to a crisp point.
γ ∈ V. The fuzzy logic system with the center of gravity is defined below:

γ*(k) = \frac{\sum_{i=1}^{n} y_i \mu_B(\gamma_i(k))}{\sum_{i=1}^{n} \mu_B(\gamma_i(k))}

(39)

\( n \) is the number of outputs. \( y_i \) is the value of the \( l \)th output. \( \mu_B(\gamma_i(k)) \) represents the membership of \( \gamma_i(k) \) in the fuzzy set \( B \). Substituting \( \gamma*(k) \) of Equation (39) in Equations (30) and (31) allows us to configure an adaptive fuzzy weighting function of the recursive least square estimator (RLSE).

V. DISCUSSION OF THE EXPERIMENT AND ESTIMATION RESULTS

To verify the performance of the proposed method, a standard heat source is modeled. The heat flux in the bottom is estimated inversely by measuring the temperature on the top. The test sample is heated by the standard heat source with the fixed power. The test sample is heated in the bottom. The inner wall and the top of the environment are insulated. The thermocouples are placed at the location, \( x = 3 \) mm on the test sample. The total time period, \( t_f = 850 \) sec. The sampling interval, \( \Delta t = 1 \) sec. The measurement temperature curves of different test samples are shown in Figure 3.

The measurement error of the thermocouple is approximately ±1% (with the measurement noise variance, \( R = 10^{-4} \)). The space step, \( \Delta x = \frac{x}{N} \) (\( N = 10 \)). The process noise covariance matrix, \( Q = 10^3 \) [12]. Since the standard heat source is not in an absolutely insulated condition in the measurement process, in order to reduce the influence of the sampling noise, the interpolation method is used to increase the samples. The two sets of chosen sampling time, \( \Delta t = 0.05 \) sec. Figure 4 shows that the heat flux in the bottom is estimated inversely by substituting the temperature data into the presented method.

The Kalman filter is operating under the processing error variance, \( Q \), and the measurement error variance, \( R \). Figure 5, 6, 7 show the estimation results of the heat flux using three difference parameters, \( Q = 10^{-1}, 10^3 \) and \( 10^5 \) to samples. We can see that the process noise variance \( Q \) increases, the error covariance matrix will increase, which causes the Kalman gain \( K(k) \) to increase.

The estimation results reveal that the maximum heat flux have consistency by the standard heat source and demonstrate that the penetration delay of temperature may exist in the estimation process. In this paper, we are not deal with the conformity decay to heat flux; but it is interesting to note the feasibility to estimate the thermal contact resistance between heat source and test sample.
VI. CONCLUSIONS

In this paper, the bottoms of three different material samples with the same 3mm thickness and are heated by applying the standard heat source, and the temperatures on the top are measured by using the thermocouples. The FWIEM is utilizing the measured temperature data to estimate the heat flux in the bottoms of samples. The results reveal that the experiment verification shows the FWIEM has the properties of target tracking capability and effective noise reduction, and that it is an efficient, adaptive, and robust inverse estimation method for the estimation of the unknown heat flux. The nonlinear influence on the estimation to IHCP will be research by utilizing the experiment verification with different heat conduction coefficients to iron and steel in the future.

ACKNOWLEDGMENTS

This work was supported by the National Science Council of the Republic of China under Grant NSC98-2221-E-606-020.

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