Identification of an Mechanism Systems by Using the Modified PSO Method
Chih-Cheng Kao Hsin-Hua Chu

Abstract—This paper mainly proposes an efficient modified particle swarm optimization (MPSO) method, to identify a slider-crank mechanism driven by a field-oriented PM synchronous motor. In system identification, we adopt the MPSO method to find parameters of the slider-crank mechanism. This new algorithm is added with “distance” term in the traditional PSO’s fitness function to avoid converging to a local optimum. It is found that the comparisons of numerical simulations and experimental results prove that the MPSO identification method for the slider-crank mechanism is feasible.

Keywords—Slider-crank mechanism, distance, system identification, modified particle swarm optimization.

I. INTRODUCTION

A slider-crank mechanism is widely used in gasoline and diesel engines, and has been studied extensively in the past three decades. The responses of the system found by Viscomi and Ayre [1] are to be dependent upon the five parameters as the length, mass, damping, external piston force and frequency. The steady-state responses of the flexible connecting rod of a slider-crank mechanism with time-dependent boundary effect were obtained by Fung [2]. A slider-crank mechanism with constantly rotating speed was controlled by Fung et al. [3], where the system is actuated by a field-oriented control permanent magnet (PM) synchronous servomotor.

Particle swarm optimization (PSO) is a stochastic population based optimization approach, and was first published by Kennedy and Eberhart in 1995 [5,6]. PSO has been shown to be an efficient, robust and simple optimization algorithm. In this paper, a modified PSO algorithm is proposed to improve the searching ability and prevent from being trapped in a local optimum. From these empirical studies it can be concluded that the MPSO is sensitive to control parameter choices.

II. DYNAMIC MODELING

Figure 1 shows the physical model of a slider-crank mechanism, where the mass center and the radius of the rigid disk are denoted as point “O” and length “r”, respectively. And “l” is denoted as the length of the connected rod AB. The angle \( \theta \) is between OA and the X-axis, while the angle \( \phi \) is between the rod AB and the X-axis. In OXY plane, the geometric positions of gravity centers of rigid disk, connected rod, and slider, respectively, are as follows:

\[
x_{1g} = 0, \quad y_{1g} = 0
\]

\[
x_{2g} = r \cos \theta + \frac{1}{2} l \cos \phi, \quad y_{2g} = \frac{1}{2} l \sin \phi
\]

\[
x_{3g} = r \cos \theta + l \cos \phi, \quad y_{3g} = 0
\]

The mechanism has a constrained condition as follows

\[
r \sin \theta = l \sin \phi.
\]

The angle \( \phi \) can be found from Eq. (4) as

\[
\phi = \sin^{-1} \left( \frac{r}{l} \sin \theta \right).
\]

By taking account of the control force and constraint force, the equation in the matrix form can be obtained as

\[
M(Q) \ddot{Q} + N(Q, \dot{Q}) + \Phi_0^T \lambda = Q^\lambda
\]

where \( M(Q), \ N(Q, \dot{Q}), \ \Phi_0^T \lambda, \) and \( Q^\lambda \) can be seen in the Appendix A.

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\[ Q = [Q_1, Q_2, \ldots, Q_m]^T = [p^T, q^T]^T \]  

(7)

where \( p = [p_1, p_2, \ldots, p_m]^T \) and \( q = [q_1, q_2, \ldots, q_m]^T \) are the \( m \) dependent and \( k \) independent coordinates, respectively. The \( m \) constraint equations are

\[ \Phi(Q) = \Phi(p, q) = 0. \]  

(8)

Numerical method may be used to solve the set of nonlinear algebraic equation (8). If the \( m \) constraint equations are independent, the existence of a solution \( p \) for a given \( q \) can be asserted by an implicit function theory.

