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Traveling wave solutions for the (3+1)-dimensional breaking soliton equation by $(\frac{G'}{G})$-expansion method and modified F-expansion method

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Abstract—In this paper, using $(\frac{G'}{G})$-expansion method and modified F-expansion method, we give some explicit formulas of exact traveling wave solutions for the (3+1)-dimensional breaking soliton equation. A modified F-expansion method is proposed by taking full advantages of F-expansion method and Riccati equation in seeking exact solutions of the equation.

Keywords—Exact solution, The (3+1)-dimensional breaking soliton equation, $(\frac{G'}{G})$-expansion method, Riccati equation, Modified F-expansion method.

I. Introduction

IN many different fields of science and engineering, it is very important to obtain exact or numerical solutions of nonlinear partial differential equations. It is well known that nonlinear phenomena are very important in a variety of scientific fields, especially in fluid mechanics, solid state physics, plasma physics, plasma waves and chemical physics. Searching for exact and numerical solutions, especially for traveling wave solutions, of nonlinear equations in mathematical physics plays an important role in soliton theory [1], [2]. Recently many new approaches to nonlinear equations were proposed, such as the homotopy perturbation method [3], [4], [5], [6], [7], the variational iteration method [8], [9], [10], parameter expansion method [11], [12], [13], [14], spectral collocation method [15], [16], [17], [18], [19], homotopy analysis method [20], [21], [22], [23], [24], [25], and the Exp-function method [26], [27], [28], [29], [30], [31]. In this paper, we solve a (3+1)-dimensional breaking soliton equation by the $(\frac{G'}{G})$-expansion method and modified F-expansion method, and obtain some exact and new solutions for it.

The(2+1)-dimensional breaking soliton equation has the following form

$$u_{xt} - 4u_{xy}u_x - 2u_{xx}u_y - u_{xxxy} = 0, \tag{1}$$

this equation describes the (2+1)-dimensional interaction of the Riemann wave propagated along the y-axis with a long wave propagated along the x-axis [32]. Wazwaz [33] presented an extension to equation (1) by adding the last three terms with y replaced by z. By his work, one enables to establish the following (3+1)-dimensional breaking soliton equation

$$\begin{aligned} u_{xt} - 4\,u_x\,(u_{xy} + u_{xz}) - 2\,u_{xx}\,(u_y + u_z) - \\ (u_{xxxy} + u_{xxxz}) = 0, \end{aligned} \tag{2}$$

where $u = u(x, y, z, t) : \mathbb{R}_x \times \mathbb{R}_y \times \mathbb{R}_z \times \mathbb{R}_t \to \mathbb{R}$.

In this paper, by means of the $(\frac{G'}{G})$-expansion method and modified F-expansion method, we obtain some exact traveling wave solutions for equation (2).
The outline of this paper is as follows. In the following section we have a brief review on the $(\frac{G'}{G})$-expansion. In Section III we apply the $(\frac{G'}{G})$-expansion method on equation (2) to obtain some traveling wave solution for the equation. In Section IV a review on the modified F-expansion method is presented. We obtain some traveling wave solutions for equation (2) by the modified F-expansion method in Section V. The paper is concluded is Section VI.

II. The $(\frac{G'}{G})$-expansion method

Wang et al. [34] proposed the $(\frac{G'}{G})$-expansion method to solve nonlinear partial differential equations, where $G = G(\xi)$ satisfies a second order linear ordinary differential equation. In this section we describe the $(\frac{G'}{G})$-expansion method to find traveling wave solutions of nonlinear evolution equations. Suppose that a nonlinear equation, say in two independent variables x, t, is given by

$$P(u, u_t, u_x, u_{tx}, u_{xx}, \cdots) = 0, \tag{3}$$

where $u = u(x, t)$ and P is a polynomial of u and its derivatives in which the highest order derivatives and nonlinear terms are involved. The main steps of the $(\frac{G'}{G})$-expansion method are as follows:

- **First.** Suppose that
$$u(x, t) = u(\xi), \quad \xi = x + wt \tag{4}$$
the traveling wave variable (4) permits us reducing (3) to an ordinary differential equation (shortly ODE) for $u = u(\xi)$ such as
$$P(u, u', u'', u''', \cdots) = 0, \tag{5}$$
- **Second.** Now, we suppose that the solution of (5) can be expressed by a polynomial in $(\frac{G'}{G})$ as
$$u(\xi) = \alpha_m(\frac{G'}{G})^m + \dots \tag{6}$$

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where $G = G(\xi)$ satisfies the second order linear ODE in the following form

$$G'' + \lambda G' + \mu G = 0 \tag{7}$$

$\alpha_m, \ldots, \lambda$ and μ are constants to be determined later, $\alpha_m \neq 0$. The unwritten part in (6) is also a polynomial in $(\frac{G'}{G})$, the degree of which is generally equal to or less than $m-1$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (5).

- **Third.** Substituting (6) into (5) and using the second order linear ODE (7), collecting all terms with the same order of $(\frac{G'}{G})$ together, the left-hand side of (5) is converted into another polynomial in $(\frac{G'}{G})$. Equating each coefficients of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, \ldots, \lambda$ and μ.

- **Fourth.** Assuming that the constants $\alpha_m, \ldots, \lambda$ and μ can be obtained by solving the algebraic equations in item **Third**. Since we know the general solutions of the second order linear ODE (7), then substituting $\alpha_m, \ldots, w$ and the general solutions of (7) into (6) yields the traveling wave solutions of the nonlinear equation (3).

III. Application of the $(\frac{G'}{G})$-expansion method for the (3+1)-dimensional breaking soliton equation

In this section, we apply the $(\frac{G'}{G})$-expansion method to construct the traveling wave solutions for (3+1)-dimensional breaking soliton equation

$$\begin{gathered} u_{xt} - 4\, u_x\,(u_{xy} + u_{xz}) - 2\, u_{xx}\,(u_y + u_z) - \\ (u_{xxxy} + u_{xxxz}) = 0. \end{gathered} \tag{8}$$

Some exact solutions for (8) were presented in [35] by the three-wave method. To apply the $(\frac{G'}{G})$-expansion method on this equation, we suppose that

$$u(x,y,z,t) = u(\xi), \qquad \xi = kx + my + nz + wt \tag{9}$$

k, m, n, w are constants that to be determined later.

By substitute eq. (9) into eq. (8), we obtain

$$w\, u'' - 6\, k(m+n)\, u'\, u'' - k^2\,(m+n) u^{(4)} = 0. \tag{10}$$

Integrating (10) once, we have

$$w\, u' - 3\, k(m+n)\,(u')^2 - k^2\,(m+n) u^{(3)} = c \tag{11}$$

where c is the integration constant that can be determined later.

Suppose that the solutions of the ODE (11) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i\,(\frac{G'}{G})^i \tag{12}$$

where a_i are constants, $G = G(\xi)$ satisfies the following second order linear ODE

$$G'' + \lambda\, G' + \mu\, G = 0, \tag{13}$$

where λ and μ are constants.

By balancing the order of $(u')^2$ and $u^{(3)}$ in eq. (11), we have $2m+2 = m+3$ then $m = 1$. So we can write

$$u(\xi) = a_1\,(\frac{G'}{G}) + a_0 \quad , \quad a_1 \neq 0 \tag{14}$$

a_1 , a_0 are constants to be determined later.

