Wavelet-Based Spectrum Sensing for Cognitive Radios using Hilbert Transform

Shiann-Shiun Jeng, Jia-Ming Chen, Hong-Zong Lin and Chen-Wan Tsung

Abstract—For cognitive radio networks, there is a major spectrum sensing problem, i.e. dynamic spectrum management. It is an important issue to sense and identify the spectrum holes in cognitive radio networks. The first-order derivative scheme is usually used to detect the edge of the spectrum. In this paper, a novel spectrum sensing technique for cognitive radio is presented. The proposed algorithm offers efficient edge detection. Then, simulation results show the performance of the first-order derivative scheme and the proposed scheme and depict that the proposed scheme obtains better performance than does the first-order derivative scheme.

Keywords—cognitive radio, Spectrum Sensing, wavelet, edge detection

I. INTRODUCTION

A cognitive radio (CR) network is a novel idea of wireless communications to solve the dynamic spectrum problem [1, 2]. The major objectives of the cognitive radio network are highly reliable communication for users and more efficient utilization of the radio spectrum [3]. Cognitive radio networks can sense and predict the environments and serve target users without interference to other users [4]. Spectrum sensing involves several tasks [3, 5], i.e. radio-spectral estimation and detection, radio-spectral resolution, channel estimation and prediction, and system reconfiguration. Therefore, one of the most important issues for cognitive radio networks is dynamic spectrum management, which has to estimate and detect for sensing and identifying radio spectrum.

There are a lot of researches for cognitive radio dynamic access techniques [5, 6], e.g. matched filter, energy detection, cyclostationary feature detection, and wavelet-based edge detection. For matched filter, it is the most accuracy but requires high computational complexity and perfect knowledge of the target users. Energy detection is the most common scheme of spectrum sensing with low computational complexity. However, there is a little high computational complexity for cyclostationary feature detection. For wavelet-based edge detection, wavelet transform is powerful to analyze local spectrum and identify characteristic and edges with low computational complexity.

There are some researches which focus on wavelet-based spectrum sensing [7, 8]. One of these researches [7] uses the wavelet transform to reduce computational complexity and utilizes the first-order derivative scheme to detect edge. The first-order derivative scheme can detect frequency boundaries accurately at high SNR but has worse performance at low SNR. Therefore, this paper proposes that the Hilbert Transform is used for edge detection. The Hilbert Transform is usually utilized to analyze the harmonic function. The average power spectrum density (PSD) within each sub-band, which is needed to be identified, is estimated by using a simple estimator, and it has to determine the occupied bands by the estimated PSD. Then, the performance of the proposed algorithm in term of edge detection is simulated and compared with that of the first-order derivative scheme [7], while the different SNR is considered.

The remainder of this paper is organized as follows. Section 2 introduces the spectrum sensing problem, the wavelet transform of the sensing signals and a simple estimator of PSD. Section 3 shows wavelet-based spectrum sensing using Hilbert Transform. Section 4 shows the simulation results of the first-order derivative scheme and the proposed scheme, and Section 5 draws the conclusion for this work.

II. PRELIMINARY

A. Problem Formulation for Spectrum Sensing

In order to identify spectrum holes, a CR system is used to sense the wireless environment. It is assumed that the observed signal received by the CR system occupies $N$ spectrum bands, whose frequency position and PSD levels have to be detected and identified. These spectrum bands locate between $f_0$ and $f_N$, and their frequency boundaries locate at $f_0 < f_1 < \ldots < f_N$. Fig. 1 shows the PSD structure of a wideband signal with the $n$-th band defined by $B_n = \{ f \in B_n : f_{n-1} \leq f < f_n \}$, $n=1, 2, \ldots, N$. The following basic assumptions are adopted in this work. First, the CR system knows the spectrum boundaries $f_0$ and $f_N$. For the CR system, the observed signal may occupy a wider band, but this work only analyzes the interesting spectrum between $f_0$ and $f_N$. Second, $N$ spectrum bands and the positions $f_1, \ldots, f_{N-1}$ are unknown to the CR system. These environment parameters keep on within a time burst but may change in next
time burst. Third, there is a jump at the edge of each band, and these edges have to be identified. Finally, The additive white Gaussian noise (AWGN) with zero mean and two-sided PSD $S_r(f)=N_0/2$ is considered in this work.

