Geometric Operators in the Selection of Human Resources

José M. Merigó, and Anna M. Gil-Lafuente

Abstract—We study the possibility of using geometric operators in the selection of human resources. We develop three new methods that use the ordered weighted geometric (OWG) operator in different indexes used for the selection of human resources. The objective of these models is to manipulate the neutrality of the old methods so the decision maker is able to select human resources according to his particular attitude. In order to develop these models, first a short revision of the OWG operator is developed. Second, we briefly explain the general process for the selection of human resources. Then, we develop the three new indexes. They will use the OWG operator in the Hamming distance, in the adequacy coefficient and in the index of maximum and minimum level. Finally, an illustrative example about the new approach is given.

Keywords—OWG operator, decision making, human resources, Hamming distance.

I. INTRODUCTION

The selection of the most appropriate human resources for the company represents a fundamental problem for its good development. With the large variety of alternatives existing in the market, the enterprise needs to know which the most appropriate person according to their interests is. In order to solve this problem, the company has to elaborate a selection process. Among the great variety of studies existing in selection, this work will focus on the methods developed in [1]–[3] about selection of human resources, the methods developed in [4]–[8] about selection of financial products and the methods developed in [9]–[10] about selection of players in sport management.

One problem about these selection indexes is that they are neutral against the attitudinal character of the decision maker. Then, when developing the selection process, we cannot manipulate the results according to the interests of the decision maker. This problem becomes important in situations where we want to under estimate or over estimate the decisions in order to be more or less prudent against the uncertain factors affecting the future. One common method for aggregating the information considering the decision attitude of the decision maker is the ordered weighted geometric (OWG) operator introduced in [11]. Since its appearance, the OWG operator has been studied by different authors such as [12]–[21].

Our objective in this paper will consist in developing new selection indexes that include the attitudinal character of the decision maker for the selection of human resources. These new indexes will consist in combining the old selection methods with the OWG operator because then, the neutrality of the old methods will be changed by the OWG operator. We will introduce in the selection of human resources, the ordered weighted geometric distance (OWGD) operator, the ordered weighted geometric adequacy coefficient (OWGAC) and the ordered weighted geometric index of maximum and minimum level (OWGIMAM).

In order to do so, this paper is organized as follows. In Section 2 we briefly describe the OWG operator. Section 3 develops the process to follow when using the OWG operator in the selection of human resources. Section 4 gives an illustrative example of the suggested methodology and in Section 5 we finish with the main conclusions found in the paper.

II. OWG OPERATOR

The OWG operator was introduced in [11] and it provides a parameterized family of aggregation operators similar to the OWA operator [22]–[27]. It consists in using the geometric mean in the OWA operator. In the following, we provide a definition of the OWG operator as introduced by [14].

Definition 1. An OWG operator of dimension $n$ is a mapping $OWG: R^n \rightarrow R^*$ that has an associated weighting vector $W$ of dimension $n$ such that $w_j \in [0, 1]$ and the sum of the weights is 1, then:

$$OWG(a_1, a_2, \ldots, a_n) = \prod_{j=1}^{n} b_j^{w_j}$$

where $b_j$ is the $j$th largest of the $a_i$.

Although the reordering step is used in most of the cases in descending order, due to the large number of different existing cases, we have to distinguish between the Descending OWG (DOWG) operators and the Ascending OWG (AOWG) operators. The weights of these two operators are related by $w_j$...
Step 1: Analysis and determination of the significant characteristics of the interesting workers for the company. That is: \( C = \{ C_1, C_2, \ldots, C_n \} \).

Step 2: Fixation of the ideal level of each significant characteristic in order to form the ideal worker. That is:

\[
\begin{array}{ccccccc}
C_1 & C_2 & \ldots & C_i & \ldots & C_n \\
p & \mu_1 & \ldots & \mu_i & \ldots & \mu_n
\end{array}
\]

Step 3: Fixation of the real level of each characteristic for all the different workers considered. That is:

\[
\begin{array}{ccccccc}
C_1 & C_2 & \ldots & C_i & \ldots & C_n \\
p_k & \mu_i^{(2)} & \ldots & \mu_i^{(k)} & \ldots & \mu_i^{(n)}
\end{array}
\]

Step 4: Comparison between the ideal worker and the different workers considered, and determination of the level of removal using the OWG operator. That is, changing the neutrality of the results to over estimate or under estimate them.