Then it follows:

$$u'(\xi) = -a_1\,(\frac{G'}{G})^2 - a_1\,\lambda\,(\frac{G'}{G}) - a_1\,\mu \tag{15}$$

$$u'' = 2\,a_1\,(\frac{G'}{G})^3 + 3a_1\lambda\,(\frac{G'}{G})^2 + a_1\,(\lambda^2 - 2\,\mu)\,(\frac{G'}{G}) + \lambda\,\mu\,a_1 \tag{16}$$

and

$$\begin{gathered} u''' = -6\,a_1\,(\tfrac{G'}{G})^4 - 12\,a_1\,\lambda\,(\tfrac{G'}{G})^3 - a_1\,(8\mu + 7\lambda^2)(\tfrac{G'}{G})^2 \\ -a_1\,(\lambda^3 + 8\lambda\mu)(\tfrac{G'}{G}) - a_1(\lambda^2\mu + 2\mu^2). \end{gathered} \tag{17}$$

Substituting eq. (14) into eq. (11) and collecting all terms with the same power of $(\frac{G'}{G})$ together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$(\tfrac{G'}{G})^4 : 6\,k^2 m - 3\,k a_1 m - 3\,k a_1 n + 6\,k^2 n = 0$$

$$(\tfrac{G'}{G})^3 : 12\,k^2 n\lambda - 6\,k a_1 n\lambda - 6\,k a_1 m\lambda + 12\,k^2 m\lambda = 0$$

$$\begin{aligned} (\tfrac{G'}{G})^2 : & -6\,k a_1 m\mu - 3\,k a_1 n\lambda^2 - 3\,k a_1 m\lambda^2 - w + 7\,k^2 m\lambda^2 \\ & +8\,k^2 m\mu - 6\,k a_1 n\mu + 8\,k^2 n\mu + 7\,k^2 n\lambda^2 = 0 \end{aligned}$$

$$\begin{aligned} (\tfrac{G'}{G})^1 : & \; 8\,k^2 m\lambda\,\mu - 6\,k a_1 m\lambda\,\mu - 6\,k a_1 n\lambda\,\mu + \\ & k^2 m\lambda^3 + k^2 n\lambda^3 - w\lambda + 8\,k^2 n\lambda\,\mu = 0 \end{aligned}$$

$$\begin{aligned} (\tfrac{G'}{G})^0 : & \; a_1\left(-3\,k a_1 m\mu^2 + k^2 m\lambda^2\mu + k^2 n\lambda^2\mu\right) - \\ & a_1\left(w\mu - 3\,k a_1 n\mu^2 + 2\,k^2 m\mu^2 + 2\,k^2 n\mu^2\right) - c = 0. \end{aligned}$$

Solving the above algebraic equations by using Maple, we get:

$$a_0 = a_0, a_1 = 2\,k, c = 0, w = -(4\,k^2\mu - k^2\lambda^2)(m+n) \tag{18}$$

where k, n, m, a_0 are arbitrary constants.
Substituting(18) into eq. (14), we have

$$u(\xi) = 2k\,(\frac{G'}{G}) + a_0 \tag{19}$$

where $\xi = kx + my + nz - (4\,k^2\mu\, - k^2\lambda^2)(m+n)\,t$.
Substituting the general solutions of eq. (13) into eq. (19), we obtain:

Case i: When $\lambda^2 - 4\mu > 0$

$$u_1(\xi) = -k\lambda + k\,\sqrt{\lambda^2 - 4\mu}\times \left(\frac{C_1 \sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right) + a_0$$

where

$$\xi = kx + my + nz - (4\,k^2\mu\, - k^2\lambda^2)(m+n)\,t,$$

and C_1, C_2, k, m, n and a_0 are arbitrary constants.
In particular, if $C_1 = 1, C_2 = 0, k = m = n = 1, \lambda = 2, \mu = 0$, then we have

$$u(x,y,z,t) = -2 + 2\tanh(x+y+z+8t) + a_0.$$

Case ii: When $\lambda^2 - 4\mu < 0$

$$u_2(\xi) = -k\lambda + k\,\sqrt{4\mu - \lambda^2}\times \left(\frac{-C_1 \sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right) + a_0$$

