Delay-range-dependent exponential synchronization of Lur’e systems with Markovian switching

Xia Zhou, Shouming Zhong

Abstract—The problem of delay-range-dependent exponential synchronization is investigated for Lur’e master-slave systems with delay feedback control and Markovian switching. Using Lyapunov-Krasovskii functional and nonsingular M-matrix method, novel delay-range-dependent exponential synchronization in mean square criteria are established. The systems discussed in this paper is advanced system, and takes all the features of interval systems, Itô equations, Markovian switching, time-varying delay, as well as the environmental noise, into account. Finally, an example is given to show the validity of the main result.

Keywords—Synchronization, Delay-range-dependent, Markov chain, Generalized Itô’s formula, Brownian motion, M-matrix.

I. INTRODUCTION

CHAOS is very interesting nonlinear phenomenon and has been intensively studied in the last three decades. It is found to be useful or has great potential in many disciplines. Since Pecora and Carroll [1] addressed the synchronization problem of chaotic systems using a drive-response conception, the subject of chaotic synchronization has received considerable attentions [2-11]. Synchronization has been widely explored in a variety of fields, such as physical, chemical and ecological systems, human heartbeat regulation, secure communications, and so on.

Recently, the effect of delay on synchronization between two chaotic systems has been reported in many literatures due to the unavoidable signal propagation delay. In [12], Yalcin ME, Suykens JAK, and Vandewalle studied the master-slave synchronization of Lur’e systems with time-delay of the form

$$\begin{align*}
\mathcal{M} : & \begin{cases} 
\dot{x}(t) = Ax(t) + B f(Cx(t)) \\
q(t) = Hx(t)
\end{cases} \\
\mathcal{G} : & \begin{cases} 
\dot{y}(t) = Ay(t) + B f(Cy(t)) + u(t) \\
q(t) = Hy(t)
\end{cases}
\end{align*}$$

$$\mathcal{G} : u(t) = K(x(t) - y(t)) + M(p(t - \tau_1) - q(t - \tau_1)).$$

with master system \(\mathcal{M}\), slave system \(\mathcal{G}\) and controller \(\mathcal{G}\), where the time delay \(\tau_1 > 0\) is constant, state vectors \(x, y \in \mathbb{R}^n\), outputs of subsystems \(p, q \in \mathbb{R}^l\), \(H, A, B, C\) are real matrices, \(f(.)\) is a sector condition. They are derived some delay-independent and delay-dependent synchronization criteria. In [13], Jinde Cao , H.X.Li b, Daniel W.C. Ho, studied the systems (1)–(3), employed model transformation, which leads to some conservative synchronization criteria for inducing additional terms. In [14], Ji Xiang, Yanjun Li, WeiWei used Integral inequality approach studied the same systems and again improved synchronization condition. In [15], Tao Li, Jianjiang Yu, and Zhao Wang, they considered the time-varying delay which often arises and may vary in a range, they studied the system of the form

$$\begin{align*}
\mathcal{M} : & \begin{cases} 
\dot{x}(t) = Ax(t) + B f(Cx(t)) \\
p(t) = Hx(t)
\end{cases} \\
\mathcal{G} : & \begin{cases} 
\dot{y}(t) = Ay(t) + B f(Cy(t)) + u(t) \\
q(t) = Hy(t)
\end{cases}
\end{align*}$$

$$\mathcal{G} : u(t) = M(p(t - d(t)) - q(t - d(t))).$$

where the time-delay \(h_1 \leq d(t) \leq h_2\) and \(\dot{d}(t) \leq \mu\). And derived the delay-range-dependent asymptotical synchronization criteria.