The PSD of the observed signal $r(t)$ in the CR system can be written as

$$S_r(f) = \sum_{n=1}^{N} \alpha_n^2 S_n(f) + S_w(f) \quad f \in [f_0, f_N]$$

where $S_n(f)$ is the $n$-th signal spectrum, and $\alpha_n^2$ is the signal power density within the $n$-th band. The PSD $S_n(f)$ is $F[R_n(\tau)]$, where the autocorrelation function $R_n(\tau)$ is equal to $E\{r(t)r(t+\tau)\}$ and $F\{\cdot\}$ is Fourier Transform. The corresponding time-domain signal $r(t)$ with $S_l(f)$ can be represented as

$$r(t) = \sum_{n=1}^{N} \alpha_n x_n(t) + w(t)$$

where $x_n(t)$ is the time-domain signal with $S_n(f)$, and $w(t)$ denotes the AWGN with $S_w(f)$. Then, $x_n(t)$ occupying the $n$-th band can be expressed as

$$x_n(t) = \sum_{k=-\infty}^{\infty} s_k p(t-kT_s)e^{j2\pi f_s t}$$

where $s_k$ is the $k$-th modulated symbol, $p(t)$ is a pulse shaper with the bandwidth $(f_{b_1} - f_{b_2})$, and $f_{b_1} = (f_{b_2} + f_0)/2$ is the center frequency of the $n$-th band. The shape of $S_n(f)$ is corresponding to $|F[p(t)]|^2$.

For the CR system, the wideband spectrum sensing problem is that the parameters characterizing the wideband spectral environment, i.e. $N, f_0$, and $\alpha_n^2$, have to be estimated.

B. Wavelet-Based Sensing Signals

The wavelet-based spectrum sensing has detailed in [7] and will be introduced in brief. Let $\phi(f)$ be a wavelet smoothing function. The extension of $\phi(f)$ by a scale factor $s$, which is the powers of 2, is given by

$$\phi_s(f) = \frac{1}{s} \phi\left(\frac{f}{s}\right)$$

Then, the continuous wavelet transform (CWT) of $S_l(f)$ can be represented as

$$W_S(f) = S_l(f) * \phi_s(f)$$

where * is convolution computation. Then, the inverse Fourier Transform of the wavelet function can be represented as

$$W_{sR}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W_S(f) e^{j2\pi f \tau} df$$

where $R(t)$ is the AWGN with $S_r(f)$, and $\Phi(t)$ is Fourier Transform of $\phi(f)$. Then, (5) can be rewritten as

$$W_{sR}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W_S(f) \phi(f) e^{j2\pi f \tau} df$$

Then, a product operation in (8) is more efficient than a convolution computation operation in (5). When the PSD $S_l(f)$ smoothed by the scaled wavelet $\phi_s(f)$, $W_S(f)$, is derived from (8), the first-order derivative scheme of $W_S(f)$ can be represented as

$$W_S'(f) = s \frac{d}{df} (S_l(f) * \phi_s(f))$$

where

$$S_l(f) * \phi_s(f) = (S_l(f) * \phi_s(f)) e^{j2\pi f \tau}$$

The local maximum of the first derivative scheme is utilized to identify the boundaries $f_0$ [9]. The local maxima of $W_S'(f)$ can be represented as

$$f_n = \max_{f} \left\{ W_S'(f) \right\}, \quad f \in (f_0, f_N)$$

Fig. 1 The PSD structure of a wideband signal with $N$ bands

$$\Phi_s(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W_{sR}(\tau) e^{j2\pi f \tau} df$$

where

$$W_{sR}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W_S(f) \phi(f) e^{j2\pi f \tau} df$$

Then, (5) can be rewritten as

$$W_{sR}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W_S(f) \phi(f) e^{j2\pi f \tau} df$$

where

$$S_l(f) * \phi_s(f) = (S_l(f) * \phi_s(f)) e^{j2\pi f \tau}$$

The local maximum of the first derivative scheme is utilized to identify the boundaries $f_0$ [9]. The local maxima of $W_S'(f)$ can be represented as

$$f_n = \max_{f} \left\{ W_S'(f) \right\}, \quad f \in (f_0, f_N)$$

9)
C. PSD Estimation and Identification

After $\hat{f}_n$ has been detected and estimated by (10), $\alpha^2_n$ has to be estimated. The average PSD within the band $B_n$ can be computed and represented as

$$\hat{\beta}_n = \frac{1}{\int_{f_n}^{f_{n+1}} S_s(f)df}$$

(11)