Decomposing \( Q \) into \( p \) and \( q \), the system equations become

\[ M^{pp} \ddot{p} + M^{pq} \dot{q} + \Phi_p^T \lambda = Q^p - N^p \]  

(9a)

\[ M^{pq} \ddot{p} + M^{qq} \dot{q} + \Phi_q^T \lambda = Q^q - N^q \]  

(9b)

\[ \Phi_p \ddot{p} + \Phi_q \ddot{q} = \gamma \]  

(9c)

By using equations (9a) and (9c) and eliminating \( \lambda \) and \( \ddot{p} \) yields

\[ \lambda = (\Phi_p^T)^{-1} [Q^p - N^p - M^{pp} \ddot{p} - M^{pq} \dot{q}] \]  

(10)

\[ \ddot{p} = \Phi_p^{-1} [\gamma - \Phi_q \dot{q}] \]  

(11)

Equations (9b), (10) and (11) can be combined in the matrix form as

\[ M(q) \ddot{q} + \dot{N}(q, \dot{q}) = \ddot{F} \]  

(12)

where

\[ \ddot{M} = M^{pp} - M^{pq} \Phi^{-1} \Phi_q \]  

(13)

\[ \ddot{N} = [N^q - \Phi_q^T (\Phi_p^T)^{-1} N^p] + [M^{pq} \Phi^{-1} - \Phi_q^T (\Phi_p^T)^{-1} M^{pp} \Phi^{-1}] \dot{q} \]  

(14)

\[ \ddot{F} = Q^q - \Phi_q^T (\Phi_p^T)^{-1} M^{pq} \dot{p} \]  

(15)

Equation (12) is a set of differential equations with only one independent generalized coordinate vector \( \theta = [\theta] \). It is seen that the entries of \( \ddot{M}, \ddot{N} \) and \( \ddot{F} \) of Eq. (12) has two independent variables \( \theta \) and \( \dot{\theta} \). By using Eq. (4) and its time derivative, we could derive the equation with only one independent variable \( \theta \) as follows

\[ \ddot{M}(\theta) \ddot{\theta} + \ddot{N}(\theta, \dot{\theta}) = \ddot{F}(\theta) \]  

(16)

where

\[ \dddot{M} = \left[2m_1 + m_2 \right] + m_2 r \cos \theta \left[ r^2 \cos \theta \sin^2 \theta \right] \]  

\[ + \left(m_2 + m_1 r^2 \sin^2 \theta \right) + \frac{1}{3} m_1 \left(r \cos \theta \right)^2 + \frac{1}{2} m_r^2 + J_m, \]  

(17)

\[ \dot{N} = \left( m_2 r^2 \sin \theta \cos \theta \left[ \frac{1}{2} \left( 1 - \frac{l^2}{3c^2} + \frac{r^4}{c^2} \right) \right] \right. \]  

\[ + \left( \frac{r^4}{c^2} - \frac{r^2}{2c} \right) \cos \theta \sin^2 \theta \right) \]  

\[ - m_2 r \sin^3 \theta \cos \theta \]  

(18)

The system becomes an initial value problem and can be directly integrated by using the fourth order Runge-Kutta method.

### III. MODIFIED PARTICLE SWARM OPTIMIZATION

3.1 Particle swarm optimization

Birds (particles) flocking optimizes a certain objective function in a PSO system. Each agent knows its best value so far \( pbest \) and its position. This information is analogy of personal experiences of each agent. Moreover, each agent knows the best value so far in the group \( gbest \) among \( pbest \). This information is analogy of knowledge of how the other agents around them have performed. The PSO concept [5-6] consists of changing the velocity of each particle toward its best position. If a particle discovers a new probable solution, a new position according to new velocity and the previous positions of the particle. This is compared with the best position generated by previous particles in the fitness function, and the best one is kept; so each particle accelerates in the direction of not only the local best solution but also the global best position. If a particle discovers a new probable solution, other particles will move closer to it to explore the region more completely in the process.