where

$$\xi = kx + my + nz - (4\,k^2\mu\, - k^2\lambda^2)(m+n)\,t,$$

and C_1, C_2, k, m, n, a_0 are arbitrary constants.
In particular, if

$$C_1 = 1, C_2 = 0, k = m = n = 1, \lambda = 0, \mu = 1,$$

then we have

$$u(x,y,z,t) = -2\tanh(x+y+z-8t) + a_0.$$

Case iii: When $\lambda^2 - 4\mu = 0$

$$u_3(\xi) = \frac{k(2\,C_2 - C_1\,\lambda - C_2\,\lambda\,\xi)}{C_1 + C_2\,\xi} + a_0$$

where

$$\xi = kx + my + nz - (4\,k^2\mu\, - k^2\lambda^2)(m+n)\,t,$$

and C_1, C_2, k, m, n, a_0 are arbitrary constants.
In particular, if

$$C_1 = 1, C_2 = 1, k = m = n = 1, \lambda = 2, \mu = 1,$$

then we have

$$u(x,y,z,t) = \frac{2}{1+x+y+z} + a_0$$

IV. The modified F-expansion method

We simply describe the modified F-expansion method. To do this we follow descriptions which has presented in [36].
Consider a given nonlinear partial differential equation with independent variables $x_1, x_2, \ldots, x_l, t$ and dependent variable u as:

$$P(u, u_t, u_{x_1}, u_{x_2}, \cdots, u_{tt}, \cdots) = 0, \qquad (20)$$

in general, the function P is a polynomial in u and its various partial derivatives, which we seek its traveling wave solutions by taking

$$u(x_1, x_2, \ldots, x_l, t) = u(\xi), \xi = k_1(x_1 + k_2\,x_2 + \ldots + k_l\,x_l + w\,t) \qquad (21)$$

where $k_1, k_2, \ldots, k_l$ and w are unknown constants to be determined. Inserting (21) into (20) yields an ODE for $u(\xi)$ as

$$P(u, u', u'', u''', \ldots) = 0 \qquad (22)$$

conversely, we suppose that $u(\xi)$ can be expressed as

$$u(\xi) = a_0 + \sum_{i=-N}^{N} a_i\,F^i(\xi), \qquad (a_N \neq 0) \qquad (23)$$

where a_0 and a_is are constants to be determined. $F(\xi)$ satisfies Riccati equation

$$F'(\xi) = A + B\,F(\xi) + C\,F^2(\xi) \qquad (24)$$

where A, B, C are constants to be determined. Integer N can be determined by considering the homogeneous balance between the governing nonlinear term(s) and highest order derivatives of $u(\xi)$ in (22).
Substituting (23) into (22), and using (24), the left-hand side of (22) can be converted into a finite series in $F^p(\xi)$ for $p = -N, \ldots, -1, 0, 1, \ldots, N$. Equating each coefficient of $F^p(\xi)$ to zero yields a system of algebraic equations for a_i, $i = -N, \ldots, -1, 0, 1, \ldots, N$, k_j, $j = 1, \ldots, l$ and w.
Then solving the obtained system of algebraic equations, with the aid of a symbolic computations like Mathematica or Maple, a_i, k_j, w can be expressed by A, B, C or the coefficients of (22). Finally, by substituting these results into (23), we can obtain the general form of traveling wave solutions to (22).
From the general form of traveling wave solutions of equation (24) listed in Table I, we can give a series of soliton-like solutions, trigonometric function solutions, and exponential function solutions to (20); cf. [36].

TABLE I
SOLUTIONS OF EQUATION (24) GIVEN IN [36].

A	B	C	$F(\xi)$
0	1	-1	$\frac{1}{2} + \frac{1}{2}\ \tanh(\frac{\xi}{2})$
0	-1	1	$\frac{1}{2} - \frac{1}{2}\ \coth(\frac{\xi}{2})$
$\frac{1}{2}$	0	$-\frac{1}{2}$	$\coth(\xi) \pm \mathrm{csch}(\xi), \tanh(\xi) \pm i\mathrm{sech}(\xi)$
1	0	-1	$\tanh(\xi)$,$\coth(\xi)$
$\frac{1}{2}$	0	$\frac{1}{2}$	$\sec(\xi) + \tan(\xi), \csc(\xi) - \cot(\xi)$
$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\sec(\xi) - \tan(\xi), \csc(\xi) + \cot(\xi)$
1(-1)	0	1(-1)	$\tan(\xi)$,$\cot(\xi)$
0	0	$\neq 0$	$-\frac{1}{C\,\xi+m}$ (m is arbitrary constant)
arbitrary constant	0	0	$A\,\xi$
arbitrary constant	$\neq 0$	0	$\frac{exp(B)-A}{B}$

