Abstract—The concept of differential protection based on current quantities has been discussed in many papers and researches. For certifying and inverting of currents and voltages through converter systems, there is no conventional current differential relay, which can compare current quantities, because they are different in form and frequencies.

An overview over a new concept of differential protection for converters based on instantaneous power quantities will be discussed in this paper. To drive the power quantities a mathematical background of the space vectors will be introduced. A simple DC-Link is preceded in this paper and a power analysis description and simulation is derived using Matlab®/Simulink™ concerning a certain construction scheme of Power Differential Relay System. Finally a complete analysis of three phase fault in DC-Link Rectifier is discussed to ensure the ability of Power Differential Protection System to detect the fault in main and selectivity protection sections.

Keywords—Space Vectors, Power Differential Relay (PDR), Short Circuit Power, Diode Recovery Energy, Detected Power Differential Signal (DPDS), Power Space Vector (PSV), Power Space Vector Protection Area (PSVPA).

I. INTRODUCTION

When we take a look on any power distribution and transmission system, it could be seen, that differential protection exists in many sections of this power system, like for generators, transformers, transmission lines, bus bars, over which current differential relay can compare the current quantities between the unit terminals.

However, there was an increase of some needs in power systems through many years, which has forced the grids to be equipped with different types of converter systems. These needs are, for example, connecting two networks operating at different frequencies (DC-Link), power flow control (UPFC), using high DC current for chemical applications (Chemical Rectifier for Al-Production), or using a DC source to supply frequency dependent loads (IGBT Inverters supplying induction motors).

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Using the conventional differential protection systems based on current quantities for these systems faces the following problems:

- The current form is unknown because of harmonics and unbalances.
- The currents are independent of each other.
- Different frequencies exist already on the primary and secondary terminals of any of these special units.

For that all the base of differential protection must be oriented to suit these special power units, which is differential protection based on power quantities.

In general the input power to any power system equipment should equal to the output power with exception of some losses in this unit. Fig. 1 and equation (1) illustrate the general case of differential power concept.

\[
\begin{align*}
 p_1 + p_2 &= \Delta p \\
 p_1 &= u_1 i_1 \\
 p_2 &= u_2 i_2
\end{align*}
\]

Fig. 1 Measuring of input and output power through none linear power unit

The point now is how to compare between the instantaneous power quantities! It is important to define the effective and average values of the instantaneous power flow. After obtaining instantaneous power signals from measured current and voltage signals, the space vectors’ components (α,β) are calculated. From the space vectors in a modified form the effective and average power quantities have to be calculated. At that moment one can compare between each average and effective power quantities calculated from the space vectors. The comparison will be proceed based on the different between similar power signal forms (average with average and effective with effective) in a percentage ratio of the input average or effective power value respectively.

Besides, the possibility of having selectivity in the power differential system is going to be declared.

A single diagram of power differential protection system
for a DC-Link will be presented. The DC-Link is connecting 60Hz-Power Supply to 50Hz-Inductive Load. This paper is based on preceding a DFG-Project (German Research Foundation) for an expected period of 4 years.

II. MAIN DIFFERENTIAL POWER CALCULATIONS

In three phase systems the general space vectors equation can be written as [1]:

\[
v_0 = \frac{1}{3}(v_R + v_S + v_T)
\]

(2)

\[
v = \frac{2}{3}(v_R + av_S + a^2 v_T)
\]

(3)

Where,

- \(v_0\): Zero component of three phase system,
- \(v\): Space vector of three phase system and
- \(v_{R,S,T}\): Instantaneous three phase signals of three phases.

By measuring the instantaneous three phase currents and voltages values in the three phases, the instantaneous power values could be calculated as follows:

\[
p_R = u_R \cdot i_R, \quad p_S = u_S \cdot i_S, \quad p_T = u_T \cdot i_T
\]

(4)

By replacing \(v_{R,S,T}\) by \(p_{R,S,T}\), the zero and space vector components of the instantaneous power quantities in a three phase system are written in a matrix form as follows:

\[
\begin{bmatrix}
q_0 \\
q_a \\
q_β
\end{bmatrix} =
\frac{1}{3}
\begin{bmatrix}
1 & 1 & 1 \\
2 & -1 & -1 \\
0 & \sqrt{3} & -\sqrt{3}
\end{bmatrix}
\begin{bmatrix}
p_R \\
p_S \\
p_T
\end{bmatrix}
\]

(5)

where,

- \(q_0\): zero component of instantaneous three phase power,
- \(q_a\): real component of power space vector and
- \(q_β\): imaginary component of power space vector.