The hybrid systems combine a part of the state that takes values continuously and another part of the state that takes discrete values, an important class of hybrid systems is the semi-linear stochastic differential equation with Markovian switching of the form

$$\begin{align*}
\mathcal{M} : & \begin{cases} 
\dot{x}(t) = A(r(t))x(t) + B(r(t))f(Cx(t)) \\
p(t) = Hx(t)
\end{cases} \\
\mathcal{G} : & \begin{cases} 
\dot{y}(t) = A(r(t))y(t) + B(r(t))f(Cy(t)) + u(t) \\
q(t) = Hy(t)
\end{cases}
\end{align*}$$

$$\mathcal{G} : u(t) = M(p(t - d(t)) - q(t - d(t))).$$

where \(r(t)\) is a Markov chain taking values in \(S = \{1, 2, ..., N\}\). Continuous-time Marlov chains are used to model the abrupt changes in system structure and parameters. If we also take the environmental noise into account, the systems (7)–(9) becomes

$$\begin{align*}
\mathcal{M} : & \begin{cases} 
dx(t) = \dA(r(t))x(t) + \dB(r(t))f(Cx(t))\,dt \\
p(t) = Hx(t)
\end{cases} \\
\mathcal{G} : & \begin{cases} 
dy(t) = \dA(r(t))y(t) + \dB(r(t))f(Cy(t)) + \,du(t) \\
q(t) = Hy(t)
\end{cases}
\end{align*}$$

$$\mathcal{G} : u(t) = M(p(t - \delta(t)) - q(t - \delta(t))).$$

In this paper, we will discuss (10)–(12), which is advanced system, and takes all the features of interval systems, Itô equations, Markovian switching, time-varying delay, as well as the environmental noise, into account. Then we will give the delay-range-dependent exponential synchronization criteria.

The rest of this paper is organized as follows. In section 2, we introduce the basic notation, lemma’s and some definitions. In section 3, give our main results and corollary’s.
In section 4, an example is given to show the effectiveness and less conservatism of the proposed criterion.

II. NOTATION AND PRELIMINARIES

Throughout this article, unless otherwise specified, we use the following notations. Let $\| \cdot \|$ be the Euclidean norm in $\mathbb{R}^n$. If $A$ is a vector or matrix, its transpose is denoted by $A^T$. If $A$ is a matrix, its trace norm is denoted by $|A| = \sqrt{\text{trace}(A^T A)}$ while its operator norm is denoted by $\| A \| = \sup \{ |A\xi| : |\xi| = 1 \}$. If $A$ is a symmetric matrix, denote by $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ its largest and smallest eigenvalue, respectively.

Let $R_+ = [0, \infty)$ and $\tau > 0$. Let $C([\tau, \tau]; R^n)$ denote the family of continuous functions $\varphi$ from $[\tau, 0]$ to $R^n$ with the norm $\| \varphi \| = \sup_{\tau \leq t \leq 0} |\varphi(t)|$. Let $\delta(t) : R_+ \rightarrow [0, \tau]$ be a continuous function which will stand for the time delay of the system discussed in this paper. As a standing hypothesis, we shall always assume that $\delta(t)$ is differentiable and its derivative is bounded by a constant less than one, namely $\delta(t) \leq \delta_0 < 1, \forall t \geq 0$. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space equipped with some filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions, i.e., the filtration is right continuous and $\mathcal{F}_0$-contains all $\mathbb{P}$-null sets. $C_{\mathbb{P}}^{\mathbb{R}}([\tau, \tau]; \mathbb{R}^n)$: the family of all bounded, $C([\tau, \tau]; \mathbb{R}^n)$-valued, $\mathbb{P}$-measurable random variables. Let $\mathcal{W}(t)$ be a standard $n$-dimensional Brownian motion defined on the probability space. Let $r(t), t \geq 0$ be right-continuous Markov chain on the probability space taking values in a finite state space $S = \{1, 2, ..., N\}$ with generator $\Gamma = (\gamma_{ij})_{N \times N}$ given by

$$P[r(t + \Delta) = j | r(t) = i] = \begin{cases} \gamma_{ij} \Delta + o(\Delta), & \text{if } i \neq j \\ 1 + \gamma_{ii} \Delta + o(\Delta), & \text{if } i = j \end{cases}$$

where $\Delta > 0$. Here $\gamma_{ij} \geq 0$ is the transition rate from $i$ to $j$ if $i \neq j$, while $\gamma_{ii} = - \sum_{j \neq i} \gamma_{ij}$. We assume that the Markov chain $r(t)$ is independent of the Brownian motion $w(t)$. It is well known that almost every sample path of $r(t)$ is a right-continuous step function with a finite number of simple jumps in any finite subinterval of $R_+$. In other words, there is a sequence of stopping times $0 = \tau_0 < \tau_1 < ... < \tau_k \rightarrow \infty$ almost surely such that $r(t) = \sum_{k=0}^{\infty} r(\tau_k)I[\tau_k, \tau_{k+1})$, where $I_{\cdot}$ denotes the indicator function of set $\cdot$.