Step 5: Adoption of decisions according to the results found in the previous steps.

In Step 4, the objective is to express numerically the removal between the ideal worker and the different workers considered. For this, it can be used the traditional selection indexes [1]–[10]. In this paper, the difference will be that they will be mixed with the OWG operator. Then, with this operator we will be able to provide a parameterized family of aggregation operators in the selection indexes such as the maximum, the minimum, the geometric mean and the weighted geometric mean. In the following, it will be shown how to use the OWG operator in the main selection indexes.

B. Using the OWGD operator

The ordered weighted geometric distance (OWGD) operator consists in combining the OWG operator with the normalized Hamming distance. It provides a parameterized family of distance operators that include the maximum distance, the minimum distance, the normalized geometric distance and the weighted geometric distance. It can be defined as follows.

Definition 2. An OWGD operator of dimension \( n \), is a mapping \( \text{OWGD}: \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R} \) that has an associated weighting vector \( W \), with \( w_j \in [0,1] \) and the sum of the weights is 1, then:

\[
\text{OWGD}(P,P_0) = \prod_{j=1}^{n} D_j^{w_j}
\]

where \( D_j \) represents the \( j \)th largest of the \( |\mu_i - \mu_i^{(k)}| \), and \( k = 1,2,\ldots,m \). Note that in distances, the best result is usually the
smallest distance. It is important to note that we will not include in the aggregation the $S_j = 0$ for all $j$.

From a generalized perspective of the reordering step we have to distinguish between the descending OWGD (DOWGD) operator and the ascending OWGD (AOWGD) operator. The DOWGD operator has the same definition than the OWGD operator. The AOWGD operator also has the same formulation with the difference that the reordering of the $D_i$ is ascendant. Note that the weights of this two operators are related by $w_j = w^{*}_{n-j+1}$, where $w_j$ is the $j$th weight of the DOWGD and $w^{*}_{n-j+1}$ the $j$th weight of the AOWGD operator. Also note that this operator is commutative, monotonic, idempotent and bounded.

By using a different manifestation of the weighting vector we are able to obtain different types of aggregation operators. For example, with the DOWGD operator the maximum distance is found when $w_j = 1$ and $w_x = 0$ for all $j \neq 1$. The minimum is found when $w_x = 1$ and $w_y = 0$ for all $j \neq x$. The normalized geometric distance is obtained when $w_j = 1/n$ for all $j$. The weighted geometric distance is found when the ordered position of $i$ is the same than the ordered position of $j$. Note that in the case of tie in the final result, especially for the maximum and the minimum, it could be used in the decision the second best or worst result, and so on.

### Definition 3. An OWGD operator of dimension $n$, is a mapping $OWGD: \mathbb{R}^n \rightarrow \mathbb{R}$ that has an associated weighting vector $W$, with $w_j \in [0, 1]$ and the sum of the weights is 1, then:

$$
OWGD(P_k \rightarrow P) = \prod_{j=1}^{n} K_j w_j
$$

where $K_j$ represents the $j$th largest of the $[1 \land (1 - \mu_j + \mu_j^{(k)})]$, and $k = 1, 2, ..., m$. The final result will be a number between [0,1], being the maximum possible result 1.

From a generalized perspective of the reordering step we have to distinguish between the descending OWGD (DOWGD) operator and the ascending OWGD (AOWGD) operator. The DOWGD operator has the same definition than the OWGD operator. The AOWGD operator also has the same formulation with the difference that the reordering of the $D_i$ is ascendant. Then, the weights of this two operators are related by $w_j = w^{*}_{n-j+1}$, where $w_j$ is the $j$th weight of the DOWGD and $w^{*}_{n-j+1}$ the $j$th weight of the AOWGD operator. Note that the OWGD operator also accomplishes the properties of monotonicity, commutativity, boundedness and idempotency.

