Vessel Inscribed Trigonometry to measure the Vessel Progressive Orientations in the Digital Fundus Image

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Abstract— In this paper, the vessel inscribed trigonometry (VITM) for the vessel progression orientation (VPO) is proposed in the two-dimensional fundus image. The VPO is a major factor in the optic disc (OD) detection which is a basic process in the retina analysis. To measure the VPO, skeletons of vessel are used. First, the vessels are classified into three classes as vessel end, vessel branch and vessel stem. And the chain code maps of VS are generated. Next, two farthest neighborhoods of each point on VS are searched by the proposed angle restriction. Lastly, a gradient of the straight line between two farthest neighborhoods is estimated to measure the VPO. VITM is validated by comparing with manual results and 2D Gaussian templates. It is confirmed that VPO of the proposed mensuration is correct enough to detect OD from the results of experiment which applied VITM to detect OD in fundus images.

Keywords—Angle measurement, Optic disc, Retina vessel, Vessel progression orientation.

I. INTRODUCTION

The optic disc (OD) detection is an essential strategy in the diagnosis of eye diseases such as glaucoma, diabetic retinopathy etc. The OD is a main feature for analyzing retinal fundus image. Especially, OD dimensions are used to diagnosis glaucoma and diabetic retinopathies [1]. OD detection method using vessel orientation is more essential and efficient than others such as image brightness, shape matching, and etc [2]–[4].

The vessel orientation is generally estimated manually or is used standard edge fitting algorithm based on the pre-defined 2D Gaussian templates [4], [5]. In the latter case, the templates are generated to model the retina vessel along all different orientations and applied to points consisting of vessels. For each point on vessels, maximum response of the templates is chosen as its direction. However, all points contained in the template are concerned in determining orientation whether they belong to same vessel or not. So, it is hard to detect an orientation of a singular vessel. Therefore, the more correct mensuration is needed.

In this paper, vessel inscribed trigonometry (VITM) to measure the vessel progressive orientation (VPO) in a digital image is proposed. The VITM consists of three strategies as vessel classification, chain coding of vessel and orientation measurement. To measure the VPO, a skeleton of vessel is used. The VPO is measured in a vessel stem (VS). There are two chain code maps because all points on VS have two neighborhoods. And then, two farthest neighborhoods of each point are searched by the proposed angle restriction. The VPO is measured by estimating the gradient of them. The VITM was validated by comparing with the tangent angle of a circle and 2D Gaussian templates. And it is confirmed that it was valuable and correct.

II. VESSEL INSCRIBED TRIGONOMETRY (VITM)

Digital image usually is acquired and is processed in a grid format with equal spacing in the x- and y-directions [6]. In digital images, it is hard to measure exact angle of a line. The problem is occurred when an orientation of a line is measured, because a pixel in the grid format has only eight neighborhoods. An orientation angle between a center point and one neighborhood is limited to eight. Therefore, it cannot be estimated precisely that an orientation angle of a line or a curve in a digital image. Thus, chain-codes and connectivity of neighborhoods were employed to solve the problem.

Let \( v(x, y) \) denote a point on vessel in a digital image. If \( v \) were on VS, \( v \) has two neighborhoods, \( v(x_{n1}, y_{n1}) \) and \( v(x_{n2}, y_{n2}) \). Each distance \( d \) between \( v \) and neighborhoods is one. The orientation \( VPO(x, y) \) on \( v(x, y) \) can be defined by,

\[
VPO(x, y) = \tan^{-1}\left(\frac{y_{n1} - y_{n2}}{x_{n1} - x_{n2}}\right)
\]

As shown in Fig. 1 (a), one pixel on a digital image has only eight neighborhoods. An angle interval between adjacent neighbors is \( \pi/4 \), when \( d=1 \). To expand the angle interval, \( d \) needs to be also extended. Fig. 1 (b) shows 16 points surround the center point, when \( d=2 \). Let 16 points be \( 2nd \)
neighborhoods. The arrow indicates one of the third neighborhoods from the center point.

Fig. 1 Sectors depended on distance of neighborhood in the grid format (dark : line, gray : neighborhood)

As shown in Fig. 3 (c), there are 24 points in 3th neighborhood, when \( d=3 \). The one of fourth points is indicated by the arrow.

The proposed angle restriction is based on the order of point and angle interval. First, the order of point is restricted in \( d \)th neighborhood. If \( d+1 \) point from the center point were in \( d+1 \)th neighborhood, it should be set the farther neighborhood candidate. Next, for the angle interval restriction, a chain code is used. Let \( CC_d \) denote the chain code between \( d-1 \) point and \( d \) point. If \( d+1 \) point was within between \( CC_{d+1} \) and \( CC_{d-1} \), it should be set also the farther neighborhood candidate. The rule mentioned above, is shown in Fig. 2.

Fig. 2 The rule of extending distance for neighborhood

If the farther neighborhood could be employed to estimate the orientation angle, it could be expanded to more degrees and its interval could be also minutely divided. Therefore, the farthest neighborhoods from the center point are searched by the proposed angle restriction.

Fig. 3 shows an example of the farthest neighborhoods estimated by the proposed angle restriction Solid arrow and dash arrow indicates original point and candidates, respectively.

Fig. 3 Examples of estimating the farthest neighborhoods both side and sequences of chain code (white circle : original point, gray circle : candidates, arrow : connectivity)

The candidates were selected by Fig. 2. If next point of \( \circ \) were suitable to Fig. 2, the distance could be extended. \( \Box \) is the end point.

### III. VALIDATION

The VITM is validated by comparing with the tangent angle of the contact line on the part of a circle where it is drawn in a digital image.

Consider that vertexes of the inscribed triangle are two farthest neighborhoods and the center point.

As shown in Fig. 4, three points are on the vessel stem, so, the VPO could be determined using the tangent line and the straight line which connects two neighborhoods. The gradient is calculated by (1).

A horizontal line is set to degree 0, and an angle range is limited in degree 0~180. Fig. 5 (a) shows the test image consisting of 2 vessels. Fig. 5 (b) and (c) show 3 different VPO of 2 vessels. As shown in Fig. 5 (b) and (c), the VITM is more similar with a manual line than 2D Gaussian templates. Therefore, the VITM is more correct and creditable than the 2D Gaussian templates.
Fig. 5 An example of VPO measurement (a) original image (b) VPO of vessel 1 (c) VPO of vessel 2

Also, the detection of OD based on [4] is processed using VPO measured by VITM. In [4], the detection method is based on the 2D Gaussian templates. STARE database was employed for experiments [7]. All optic discs in STARE were detected and performance was similar with [4].

IV. CONCLUSION

The VPO is major factor to detect OD in a digital fundus image. The Gaussian templates were the common mensuration of VPO. But, there are some problem such as vessel overlaps and the range limitation that is based on the number of templates. Moreover, it is hard to measure exact angle of a line because of the grid format. Therefore, the new mensuration of VPO in a digital image was proposed in this paper. The two farthest neighborhoods of the center point on the vessel stem are searched by the proposed angle restriction and the VPO is measured from the gradient of the straight line which connects two neighborhoods. By experiment and validation, it is confirmed that the proposed the VITM is correct and creditable. And, OD was correctly detected by VPO measured by VITM. Therefore, VPO measured by VITM can be useful to analyzing vessel on the digital images such as angiography and fundus image.

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