If $A$ and $B$ are symmetric matrix, by $A \geq B$ and $A > B$ we means that $A - B$ is positive and nonnegative definite, respectively. If $A_1$ is a vector or matrix, by $A_1 \geq 0$ we mean all elements of $A_1$ are positive. If $A_1$ and $A_2$ are vectors or matrices with same dimensions, we write $A_1 \geq A_2$ if and only if $A_1 - A_2 \geq 0$.

III. MAIN RESULTS

**Theorem 3.1.** Assume that there are two constants $\lambda_1$ and $\lambda_2$ such that

$$\lambda_1 > \tau \lambda_2$$

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Assume also that there are symmetric matrices \( Q_i > 0, E_i \geq 0 \) and constants \( \varepsilon_i > 0 (1 \leq i \leq N) \) such that
\[
\lambda_{\max}(Q_iA_i + A_i^TQ_i + 2Q_iB_iKC + D_i^TQ_iD_i) + \sum_{j=1}^N r_{ij}Q_j + E_i + \varepsilon_i Q_i \leq -\lambda_1
\]
(17)
\[
\lambda_{\max}\left(\sum_{j=1}^N r_{ij}E_j\right) \leq \lambda_2
\]
(18)
\[
E_i \geq \frac{1}{1-\delta_0}\varepsilon_i^{-1}H^TM^TQ_iMH
\]
(19)
for all \( i \in S \). Then, for any initial data \( \xi \in C_{\delta_0}^\infty([-\tau, 0]; R^n) \), the solution of (13) has the property of (20). Fix any initial data
\[
\text{Write}
\]
Thus
\[
\text{Choose}
\]
Let us first show that (21) does has a unique root
\[
0 \leq \lambda \leq \lambda_2
\]
(22)
while by (14)
\[
\Sigma V(e(t), i, t) = 2(e(t)^TQ_i[A(r(t))e(t) + B(r(t))\eta(Ce, x)]
\]
\[
+ MH e(t - \delta(t)) + \sum_{j=1}^N r_{ij}e(t)^TQ_je(t))
\]
\[
+ (D(r(t))e(t)^TQ_iD(r(t))e(t)
\]
\[
- (1 - \delta(t))e(t) - (\delta(t))^T E_i e(t - \delta(t))
\]
\[
+ \sum_{j=1}^N r_{ij} \int_{t-\delta(t)}^t e(\theta)^T E_i e(\theta) d\theta
\]
\[
+ e(t)^T E_i e(t)
\]
Using the assumptions (17), (18), (19), and lemma 2.2, we compute
\[
\Sigma V(e(t), i, t) \leq e(t)^TQ_iA_i + A_i^TQ_i e(t) + e(t)^T E_i e(t)
\]
\[
+ 2\varepsilon_i(e(t)^TQ_iB_iKC e(t) + e(t)^T(D_i^TQ_iD_i)e(t)
\]
\[
+ 2(e(t)^TQ_iMHe(t - \delta(t))
\]
\[
+ e(t)^T \sum_{j=1}^N r_{ij}E_j e(t)
\]
\[
- (1 - \delta_0)e(t - \delta(t))E_i e(t - \delta(t))
\]
\[
+ \sum_{j=1}^N r_{ij} \int_{t-\delta(t)}^t e(\theta)^T e(\theta) d\theta
\]
\[
\leq e(t)^TQ_iA_i + A_i^TQ_i + 2Q_iB_iKC
\]
\[
+ D_i^TQ_iD_i \sum_{j=1}^N r_{ij}Q_j + E_i + \varepsilon_i Q_i e(t)
\]
\[
+ e(t - \delta(t))^T Q_i e(t) - (1 - \delta_0)e(t - \delta(t))
\]
\[
+ \sum_{j=1}^N r_{ij} \int_{t-\delta(t)}^t e(\theta)^T e(\theta) d\theta
\]
\[
\leq -\lambda_1 |v| e(t)^2 + \lambda_2 \int_{t-\delta(t)}^t |e(\theta)|^2 d\theta
\]
(23)
Let us define the Lyapunov functional \( V_1: C([-\tau, 0]; R^n) \times S \times R_+ \rightarrow R \)
by
\[
V_1(e(t), i, t) = e^{\lambda t} V(e(t), i, t)
\]
By the generalized Itô formula, we have
\[
EV_1(e(t), i, t) = EV_1(\xi, r(0), 0) + E \int_0^t \Sigma V_1(e(s), r(s), s) ds
\]
and it is straightforward to see that
\[
\Sigma V_1(e(t), i, t) = e^{\lambda t}[\lambda V(e(t), i, t) + \Sigma V(e(t), i, t)]
\]
we note that
\[
V(e(t), i, t) \leq \alpha |e(t)|^2 + \alpha_1 \int_{t-\delta(t)}^t |e(\theta)|^2 d\theta
\]
(24)
(25)
By (28), (29), we know that