In general, there are three attributes, current position \( x_j \), current velocity \( v_j \), and best past position \( pbest_j \), for particles in the search space to present their features. Each particle in the swarm is iteratively updated according to the aforementioned attributes. For example [5-11], the \( j \)th particle is represented as \( x_j = (x_{j1}, x_{j2}, \ldots, x_{jd}) \) in the \( d \)-dimensional space. The best previous position of the \( j \)th particle is recorded and represented as \( pbest_j = (pbest_{j1}, pbest_{j2}, \ldots, pbest_{jd}) \). The index of best particle among all particles in the group is represented by the \( gbest \). The rate of the position change (velocity) for particle \( j \) is represented as \( v_j = (v_{j1}, v_{j2}, \ldots, v_{jd}) \). The modified velocity and position of each particle can be calculated using the current velocity and distance from \( pbest_j \) to \( gbest \) as shown in the following formulas[11]:

\[ v_j^{(t+1)} = v_j^{(t)} + \omega (pbest_j - x_j) \]  

(19)
where \( n \) is the number of particles in a group; \( m \) is the number of members in a particle; \( t \) is the pointer of iterations (generations); \( v^{(i)}_{jg} \) is the velocity of the particle \( j \) at iteration \( t \); \( v^{\min}_{g} \leq v^{(i)}_{jg} \leq v^{\max}_{g} \); \( w \) is the inertia weighting factor; \( c_1 \), \( c_2 \) are the acceleration constants; \( Rand() \), \( Rand'() \) are random numbers between 0 and 1; \( x^{(i)}_{jg} \) is the current position of particle \( j \) at iteration \( t \); \( pbest_j \) is the \( pbest \) of particle \( j \); \( gbest_j \) is the \( gbest \) of the group \( g \).

In the above procedures, the parameter \( V_g^{\max} \) determine the resolution or fitness, with which regions are searched between the present position and the target position. If \( V_g^{\max} \) is too high, particles might fly past good solutions. If \( V_g^{\max} \) is too low, particles may not explore sufficiently beyond local solutions.

The constants \( c_1 \) and \( c_2 \) represent the weighting of the stochastic acceleration terms that pull each particle toward pbest and gbest positions. Low values allow particles to roam far from the target regions before being tugged back. On the other hand, high values result in abrupt movement toward or past target regions.

Suitable selection of inertia weighting factor \( w \) provides a balance between global and local explorations, thus requiring less iteration on average to find a sufficiently optimal solution. As originally developed, \( w \) often decreases linearly from about 0.9 to 0.4 during a run. In general, the inertia weighting factor \( w \) is set according to the following equation [6][9]:

\[
w = \frac{w_{max} - w_{min} \times iter}{iter_{max}}
\]

where \( iter_{max} \) is the maximum number of iterations (generations), and \( iter \) is the current number of iterations.

### 3.2 Modified particle swarm optimization

The main point of the MPSO differs from the PSO is to consider the “distance” in its fitness function to avoid converging to a local optimum. Assign a rank (i.e., the number place 1, 2, 3, ..., etc.) \( RE_k \) to the calculated error of each new individual, \( v_k \), \( k = 1, \ldots, PS \), \( PS \) is the population size. A combined population with \( 2 \times PS \) individuals is formed. Unlike previously developed statistic methods, the concept of “distance” is added to the fitness function to prevent from being trapped in a local minimum. The fitness score of the \( k \)th individual is modified by [12,13]

\[
F_k = RE_k + \rho \times RD_k, \quad k = 1, \ldots, 2 \times PS.
\]

where \( \rho \) is an adaptive decay scale, \( \rho_{\text{max}} \) is set as 0.7 and \( \rho_{\text{min}} \) is set as 0.005 in this paper. \( RD_k \) is the rank of \( D_k \) assigned to the \( k \)th individual, where \( D_k \) is the distance from the individual to the current best solution vector, and is given by

\[
D_k = \| v_k - v_{\text{best}} \|	ag{23}
\]

where \( v_k \) is the vector of the \( k \)th individual in the combined population, and \( v_{\text{best}} \) is the current best solution vector.