When it is assumed that the AWGN PSD $S_s(f)$ can be measured, it is foreseen that $\hat{\beta}_n$ is related to the unknown $\alpha^2_n$ by $\hat{\beta}_n \approx \alpha^2_n + N_0/2$ and $\hat{\beta}_{n,\min} = N_0/2$ is minimum for all $\hat{\beta}_n$. Therefore, a simple estimator for $\alpha^2_n$ can be represented as

$$\alpha^2_n = \hat{\beta}_n - \min_{\alpha} \hat{\beta}_n.$$  

(12)

The simple estimator in (12) is enough to solve the sensing problem in this work. The major sensing problem in this work may be categorized into three spaces, i.e. white, gray, and black. Therefore, a simple estimator for $\alpha^2_n$ is enough to detect $[3, 9]$.

III. Wavelet-Based Spectrum Sensing Using Hilbert Transform

In recent years, the Hilbert transform is more commonly utilized to analyze the harmonic function by using a convolution computation operation with the Cauchy kernel. The Hilbert transform is a linear operation which transforms a desired function $f(s)$ into another function $g(s)$ with the same domain. For example, a square wave function is transformed and shown in Fig. 2. When the desired function $S(f)$ is determined, the transformed function, the Hilbert transform of $S(f)$, can be represented as

$$H(f) = H(f) * S(f)$$

(13)

where $H(f)$=p.v.1/$\pi f$. p.v. is the Cauchy principal value. When the continuous wavelet transform (CWT) of $S(f)$ is first used, $W_S(f)$ can be obtained. Then, the Hilbert transform of $W_S(f)$ can be represented as

$$HW_S(f) = H(f) * W_S(f)$$

(14)

The inverse FT of $HW_S(f)$, $HW_S(\tau)$, can be represented as

$$HW_S(\tau) = h(\tau) \cdot R_1(\tau) \cdot \Phi_s(\tau)$$

(15)

where $h(\tau)$=F^{-1}[H(f)]. Then, substituting (16) into (15) yields

$$HW_S(\tau) = F\{HW_S(\tau)\} = F\{h(\tau) \cdot R_1(\tau) \cdot \Phi_s(\tau)\}$$

(16)

When the Hilbert transform is used, the computation of the $HW_S(f)$ involves either convolution computation and Fourier Transform operations as in (14) or product computation and Fourier Transform operations as in (16). Similar to (8), a product operation in (16) is easier to operate than a convolution computation operation in (14). Then, the local maximum of the proposed scheme is utilized to identify the boundaries $f_n$ [9]. The local maxima of $HW_s(f)$ can be represented as

$$\hat{f}_n = \max_f \{HW_s(f)\}$$

(17)

After $\hat{f}_n$ has been detected and estimated by (17), (11) and (12) can be utilized to estimate $\alpha^2_n$.