\[ EV_1(e(t), i, t) \leq EV_1(\xi, r(0), 0) + E \int_{t_0}^{t} e^{\lambda(s)}|e(s)|^2 ds \]

\[ + \alpha_1 \int_{s-\delta(s)}^{s} |e(\theta)|^2 d\theta ds \]

\[ + E \int_{0}^{t} e^{\lambda s}(-|e(s)|^2) ds \]

\[ + \lambda_2 \int_{s-\delta(s)}^{s} |e(\theta)|^2 d\theta ds \]

\[ = (\lambda_2 + \alpha_1 \lambda) E \int_{0}^{t} e^{\lambda s} \int_{s-\delta(s)}^{s} |e(\theta)|^2 d\theta ds \]

\[ - (\lambda_1 - \alpha \lambda) E \int_{0}^{t} e^{\lambda s} |e(s)|^2 ds \]

\[ + EV_1(\xi, r(0), 0) \]

We compute

\[ EV_1(e(t), i, t) \leq EV_1(\xi, r(0), 0) \]

\[ + (\lambda_2 + \alpha_1 \lambda) \tau e^{\lambda t} \int_{t-\delta(t)}^{t} e^{\lambda s} |\xi(s)|^2 ds \]

By (28), (29), we know that

\[ \lim_{t \to \infty} \frac{1}{t} \ln(E(|e(t; \xi)|^2)) < -\lambda < 0 \]

In other words, (13) is exponentially stable in mean square and Lyapunov exponent is greater than \(-\lambda\).

Assume also that there are symmetric matrices \(Q_i > 0\), and \(E_i \geq 0(1 \leq i \leq N)\) such that

\[ \lambda_{\text{max}}(Q_iA_i + A_i^T Q_i + 2Q_iB_iKC) \]

\[ + D_i^T Q_iD_i + \sum_{j=1}^{N} r_{ij}Q_j + E_i + Q_i) \leq -\lambda_1 \]

\[ \lambda_{\text{max}}(\sum_{j=1}^{N} r_{ij}E_j) \leq \lambda_2 \]

\[ E_i \geq \frac{1}{1 - \delta_0} H^T M^T Q_i MH \]

for all \(i \in S\). Then, for any initial data \(\xi \in C_{\text{loc}}^b([\tau, 0]; R^n)\), the solution of (13) is exponentially stable in mean square and Lyapunov exponent is not greater than \(-\lambda\).

Remark 3.1. Corollary 3.1 is stated without \(\varepsilon_i\), so it looks neat, but Theorem 3.1 is more general since it allows to choose different \(\varepsilon_i\) for different situations in practice, for example, if we choose \(\varepsilon_i = \|HM\|_i\), we can get corollary 3.2.