If equation (5) is multiplied by 3 the result will release a modified power zero and space vector components, which indicate the total instantaneous power flow as quazi-zero component of the total instantaneous power quantities and analogy the quazi-space vectors. In that way equation (5) could be rewritten as follows [2]:

\[
\begin{bmatrix}
q_0' \\
q_a' \\
q_β'
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 \\
2 & -1 & -1 \\
0 & \sqrt{3} & -\sqrt{3}
\end{bmatrix}
\begin{bmatrix}
p_R \\
p_S \\
p_T
\end{bmatrix}
\]

(6)

By referring to the general equation to calculate the instantaneous RMS value of any signal using the zero and space vector components as follows:

\[
V_{\text{eff}}^2 = \frac{1}{2}(V_{0,\text{eff}}^2 + V_{a,\text{eff}}^2 + V_{β,\text{eff}}^2)
\]

(7)

The active, reactive and apparent power could be presented as follows:

\[
S = \sqrt{\frac{1}{2}(q_{0,\text{eff}}^2 + q_{a,\text{eff}}^2 + q_{β,\text{eff}}^2)}
\]

(8)

\[
P = \frac{1}{T} \int_0^T p_0 dt,
\]

(9)

\[
Q = \sqrt{S^2 - P^2}.
\]

(10)

Now it is possible to have the instantaneous RMS and average values of the instantaneous power flow but on AC-Sides. As the common instantaneous value between AC- and DC-sides is the average value, it is then easier to think only about calculating the instantaneous average value of power flow on DC-Sides as follows:

\[
p_{DC} = u_{DC} \cdot i_{DC} \Rightarrow p_{DC} = \frac{1}{T} \int_0^T p_{DC} dt
\]

(11)

The Detected Power Differential Signal (DPDS) between power flow quantities through any power unit equipment can be percentage calculated as follows:

\[
\text{DPDS} = \frac{P_{in} - P_{out}}{P_{in}} \times 100
\]

(12)

Where,

- \(P_{in}\): \(S\), \(P\), \(Q\) or \(D\) CP from equations (8-11) at the input of a power unit, Fig. 1.

- \(P_{out}\): \(S\), \(P\), \(Q\) or \(D\) CP from equations (8-11) at the output of a power unit, Fig. 1.

III. THE DC-LINK IN NORMAL OPERATION AND FAULT CONDITIONS

Fig. 2 shows the measuring points considered in Matlab ® /Simulink™ (circles). The power differential calculation is done according to the equations (2-11) to get the instantaneous RMS and average values of three phase power flow. Finally the differential process is achieved according to equation (12). The investigated DC-Link is built from 6-pulse
diode rectifier supplied by 60 Hz main source through a step down transformer. The rectifier is connected on the DC-side to IGBT-inverter through a capacitor bank to supply 50Hz-50kVA load.

From this scheme six possibilities for power differential protection are available:

1. Power differential protection over the rectifier based on the instantaneous average power flow (PDR1).
2. Power differential protection over the capacitor bank based on the instantaneous average power flow (PDR2).
3. Power differential protection over the inverter based on the instantaneous average power flow (PDR3).
4. Power differential protection over the DC-Link based on the instantaneous active power flow (PDRP).
5. Power differential protection over the DC-Link based on the instantaneous reactive power flow (PDRQ).
6. Power differential protection over the DC-Link based on the instantaneous apparent power flow (PDRS).

In this way, and without building any extra selectivity protection scheme, it is clearly seen that the last three PDR can be recognized as selective protection system, which is discussed in details later.

A. Normal Operation

The model of DC-Link according to Fig. 2 has been built in Matlab®/Simulink™ and simulated with no fault conditions for 10 seconds with time step of 20µs to ensure the accuracy of the power differential results. Figs. 3 and 4 show the resulting DPDS of the power differential relays PDR1,2,3 and PDRP,0,3 respectively according to equation (12). One can notice the following:

1. DPDS value ranges are not the same for each power unit.
2. DPDS value is not constant with time.
3. DPDS value could lie in the negative range.