Different types of aggregation operators can be obtained by choosing a different manifestation of the weighting vector. For example, the maximum is obtained when $w_j = 1$ and $w_x = 0$ for all $j \neq 1$. The minimum is found when $w_x = 1$ and $w_y = 0$ for all $j \neq x$. The normalized geometric adequacy coefficient (GAC) is obtained when $w_j = 1/n$ for all $j$. The weighted geometric adequacy coefficient (WGAC) is found when the ordered position of $i$ is the same than the ordered position of $j$. Note that in the case of tie in the final result, especially for the maximum and the minimum, it could be used in the decision the second best or worst result, and so on.

Analogously to the OWGD operator, we can suggest an equivalent removal index that it is a dual of the OWGDAC because $Q(P_j \rightarrow P) = 1 - K(P_j \rightarrow P)$. Note that this index has been studied for the selection of financial products in [7]. We will call it the ordered weighted geometric dual adequacy coefficient (OWGDAC). It can be defined as follows.

### Definition 4. An OWGDAC operator of dimension $n$, is a mapping $OWGDAC: \mathbb{R}^n \rightarrow \mathbb{R}$ that has an associated weighting vector $W$, with $w_j \in [0, 1]$ and the sum of the weights is 1, then:

$$
OWGDAC(P_k \rightarrow P) = \prod_{j=1}^{n} Q_j w_j
$$

where $Q_j$ represents the $j$th largest of the $[0 \lor (\mu_j - \mu_j^{(k)})]$, and $k = 1, 2, ..., m$. The final result will be a number between [0,1]. Note that in this case we usually select the lowest value as the best result.
also use the same policy about ties in the final result as it has been explained for the OWGAC operator.

Another interesting issue to consider is the unification point in the selection of human resources. As it has been explained in [5], the unification point appears when the results obtained in the Hamming distance are the same than the results obtained in the adequacy coefficient. In the new methods suggested in this paper, we also find the unification point when the OWGD and the OWGAC accomplish the theorems explained in [5]. Note that it is possible to find a total unification point or a partial unification point [5].

**Theorem 1.** Assume \( \text{OWGD}(P, P_\lambda) \) is the selection of human resources with the OWGD operator and \( \text{OWGDAC}(P_k \rightarrow P) \) the selection of human resources with the OWGDAC operator. If \( \mu_i \geq \mu_i^{(k)} \) for all \( i \), then:

\[ \text{OWGD}(P, P_\lambda) = \text{OWGDAC}(P_k \rightarrow P) \]  

(5)

**Proof.** Let

\[ \text{OWGD}(P, P_\lambda) = \sum_{j=1}^{n} w_j | \mu_j - \mu_j^{(k)} | \]  

and

\[ \text{OWGDAC}(P_k \rightarrow P) = \sum_{j=1}^{n} w_j [0 \vee (\mu_j - \mu_j^{(k)})] \]

Since \( \mu_i \geq \mu_i^{(k)} \) for all \( i \), \( [0 \vee (\mu_j - \mu_j^{(k)})] = (\mu_j - \mu_j^{(k)}) \) for all \( i \), then

\[ \text{OWGDAC}(P_k \rightarrow P) = \sum_{j=1}^{n} w_j (\mu_j - \mu_j^{(k)}) = \text{OWGD}(P, P_\lambda) \]  

Analysing this theorem, we could generalize it for all the human resources considered in the decision problem. The theorem that explains this generalization is very similar to Theorem (1) with the difference that now we consider all the characteristics \( i \) and all the human resources \( k \).

**D. Using the OWGIMAM operator**

In this subsection we study the use of the OWG operator in the index of maximum and minimum level. We will call this operator as the ordered weighted geometric index of maximum and minimum level (OWGIMAM). This operator also provides a parameterized family of aggregation operators that include the maximum, the minimum, the normalized geometric index of maximum and minimum level and the weighted geometric index of maximum and minimum level. It can be defined as follows.

**Definition 5.** An OWGIMAM operator of dimension \( n \), is a mapping \( \text{OWGIMAM} : R^n \times R^n \rightarrow R \) that has an associated weighting vector \( W \), with \( w_j \in [0,1] \) and the sum of the weights is 1, then:

\[ \text{OWGIMAM}(P_k \rightarrow P) = \prod_{j=1}^{n} S_j^{w_j} \]  

(6)

where \( S_j \) represents the \( j \)th largest of all the \( [\mu_j - \mu_j^{(k)}] \) and the \( [0 \vee (\mu_j - \mu_j^{(k)})] \), with \( k = 1,2,\ldots,m \). It is important to note that we will not include in the aggregation the \( S_1 = 0 \), for all \( j \), as it gives inconsistent results.