IV. Performance Evaluation

This work considers an interesting spectrum between 50 MHz and 200 MHz, i.e. $f_0=50$ MHz and $f_6=200$ MHz. Fig. 3 shows that the PSD of the observed signal has 7 bands with the frequency boundaries $f_i$ at [50, 100, 125, 150, 165, 175, 200] MHz when $S_s(f)=-20$ dB. $B_1$, $B_2$, and $B_7$ are not occupied. As shown in Fig. 3, $B_2$ is considered that $B_2$ is equal to $S_s(f)$, and then $B_3$, $B_4$, and $B_5$ have corresponding PSDs with 0 dB, 4 dB and 10 dB, respectively. Figs. 4 and 5 are simulated with different $S_s(f)$ under $s=1$, and Figs. 6 and 7 are simulated with different $S_s(f)$ under $s=2$. Fig. 4 shows the edge detection of the first-order derivative scheme and the proposed scheme at $S_s(f)=-10$ dB when $s=1$. As shown in Figs. 4 (a) and (b), all frequency boundaries, $f_3$, $f_4$, $f_5$, $f_6$, $f_7$, and $f_n$ are local maxima for both schemes, so both schemes can easily detect frequency boundaries. Fig. 5 shows the edge detection of the first-order derivative scheme and the proposed scheme at $S_s(f)=0$ dB when $s=1$. As shown in Fig. 5 (a) for the referred scheme, $f_i$ and $f_2$ are difficult to identify, $f_1$ can be easily detected, but $f_4$ is difficult to detect. Because $B_4$ has higher power than $B_2$ and $B_6$, $f_4$ and $f_n$ can be identified. As shown in Fig. 5 (b) for the proposed scheme, $f_3$, $f_4$, $f_6$, and $f_n$ are local maxima. Although $B_4$ is interfered by AWGN noise, $f_1$ and $f_2$ can be identified. Fig. 6 shows the edge detection of the first-order derivative scheme and the proposed scheme at $S_s(f)=-10$ dB when $s=2$. As shown in Figs. 6 (a) and (b), all frequency boundaries are
local maxima for both schemes, and both schemes can also obtain frequency boundaries. Fig. 7 display the edge detection of the first-order derivative scheme and the proposed scheme at $S_s(f)=0$ dB when $s=2$. As shown in Fig. 7 (a) for the referred scheme, $f_1$ and $f_2$ are difficult to identify. $f_s$ and $f_d$ can be detected. $B_6$ has higher power than $B_2$ and $B_4$, so $f_4$ and $f_6$ can be identified. As shown in Fig. 7 (b) for the proposed scheme, $f_5$, $f_6$, $f_7$, and $f_8$ are local maxima. Although $B_1$ is interfered by AWGN noise, $f_1$ and $f_2$ can also be identified. After frequency boundaries are identified, a simple estimator (12) is used to determine the occupancy of the bands. First, $B_1$, $B_2$, $B_3$, $B_4$, and $B_7$ are considered herein. $B_2$ is occupied, and $B_1$, $B_3$, $B_5$, and $B_7$ are not occupied. $S_s(f)$ can be obtained from $\beta_{n_{\text{min}}}$, and $\alpha_S^2$ can be calculated by (12). Then, Fig. 8 demonstrates the receiver operating characteristic (ROC) curves of the referred scheme and the proposed scheme when $S_s(f)/S_s(f)=10$ dB. The ROC shows the relationship between the probability of detection $P_d$ and the probability of false alarm $P_{fa}$, and an optimum threshold $\lambda$ is selected for finding a good trade-off between $P_d$ and $P_{fa}$ [3, 5]. Fig. 8 depicts that the proposed scheme outperforms the referred scheme. Similarly, $\alpha_S^2$ and $\alpha_S^2$ can be obtained from $B_2$ and $B_6$, respectively. When $S_s(f)$ or $S_s(f)$ is considered, the proposed scheme also outperforms the referred scheme, and $P_{fa}$ of the proposed scheme is lower than that of the referred scheme.

Simulation results depict that the referred scheme and the proposed scheme can obtain better performance under low $S_s(f)$, and frequency boundaries for both schemes can be identified. However, the referred scheme cannot identify the frequency boundaries under high $S_s(f)$. When high $S_s(f)$ is considered, the proposed scheme can identify the frequency boundaries. Then, a simple estimator is used to determine the occupancy of the bands, and simulation results demonstrate the proposed scheme outperforms the referred scheme.

V. CONCLUSION

For the CR system, the wideband spectrum sensing problem is that the parameters characterizing the wideband spectral environment have to be estimated. The first-order derivative scheme is usually used to detect the edge of the spectrum. This work proposes a novel spectrum sensing algorithm for cognitive radio. As shown in the simulation results, the proposed scheme can obtain better efficient estimation and identification of frequency boundaries than the referred scheme. The simulation results also demonstrate that the proposed scheme performs better $P_d$ and lower $P_{fa}$ than does the first-order derivative scheme. Furthermore, a good estimator will perform better than a simple estimator. Estimation and identification of frequency boundaries is focused in this paper. In the future, a good estimator will be used to enhance this work.

REFERENCES

Fig. 5 Edge detection of the referred and proposed schemes at $S_n(f)=0$ dB when $s=1$

Fig. 6 Edge detection of the referred and proposed schemes at $S_n(f)=-10$ dB when $s=2$

Fig. 7 Edge detection of the referred and proposed schemes at $S_n(f)=0$ dB when $s=2$

Fig. 8 ROC curves of the referred scheme and the proposed scheme