These remarks make it difficult to define a single tolerance range of DPDS oscillations in normal operation conditions, specially when the load conditions are changing from time to time, which is normally the case in reality. The negative range of DPDS values in the differential process of reactive and apparent power flow indicates that an amount of reactive power has been generated because of distortions caused by converters.

Besides, the existence of the capacitor bank as an energy store circuit makes it possible to get power delivery to the inverter according to system conditions. That means, the concept of comparing the differentiated signals to a single error value is not valid anymore. There should be then other criteria to detect the fault without any confusion with any other system conditions. This could be declared after analyzing some fault conditions and see how the DPDS values will react against them concerning all instantaneous power flow quantities.
Fig. 6 Power space vector diagram of the total instantaneous output power from the inverter

Figs. 5 and 6 show the power space vector diagrams of the instantaneous total power at the rectifier input side and inverter output side respectively. After passing the transient simulation period (around 50ms), the space vector shape for normal operation could be bordered by certain coordinates of $q'_\alpha$ and $q'_\beta$.

The power space vector offers in this way an additional protection algorithm by observing the instantaneous coordinates of space vector components, which must lie in the shape of normal operation.

Using basic triangle calculus in Fig. 5, for example, one can detect the fault condition by observing the instantaneous values of $q'_\alpha$ and $q'_\beta$. Concerning the read vertices of the intersections of the space vector borders in Fig. 5, the following formulas for $q'_\alpha$ and $q'_\beta$ were deduced:

\[
(q'_\alpha + 0.67)(q'_\alpha + 1.93q'_\beta + 3.06) \geq 0
\]

\[
0.58q'_\alpha + q'_\beta + 1.63 \leq 2.43
\]

For normal operation of the rectifier equations (13) and (14) must be fulfilled.

### B. Short Circuit Analysis and Simulation

This paper discusses symmetrical short circuit in all available positions on AC- and DC-sides. Before going in details in simulation results in fault conditions, an imagination about the behavior of the investigated system and a general view of the power flow through the system in such cases will be discussed.

As the basis of the protection system is the instantaneous power flow, one of the easiest and fastest ways for short circuit calculations is the “equivalent kVA method”, which has been introduced in book markets [5]. According to what have been stated in this book, this method is used for three phase faults only. However with some modifications and derivations it could deduce the relations between the different fault types concerning the power flow at fault points. The deduced relations are:

\[
kVA_{LG} = \frac{3}{2k+1} kVA_{3-\varphi}
\]

\[
kVA_{DLG} = \frac{3}{k+2} kVA_{3-\varphi}
\]

\[
kVA_{DL} = \frac{\sqrt{3}}{2} kVA_{3-\varphi}
\]

\[
k = \frac{kVA_{3-\varphi}}{kVA_0}
\]

where,

- $kVA_{3-\varphi}$: three phase fault power at fault point
- $kVA_{LG}$: single line to ground fault power at fault point
- $kVA_{DLG}$: double line to ground fault power at fault point
- $kVA_{DL}$: line to line fault power at fault point
- $kVA_0$: zero sequence fault power at fault point

These amounts of fault power are calculated according to the permissible $kVA$ flow in any three phase system during a fault. Equation (18) gives the value of $k$ which is the ration between the equivalent $kVA$ of the system in case of three phase fault to the equivalent $kVA$ of the zero sequence circuit of the system, which must be included in fault calculation in case of unsymmetrical faults.

The approximations considered in equations (15) to (18) are:

1. Positive and negative impedances are equal.
2. Fault and ground resistances equal to zero.

### C. Three Phase Fault on AC-Rectifier Side (PDR1)

1) Proposed theoretical analysis

As the concept of protection system is based on the power flow, the short circuit analysis and simulation will concentrate on the behavior of the power flow during the fault. Referring to figure 2 the short circuit power contributed from the source side could be calculated as follows:

\[
kVA_{3-\varphi(s-T)} = \frac{kVA_S \cdot kVA_T}{kVA_S + kVA_T} / z\%
\]