V. New exact solutions to the (3+1)-dimensional breaking soliton equation by the modified F-expansion method

In this section, we apply the F-expansion method to construct the traveling wave solutions for the mentioned (3+1)-dimensional breaking soliton equation, that is,

$$\begin{aligned} u_{xt} - 4\,u_x\,(u_{xy} + u_{xz}) - 2\,u_{xx}\,(u_y + u_z) - \\ (u_{xxxy} + u_{xxxz}) = 0, \end{aligned} \tag{25}$$

we suppose that

$$u(x, y, z, t) = u(\xi), \qquad \xi = k(x + my + nz + wt) \tag{26}$$

k, m, n, w are constants that to be determined later. By substituting eq. (26) into eq. (25), we obtain

$$w\,u'' - 6\,k(m+n)\,u'\,u'' - k^2\,(m+n)u^{(4)} = 0. \tag{27}$$

By balancing the order of $u'\,u''$ and $u^{(4)}$ in eq. (27), we have $2N + 3 = N + 4$ then $N = 1$. So we can write

$$u(\xi) = a_0 + a_{-1}\,F^{-1}(\xi) + a_1\,F(\xi) \tag{28}$$

a_0 , a_1 , a_{-1} are constants to be determined later. Substituting (28) into (27), and using (24), the left-hand side of (27) can be converted into a finite series in $F^p(\xi)$, for $p = -5, \ldots, -1, 0, 1, \ldots, 5$. Equating each coefficient of $F^p(\xi)$ to zero yields a system of algebraic equations for $a_{-1}, a_0, a_1, m, n, w$. In fact, we have

$$\begin{aligned} F^5 :\ & -12\,kma_1{}^2C^3 + 24\,k^2na_1C^4 - 12\,kna_1{}^2C^3 + \\ & 24\,k^2ma_1C^4 \end{aligned}$$

$$\begin{aligned} F^4 :\ & 60\,k^2na_1C^3B - 30\,kma_1{}^2BC^2 + 60\,k^2ma_1C^3B - \\ & 30\,kna_1{}^2BC^2 \end{aligned}$$

$$\begin{aligned} F^3 :\ & -24\,kma_1{}^2B^2C - 24\,kna_1{}^2B^2C + 50\,k^2na_1C^2B^2 + \\ & 2\,wa_1C^2 - 24\,kna_1{}^2AC^2 + 50\,k^2ma_1C^2B^2 - \\ & 24\,kma_1{}^2AC^2 + 40\,k^2na_1C^3A + 12\,kna_1a_{-1}C^3 + \\ & 40\,k^2ma_1C^3A + 12\,kma_1a_{-1}C^3 \end{aligned}$$

$$\begin{aligned} F^2 :\ & 60\,k^2ma_1C^2AB - 36\,kna_1{}^2ABC + 3\,wa_1BC + \\ & 24\,kna_1a_{-1}BC^2 - 36\,kma_1{}^2ABC + \\ & 24\,kma_1a_{-1}BC^2 + \\ & 60\,k^2na_1C^2AB + 15\,k^2ma_1CB^3 + 15\,k^2na_1CB^3 - \\ & 6\,kma_1{}^2B^3 - 6\,kna_1{}^2B^3 \end{aligned}$$