An adaptive scheme is defined as [12]

\[
\rho_k = R \times (\rho_{\text{max}} - \rho_{\text{min}}) / g_{\text{max}}
\]

and

\[
\rho(g + 1) = \begin{cases} 
\rho(g) - \rho_k, & F_{\text{min}}(g) = F_{\text{min}}(g - 1) \\
\rho(g), & F_{\text{min}}(g) < F_{\text{min}}(g - 1)
\end{cases}
\]

where \( \rho_k \) is the step size; \( F_{\text{min}} \) is the minimum value of fitness functions; \( R \) is the regulating scale and is set as 1.25 in this paper, and \( g_{\text{max}} \) is the maximum allowable number of iterations.

Individuals will be ranked in ascending according to their fitness scores by a sorting algorithm. The \( PS \) individuals are transcribed along with their fitness for the next generation. If the new population does not include the current best solution, the best solution must be replaced with the last individual in the new population. In addition, a gradually decreased decay scale can satisfy a successive statistic searching process by first using the diversification (bigger \( \rho \)) to explore more regions, and then the intensification (smaller \( \rho \)) to exploit the neighborhood of an elite solution. The current best solution (point \( A \)) for a minimum fitness problem as shown in the Fig. 2 may not reach the global optimum [12,13], and there are three electable solutions exist. Generally, solutions with slightly better fitness (point \( C \) or \( B \)) prevailed, so the solution trapped into the valley prematurely. The more attractive solution (point \( G \)) is relatively far away from point \( A \), but it nears the global optimal. To prevent prematurity, point \( G \) with slightly worse fitness than \( C \), it needs a higher rank to be selected. That is, a higher \( RD_k \) is awarded to a longer \( D_k \).
Stopping Criteria: Stopping criteria is given in the following order:
1) maximum allowable number of iterations reached.
2) number of iterations reached without improving the current best solution.

Fig. 3 shows the flow chart of the proposed algorithm.

| Step 1 | Data Collection |
| Step 2 | Production of initial population |
|        | - particle: including $m_1$, $m_2$, $m_3$, $r$, $l$ |
|        | - parameter of PSO: $c_1$, $c_2$, $w$, $R$, $p_{max}$ |
| Step 3 | Evaluation |
|        | - Calculate Error and Distance |
|        | - Rank and Sort the Error and Distance |
|        | - Evaluate the Fitness Function |
|        | - Find the $g_{best}$ and $p_{best}$ |
| Step 4 | Position Change |
|        | Calculate velocity and position |
|        | - Generate the new particle |
|        | - update all parameter mentioned in Step 2 |
| Step 5 | Converge in optimum |

reach max. iteration number $N$ → Step 3

Fig. 3 The flow chart of the MPSO

3.3 Parameter identifications

How to define the fitness function is the key point of the MPSO, since the fitness function is a figure of merit, and could be computed by using any domain knowledge. In this paper, we adopt the fitness function as follows [14,15]:

$$F(\text{parameters}) = \sum_{i=1}^{n} E_{i}$$  \hspace{1cm} (27)

$$E_{i} = x^{(i)} - x^{(i)}$$  \hspace{1cm} (28)

where $n$ is the total number of samples and $E_{i}$ is the calculated error of the $i$ th sampling time, $x^{(i)}$ is a solution by using the fourth-order Runge-Kutta method to solve the dynamic Equations (12) for the PM synchronous servomotor drive coupled with a slider-crank mechanism with the parameters identified from these two methods, and $x^{(i)}$ is the displacement measured experimentally at the $i$ th sampling time.

IV. COMPARISONS BETWEEN PSO AND MPSO METHODS

The physical model of the slider-crank mechanism driven by a servomotor. In the parameter identification, we utilize the MPSO and PSO methods to identify the 5 parameters $m_1$, $m_2$, $m_3$, $r$, and $l$ simultaneously, and the fitness function is described as Eq. (27). The identified results are given in Table 1.