The parameter $z\%$ is the nameplate impedance of the transformer in per unit value, which equal to ratio between the rated power of the transformer ($kVA_T$) to the maximum short circuit power flow through the transformer. The amount of short circuit power contributed from rectifier side will depend on the energy amount stored in the diode junction capacitance, which depends on the value of the reverse
voltage applied on the diode in the fault period. This junction capacitance will discharge in two phases: the first one is to reach a junction voltage equal to zero, where the reverse current is decaying with the decaying of the reverse voltage. The second one is to charge the diode depletion capacitance to the reverse voltage value [10]. As the reverse voltage is decaying, the diodes will be again forward biased because of the charge increase on the depletion capacitance. This process will continue till the total stored energy in diode capacitance is dissipated in diode resistances. Concerning the power flow to the fault point, the maximum power fault contributed from the diode rectifier diodes is:

\[ kVA_{3-phi(t)} = V_{max} \cdot I_{max} = 0.5kVA_{dc} - P_{df} \]  \hspace{1cm} (20)

Where,

- \( V_{max} \): maximum reverse voltage at fault stating time
- \( I_{max} \): maximum reverse current
- \( P_{df} \): maximum diode losses in forward biased direction

This amount of power will flow through the fault path till the reverse voltage reaches zero. Before the fault appears PDR1 was measuring DPDS relative to:

\[ \Delta kVA = kVA_f - kVA_{dc} \]  \hspace{1cm} (21)

After the fault appears:

\[ \Delta kVA = \frac{kVA_f \cdot kVA_f}{z\%} + kVA_s \cdot kVA_f / z\% - 0.5kVA_{dc} + P_{df} \]  \hspace{1cm} (22)

The factor 0.5 in equation (20) caused by reducing the output voltage of the rectifier by its half value at the first instant of the fault. The difference between equation (21) and (22) is quite clear and must be recognized from PDR1. From the other side it is also expected to have zero power flow through the capacitor bank which could be realized by PDR2 and PDR3 as a selective protection.

2. Simulation results

The fault has been simulated to start after 7 seconds of normal operation till the end of the 10 seconds simulation time. The results are shown in the figures from 7.a to 7.f. From these result one should think now about, how the protection system can realized the fault, if DPDS value depends mainly on the losses in power unit equipments, which are different from one to another unit. The succeeded way is to observe the difference value (DV) between two DPDS values in a certain time interval to detect abnormal jumps in the positive or negative direction of DPDS value. A software program has been built, which has detect the fault successfully for a time interval of 10ms according to the following equation:

\[ DV = \left| DPDS(t_2) - DPDS(t_1) \right| \]  \hspace{1cm} (23)

Where,

- \( DPDS(t_1) \): DPDS at time \( t_1 \)
- \( DPDS(t_2) \): DPDS at time \( t_2 \)
- \( t_2 - t_1 = 10\text{ms} \)

Regarding Fig. 7.a, after the first 10ms of the fault beginning the protection system will keep DPDS value at \( t_1 \) to compare it with the next value after 20ms according to equation (23). The reason is that the protection system must be sure that the case is fault and not just a change of load or any damage in capacitor bank. This process will continue during the fault till reaching a fault time between 30 and 50ms. After this period of time with the same DPDS behavior, the protection system must trip in less than 150ms for a maximum fault time of 200ms.

Fig. 7.b shows the observation of DPDS over the capacitor bank before and after the fault occurrence. The zooming is not showing the 10ms-steps as in Fig. 7.a; however, it is already indicated by the range of DV. Two things can be noticed:

- The first one is that even the fault has happened in the rectifier and on its AC-side; PDR2 can detect the fault also in the similar way like PDR1. That means PDR2 can be used as a selective protection as well as a primary one for the capacitor bank. This shows the advantage of using the power flow as a base for this differential protection system and PDR3.

- The second one is that after around 100ms the value of DPDS has became negative. In this case the capacitor bank is behaving as a temporary DC-source for the inverter and trying to keep the power flow level the same over the inverter, which makes PDR2 not able to realize in fault in the system. One of the opportunities to have selectivity in the protection system is the behavior of DPDS of the active power flow through DC-Link shown in Fig. 7.c. To ensure the selectivity using the DPDS value of the active power flow, the tripping time of PDR3 must be adjusted to shut down the system after 200ms to back up the tripping of PDR1 if it does not react on time for any reason. From Fig. 7.e one can see that till 300ms the DV value concerning the active power flow has increased by around 0.3% which means that the change rate in DV range after the fault appears is less than that of the DPDS in PDR1 and PDR2 regions.