$$\begin{aligned} F^1 :\ & k^2na_1B^4 + 12\,kma_1a_{-1}B^2C + k^2ma_1B^4 + \\ & 22\,k^2na_1CB^2A + 2\,wa_1CA + 12\,kna_1a_{-1}B^2C + \\ & 16\,k^2na_1C^2A^2 - 12\,kma_1{}^2AB^2 + 16\,k^2ma_1C^2A^2 + \\ & 12\,kna_1a_{-1}AC^2 - 12\,kna_1{}^2A^2C + wa_1B^2 \\ & +22\,k^2ma_1CB^2A - 12\,kma_1{}^2A^2C + \\ & 12\,kma_1a_{-1}AC^2 - 12\,kna_1{}^2AB^2 \end{aligned}$$

$$\begin{aligned} F^0 :\ & 6\,kma_{-1}{}^2BC^2 + wa_{-1}BC + k^2ma_1B^3A + \\ & 8\,k^2ma_1BCA^2 + k^2ma_{-1}B^3C + 8\,k^2ma_{-1}ABC^2 + \\ & 8\,k^2na_1BCA^2 + 6\,kna_{-1}{}^2BC^2 - 6\,kma_1{}^2A^2B \\ & +wa_1BA - 6\,kna_1{}^2A^2B + 8\,k^2na_{-1}ABC^2 + \\ & k^2na_1B^3A + k^2na_{-1}B^3C \end{aligned}$$

$$\begin{aligned} F^{-1} :\ & -12\,kna_1a_{-1}A^2C + 12\,kna_{-1}{}^2B^2C - \\ & 12\,kma_1a_{-1}AB^2 + k^2ma_{-1}B^4 - 12\,kma_1a_{-1}A^2C + \\ & 12\,kma_{-1}^2AC^2 - 12\,kna_1a_{-1}AB^2 + wa_{-1}B^2 + \\ & 22\,k^2ma_{-1}AB^2C + 12\,kna_{-1}^2AC^2 + \\ & 12\,kma_{-1}^2B^2C + 16\,k^2ma_{-1}A^2C^2 + k^2na_{-1}B^4 + \\ & 22\,k^2na_{-1}AB^2C + 16\,k^2na_{-1}A^2C^2 + 2\,wa_{-1}AC \end{aligned}$$

$$\begin{aligned} F^{-2} :\ & -24\,kna_1a_{-1}A^2B - 24\,kma_1a_{-1}A^2B + \\ & 15\,k^2na_{-1}AB^3 + 3\,wa_{-1}AB + 15\,k^2ma_{-1}AB^3 + \\ & 60\,k^2na_{-1}A^2BC + 60\,k^2ma_{-1}A^2BC + \\ & 36\,kma_{-1}^2ABC + 6\,kma_{-1}^2B^3 + 6\,kna_{-1}^2B^3 + \\ & 36\,kna_{-1}^2ABC \end{aligned}$$

$$\begin{aligned} F^{-3} :\ & 50\,k^2ma_{-1}A^2B^2 + 24\,kma_{-1}^2AB^2 + 24\,kna_{-1}^2AB^2 + \\ & 40\,k^2ma_{-1}A^3C - 12\,kna_1a_{-1}A^3 + 24\,kna_{-1}^2A^2C - \\ & 12\,kma_1a_{-1}A^3 + 2\,wa_{-1}A^2 + 50\,k^2na_{-1}A^2B^2 + \\ & 24\,kma_{-1}^2A^2C + 40\,k^2na_{-1}A^3C \end{aligned}$$

$$\begin{aligned} F^{-4} :\ & 60\,k^2ma_{-1}A^3B + 60\,k^2na_{-1}A^3B + \\ & 30\,kma_{-1}{}^2A^2B + 30\,kna_{-1}^2A^2B \end{aligned}$$

$$\begin{aligned} F^{-5} :\ & 12\,kna_{-1}^2A^3 + 24\,k^2ma_{-1}A^4 + 12\,kma_{-1}^2A^3 + \\ & 24\,k^2na_{-1}A^4. \end{aligned}$$

Solving the obtained algebraic equations by using Maple, we have the following solutions:

Case 1: when $A = 0$, we have

$$\begin{aligned} & a_0 = a_0, a_{-1} = 0, a_1 = 2\,C\,k, m = m, \\ & n = n, w = -k^2B^2\,(n+m) \end{aligned} \tag{29}$$

Case 2: when $B = 0$, we have

$$\begin{aligned} & a_0 = a_0, a_{-1} = -2\,kA, a_1 = 2\,kC, m = m, \\ & n = n, w = 16\,k^2AC(m+n) \\ & \\ & a_0 = a_0, a_{-1} = -2\,kA, a_1 = 0, m = m, \\ & n = n, w = 4\,k^2AC(m+n) \end{aligned} \tag{30}$$

Case 3: when $A = B = 0$, we have

$$\begin{aligned} & a_0 = a_0, a_{-1} = -\tfrac{w}{6Ck(m+n)}, a_1 = 2\,C\,k, \\ & m = m, n = n, w = w \end{aligned} \tag{31}$$

A. *The soliton-like solutions:*

1) When $A = 0, B = 1, C = -1$, from Table I and Case 1, we have

$$u_1 = a_0 - k - k\tanh[\frac{1}{2}k(x + my + nz - k^2\,(n+m)\,t)]$$

2) When $A = 0, B = -1, C = 1$, from Table I and Case 1, we have

$$u_2 = a_0 + k - k\coth[\frac{1}{2}k(x + my + nz - k^2(n+m)t)]$$

3) When $A = \frac{1}{2}, B = 0, C = -\frac{1}{2}$, from Table I and Case 2, we have

$$u_3 = a0 - k\coth[k(x + my + nz - k^2(n+m)t)]$$

$$\mp k\mathrm{csch}[k(x + my + nz - k^2(n+m)t)]$$

$$u_4 = a0 - k\tanh[k(x + my + nz - k^2(n+m)t)]$$

$$\mp k\, i\,\mathrm{sech}[k(x + my + nz - k^2(n+m)t)]$$

4) When $A = 1, B = 0, C = 1$, from Table I and Case 2, we have

$$u_5 = a0 + 2k\tanh[k(x + my + nz + 16k^2(n+m)t)]$$

$$-2k\coth[k(x + my + nz + 16k^2(n+m)t)]$$

B. The trigonometric function solutions:

1) When $A = C = \frac{1}{2}, B = 0$, from Table I and Case 2, we have

$$u_6 = a0 + k\sec[k(x + my + nz + k^2(n+m)t)]$$

$$+k\tan[k(x + my + nz + k^2(n+m)t)]$$

$$u_7 = a0 + k\csc[k(x + my + nz + k^2(n+m)t)]$$

$$-k\, i\cot[k(x + my + nz + k^2(n+m)t)]$$

2) When $A = C = -\frac{1}{2}, B = 0$, from Table I and Case 2, we have

$$u_8 = a0 - k\sec[k(x + my + nz + k^2(n+m)t)]$$

$$+k\tan[k(x + my + nz + k^2(n+m)t)]$$

$$u_9 = a0 - k\csc[k(x + my + nz + k^2(n+m)t)]$$

$$-k\, i\cot[k(x + my + nz + k^2(n+m)t)]$$

C. The rational solution:

When $A = B = 0, C \neq 0$, from Table I and Case 3, we have

$$u_{10} = a_0 + \frac{w}{6Ck(m+n)}[Ck(x + my + nz + wt) + m] -$$

$$\frac{2Ck}{Ck(x+my+nz+wt)+m}$$

VI. Conclusions

In this paper, by means of the $(\frac{G'}{G})$-expansion method and modified F-expansion method, some exact traveling wave solutions for the (3+1)-dimensional breaking soliton equation are obtained. These methods are very simple and with the aid of a symbolic computation like Maple or Mathematica are easy and straightforward methods which can be applied to other nonlinear partial differential equations. It must be noted that, all obtained solutions have checked in the (3+1)-dimensional breaking soliton equation. All solutions satisfy in the equation.

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