<table>
<thead>
<tr>
<th>m_1 (kg)</th>
<th>m_2 (kg)</th>
<th>m_3 (kg)</th>
<th>r (m)</th>
<th>l (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000-1.000</td>
<td>0.000-1.000</td>
<td>0.000-1.000</td>
<td>0.000-1.000</td>
<td>0.000-1.000</td>
</tr>
</tbody>
</table>

The identified results are given in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>$m_3$</th>
<th>$r$</th>
<th>$l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual value</td>
<td>0.232</td>
<td>0.332</td>
<td>0.66</td>
<td>0.026</td>
<td>0.217</td>
</tr>
<tr>
<td>Identified value of the PSO method</td>
<td>0.313</td>
<td>0.304</td>
<td>0.752</td>
<td>0.024</td>
<td>0.325</td>
</tr>
<tr>
<td>Identified value of the MPSO method</td>
<td>0.234</td>
<td>0.331</td>
<td>0.603</td>
<td>0.026</td>
<td>0.216</td>
</tr>
</tbody>
</table>

Figure 4 shows the convergence characteristics in PSO and MPSO methods of the slider-crank mechanism system. It is seen from Fig. 4 that the proposed MPSO method is superior to the PSO method.

![Figure 4](image_url)

Fig. 4 Comparison of convergence characteristics in PSO and MPSO methods of the slider-crank mechanism system

V. CONCLUSIONS

The dynamic formulations of a slider-crank mechanism driven by a field-oriented PM synchronous motor have been successfully formulated with only one independent variable. Furthermore, the main objective of this study is to utilize PSO and MPSO methods to identify a slider-crank mechanism driven by a servomotor. According to the comparisons between identified results and displacement errors, it is found that MPSO method has the best matching with the...
experimental results.

It is concluded that the implementations of MPSO are different from the PSO in five aspects. Firstly, its fitness function considers the distance to avoid converging to a local optimum. Secondly, for the MPSO, vectors with good enough fitness scores would be used as candidates to create new solutions. Thirdly, it has the advantage of the MPSO to conquer various constraints without using the fitness function with penalties, and can perform better. Fourthly, the solution is coded with a decimal representation, and saves computer memory. Finally, the gradually decaying parameters can satisfy the diversified (bigger parameters) to reserve the larger attractive region. Then, the intensification (smaller parameters) is used to search the small neighborhood of an elite solution.

APPENDIX A

We can obtain the Euler-Lagrange equation as follows

$$M(Q)\dot{Q} + N(Q, \dot{Q}) + \Phi_0^T\lambda = Q^\Lambda$$

where

$$M = \begin{bmatrix} A & E \\ B & D \end{bmatrix}, \quad N = \begin{bmatrix} K_w \\ P_w \end{bmatrix}$$

and

$$A = \frac{1}{3}m_2 l^2 + m_1 l^2 \sin^2 \phi,$$

$$B = \frac{1}{2}m_2 r^2 + (m_2 + m_1) r^2 \sin^2 \theta,$$

$$E = \left( \frac{1}{2}m_2 + m_3 \right) r l \sin \theta \sin \phi,$$

$$K_w = m_3 l^2 \dot{\phi}^2 \sin \phi \cos \phi$$

$$+ \left( \frac{1}{2}m_2 + m_3 \right) \frac{r l }{ \dot{\theta}} \cos \theta \sin \phi$$

$$+ \frac{1}{2}m_2 g \cos \phi,$$

$$P_w = \left( \frac{1}{2}m_2 + m_3 \right) r \dot{\phi} \sin \theta \cos \phi$$

$$+ (m_2 + m_3) \frac{r^2 \dot{\theta}^2 \sin \theta \cos \theta}{ \dot{\phi}},$$

$$Q^\Lambda = \begin{bmatrix} (F_k + F_\mu) l \sin \phi \\ (F_k + F_\mu) r \sin \theta - \tau \end{bmatrix}$$

$$\Phi_0 = \frac{\partial \Phi(Q)}{\partial Q} = [-l \cos \phi, r \cos \theta]$$

and $\lambda$ is the Lagrange multiplier.