Corollary 3.2. Assume there are symmetric matrices \(Q_i > 0\), and \(E_i \geq 0(1 \leq i \leq N)\) such that

\[ E_i \geq \frac{1}{1 - \delta_0} \|HM\|^{-1} H^T M^T Q_i MH \]

Then, for any initial data \(\xi \in C_{\text{loc}}^b([\tau, 0]; R^n)\), the solution of (13) is exponentially stable in mean square and Lyapunov exponent is not greater than \(-\lambda\). And the \(\lambda\) can be determined in the same way as stated in Theorem 3.1. We define

\[ \beta = \frac{1}{1 - \delta_0} \max_{1 \leq i \leq N} \{\lambda_{\text{max}}(Q_iA_i + A_i^T Q_i + 2Q_iB_iKC) \]

\[ + D_i^T Q_iD_i + \|HM\|Q_i + \sum_{j=1}^{N} r_{ij}Q_j + E_i)\}

\[ \lambda_2 = \max_{1 \leq i \leq N} \lambda_{\text{max}}(\sum_{j=1}^{N} r_{ij}E_j) \]

If we choose \(E_i = \beta I\) in corollary 3.2, we can easily get corollary 3.3.

Corollary 3.3. Assume there are symmetric matrices \(Q_i > 0(1 \leq i \leq N)\) such that

\[ \lambda_{\text{max}}(Q_iA_i + A_i^T Q_i + 2Q_iB_iKC) \]

\[ + D_i^T Q_iD_i + \|HM\|Q_i + \sum_{j=1}^{N} r_{ij}Q_j) < -\beta \]

Then the solution of (13) has the property

\[ \lim_{t \to \infty} \frac{1}{t} \ln(E(|e(t; \xi)|^2)) < -\lambda < 0 \]

That is the solution of (13) exponentially stable in mean square and Lyapunov exponent is not greater than \(-\lambda\). The \(\lambda\) and 2.2
is the unique root to
\[ \lambda(\alpha + \beta r e^{\lambda T}) = \lambda_1 \]

Where \( \alpha \) is the same as defined in Theorem 3.1 but
\[ \lambda_1 = - \max_{1 \leq i \leq N} \{ \lambda_{\max}(Q_i A_i + A_i^T Q_i + 2Q_i B_i KC) + D_i^T Q_i D_i + \|HM\| \|Q_i + \sum_{j=1}^{N} r_{ij} Q_j\| \} - \beta \]

**Theorem 3.2.** Define the matrix
\[ K = \text{diag}(\lambda_{\max}(A_1 + A_1^T + 2B_1 KC) - \|HM\| - \|D_1\|^2, \ldots - \lambda_{\max}(A_N + A_N^T + 2B_N KC) - \|HM\| - \|D_N\|^2) \]
and the vector \( \kappa = (1 - \delta_0) \begin{bmatrix} \|HM\|^{-1} \\ \vdots \\ \|HM\|^{-1} \end{bmatrix} \)
(Set \( a^{-1} = \infty \) when \( \alpha = 0 \) as usual). If \( K - \Gamma \) is a nonsingular \( M \)-matrix and
\[ \kappa \gg (K - \Gamma)^{-1} \tilde{I} \]
where \( \tilde{I} = (1, \ldots, 1)^T \), then the solution of (13) exponentially stable in mean square.

**Proof.** Since \( K - \Gamma \) is a nonsingular \( M \)-matrix, by lemma 2.1, we observe that \( K - \Gamma^{-1} \) exist and all the elements of \( K - \Gamma^{-1} \) are nonnegative. \( K - \Gamma^{-1} \) is invertible, its each row must have at least one nonzero, and hence positive element. Let
\[ \bar{q} = (q_1, q_2, \ldots, q_N)^T = (K - \Gamma)^{-1} \tilde{I} \]
then \( \bar{q} \gg 0 \). By (30)
\[ q_i \|HM\| < 1 - \delta_0, \quad \forall \ i \in S \]
Let \( Q_i = qI \) for \( i \in S \), so,
\[ \lambda_{\max}(\|HM\|^{-1} H^T M^T Q_i MH) \leq q_i(\|HM\|) < 1 - \delta_0 \]
therefore \( \beta < 1 \). On the other hand
\[ \lambda_{\max}(Q_i A_i + A_i^T Q_i + 2Q_i B_i KC + D_i^T Q_i D_i + \|HM\| Q_i + \sum_{j=1}^{N} r_{ij} Q_j) \leq q_i \lambda_{\max}(A_i + A_i^T + 2B_i KC) + \|D_i\|^2 I \]
\[ + \|HM\| I + \sum_{j=1}^{N} r_{ij} q_j \]
\[ = -[(K - \Gamma)\bar{q}]_i \]
where \( [(K - \Gamma)\bar{q}]_i \) stands for the \( i \)th element of the vector \((K - \Gamma)\bar{q})\). Then
\[ \lambda_{\max}(Q_i A_i + A_i^T Q_i + 2Q_i B_i KC + D_i^T Q_i D_i + \|HM\| Q_i + \sum_{j=1}^{N} r_{ij} Q_j) \leq -1 < -\beta \]
for all \( i \in S \). Therefore the result now follows from corollary 3.3.