Concerning Fig. 7.d, it could be seen, that the value of DPDS regarding the instantaneous reactive power flow has taken negative value before and after the fault. Besides, both values of DPDS did not reach a stable value in the same time as that of reactive power flow. The negative value of DPDS means that the amount of the reactive power supplied to the load through the inverter is more than the amount absorbed by the rectifier, if one supposes that there is no reactive power transferred through the capacitor bank. In reality the amount of the reactive power flow is decided by two values: displacement reactive power needed by the system and the distortion reactive power caused by the harmonics generated by the converters systems in AC and DC directions. That means there could be an amount of this distortion reactive power transferred from the rectifier to the inverter through the capacitor bank. All these operating factors would cause an increase in the amount of the instantaneous reactive power flow from the inverter AC side [3].

It is well known that the short circuit current used to be reactive current in most cases. This explains why the value of DPDS starts to increase immediately after the fault starts. Fig. 7.d shows a symmetrical behavior of DPDS after and before the fault instant. To ensure recognizing the fault selectivity by
PDRQ, the slope of DPDS should be observed aside checking of equation (23). To ensure the selectivity of PDRQ, the slope (DPDS) and equation (23) should be checked for t2–t1 > 10ms, so that PDRQ will definitely need more time than that for PDR, to trip. From Fig. 7.d, t2-t1=20ms and DPDS must be less than 0.2%.

\[
DPDS_s = \frac{DPDS(t_2) - DPDS(t_1)}{t_2 - t_1} \quad (24)
\]

Equation (24) could be always considered for the reactive and apparent power flow as it is always expected to have a certain slope of DPDS during normal operation because the reactive power absorption of the rectifier may decrease with time with respect to the load absorption of the reactive power through the inverter.

Concerning the power space vector diagrams in Figs. 7.e and 7.f, it is clear that the power space vector components have moved out of the normal operation borders marked by solid light lines. Concerning Figs. 5 and 6, the trace of the space vector has already crossed the normal operation borders but this was because of the transient process at the beginning of the simulation. The enlargement of the space vector diagram shown in Fig. 7.e happened symmetrically as expected because the fault is already symmetrical one. On the inverter side the disturbance on the supply from the capacitor bank side will cause unbalance operation of the inverter which will cause a deviation on the space vector diagram from normal operation borders as shown in Fig. 7.f. In all cases the using of equations (13) and (14) for Fig. 7.e and the corresponding equations for Fig. 7.f, which can be deduced in the same way, will result in detecting a fault in the DC-Link system, which could be also used as another selective protection system for the DC-Link.

This way of protection could also be called “Power Space Vector Protection Area”, as there is a certain XY area where the power space vector components are allowed to move during normal operation of the DC-Link [4]. The reason that the PSV has in this case not defined frequency working at but holding the entire main and distortion power flow through the DC-Link system. Nevertheless the results show that the concept of PSVPA could be used to observe the total instantaneous power flow using the space vector components, besides, to detect the fault as by using equation (13) and (14).
Then these systems of protection sections. There will be no constant error in normal operation, because each unit in the DC-Link has its own losses. However, the concept of observing the DPDS values in a certain time interval, which will depend on the location of the relay and the instantaneous power quantity. Also as a selective protection system one can used the observation of the PSV components to detect, if the system is working in the normal operation area or not.

Actually this is not the only case considered in this project. All types of faults on the AC-Sides of the rectifier and the inverter have been studied. Also a wide discussion on the capacitor bank itself is showing how the power protection system is reacting in case of fault or just damage in single or several capacitors, if the situation does not need to switch off the DC-Link.

Furthermore a complete fault analysis is under study for all of UPFC, frequency dependent operating motors and chemical rectifier, whose construction used to be applied for Aluminium production. This project is organized to go in this direction.

The new concept of differential protection has the advantage to be valid for all power unit equipments, as it is related to the power flow, which is holding all the information about any network. The protection system shown in figure 2 could be therefore extended to cover all the units building the DC-Link as well as any power distribution system with any kinds of power equipments. Besides, the problem of inrush current could be eliminated because the protection system is based on power flow.

**REFERENCES**


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