**Remark 3.2.** In [15], the authors studied the global asymptotically synchronization results for Lur’e systems with delay feedback control. However, the stochastic term and Markovian switching were not taken into account in the models. Therefore, our developed results in this paper are more general than reported in [12].

**Remark 3.3.** In [16], the author studied the exponential stability of stochastic delay interval systems with Markovian switching using Lyapunov-Krasovskii functional and nonsingular M-matrix methods which the same in this paper. But he studied the system is linear system, while we studied the system is nonlinear.

**Remark 3.4.** In [15], the authors studied the global asymptotically synchronization results for Lur’e systems with delay feedback control. But we studied the exponential synchronization in mean square. So, our results are better than reported in [12].

**IV. AN ILLUSTRATIVE EXAMPLE**

In this section we shall present one example to illustrate our theory.

Example 4.1. Let \( w(t) \) be a 2-dimensional Brownian motion, let \( r(t) \) be a right-continuous Markov chain taking values in \( S = \{1, 2\} \) with generator \( \Gamma = (r_{ij})_{2 \times 2} : \)
\[ -r_{11} = r_{12}, \quad -r_{22} = r_{21} > 0 \]
of course \( w(t) \) and \( r(t) \) are assumed to be independent. Consider the Lur’e systems with delay feedback control and Markovian switching of the form
\[ \dot{\mathbf{x}}(t) = A(r(t))\mathbf{x}(t)dt + D(r(t))\mathbf{x}(t)dw(t) \]
\[ p(t) = H\mathbf{x}(t) \]
\[ y(t) = \psi(t), t \in [-\tau, 0] \]
\[ \mathcal{G}: \begin{cases} dy(t) = \bar{A}(r(t))y(t) + u(t)dt + D(r(t))y(t)dw(t) \\ q(t) = H\psi(t) \end{cases} \]
\[ \mathcal{M}: \begin{cases} dx(t) = A(r(t))x(t)dt + D(r(t))x(t)dw(t) \\ p(t) = H\mathbf{x}(t) \end{cases} \]
\[ \mathcal{G}: \begin{cases} dy(t) = \bar{A}(r(t))y(t) + u(t)dt + D(r(t))y(t)dw(t) \\ q(t) = H\psi(t) \end{cases} \]
\[ \mathcal{M}: \begin{cases} dx(t) = A(r(t))x(t)dt + D(r(t))x(t)dw(t) \\ p(t) = H\mathbf{x}(t) \end{cases} \]
where \( \bar{A}(t) = 0.1 \sin^2 t \) and \( \delta(t) = 0.2 \sin(t) \cos(t) \leq 0.1 = \delta_0 \),
\[ A_1 = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2 & -1 \\ 0 & -1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad M = \begin{bmatrix} 1/\pi & -1/\pi \\ 0 & 1/\pi \end{bmatrix}, \quad H = \begin{bmatrix} 9 & 0 \\ 0 & 7 \end{bmatrix} \]
Then the solution of Lur’e systems with delay feedback control and Markovian switching is exponentially synchronization in mean square.

**V. CONCLUSION**

The problem of master–slave exponential synchronization for Lure systems has been addressed by employing time-delay feedback control techniques. By using the methods of Lyapunov-Krasovskii functional and nonsingular M-matrix,
some effective criterions for achieving synchronization have been derived. Finally, an example has been given to illustrate the validity theoretical results obtained in this paper.

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