From a generalized perspective of the reordering step we have to distinguish between the descending OWGIMAM (DOWGIMAM) operator and the ascending OWGIMAM (AOWGIMAM) operator. The DOWGIMAM operator has the same definition than the OWGIMAM operator. The AOWGIMAM operator also has the same formulation with the difference that the reordering of the \( D_j \) is ascendant. Then, the weights of this two operators are related by \( w_j = w^*_{\text{ord} + j} \), where \( w_j \) is the \( j \)th weight of the DOWGIMAM and \( w^*_{\text{ord} + j} \) the \( j \)th weight of the AOWGIMAM operator. Note that the OWGIMAM operator is commutative, monotonic, bounded and idempotent.

By choosing a different manifestation of the weighting vector, we are able to obtain different types of aggregation operators. For example, the maximum is obtained when \( w_j = 1 \) and \( w_j = 0 \) for all \( j \neq 1 \). The minimum is found when \( w_n = 1 \) and \( w_j = 0 \) for all \( j \neq n \). The normalized geometric index of maximum and minimum level (GIMAM) is obtained when \( w_j = 1/n \) for all \( j \). The weighted geometric index of maximum and minimum level (WGIMAM) is found when the ordered position of \( i \) is the same than the ordered position of \( j \). Note that in the case of tie in the final result, especially for the maximum and the minimum, it could be used in the decision the second best or worst result, and so on.

In this case, we could also analyse the unification point. The unification implies that the OWGIMAM operator becomes the OWGD operator as it has been explained in [5] for the index of maximum and minimum level. The conditions to enter in a situation of unification point follow the same policy as the basic cases explained in [5]. Note that in this case we also have to distinguish between total unification point and partial unification point.

**Theorem 2.** Assume \( \text{OWGD}(P, P_\lambda) \) is the selection of human resources with the OWGD operator and \( \text{OWGIMAM}(P_k \rightarrow P) \) the selection of human resources with the OWGIMAM operator. If \( \mu_i \geq \mu_i^{(k)} \) for all \( i \), then:

\[ \text{OWGD}(P, P_\lambda) = \text{OWGIMAM}(P_k \rightarrow P) \]  

(7)

**Proof.** Let

\[ \text{OWGD}(P, P_\lambda) = \sum_{j=1}^{n} w_j | \mu_j - \mu_j^{(k)} | \]  

and

\[ \text{OWGIMAM}(P_k \rightarrow P) = \sum_{j=1}^{n} [w_j \vee (\mu_j - \mu_j^{(k)})] + w_j [\mu_j - \mu_j^{(k)}] \]
Since $\mu_i \geq \mu_i^{(k)}$ for all $i$, $[0 \lor (\mu_i - \mu_i^{(k)})] = (\mu_i - \mu_i^{(k)})$ for all $i$, then

$$\text{OWGIMAM}(P_k \rightarrow P) = \sum_{j=1}^{n} w_j (\mu_i - \mu_i^{(k)}) = \text{OWGD}(P, P_k)$$

Note that $w_i^* + w_j^* = w_j$.

Analysing this theorem, we could generalize it for all the human resources considered in the decision problem. The theorem that explains this generalization is very similar to theorem (2) with the difference that now we consider all the characteristics $i$ and all the human resources $k$.

**IV. ILLUSTRATIVE EXAMPLE**

In the following we are going to develop an illustrative example in order to understand numerically the new approaches commented above.

**Step 1:** Analysis and determination of the significant characteristics for the company.

Assume that a company wants to select a worker for a vacant and it has 3 candidates $P_1$, $P_2$, $P_3$, with different characteristics.

**Step 2:** Fixation of the ideal level for each significant characteristic. It is defined the ideal worker as Table III:

<table>
<thead>
<tr>
<th>CHARACTERISTICS OF THE IDEAL WORKER</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P^*$</td>
</tr>
<tr>
<td>$C_1$</td>
</tr>
<tr>
<td>0.9</td>
</tr>
</tbody>
</table>

**Step 3:** Fixation of the real level of each characteristic for all the different candidates considered. For each of these characteristics, it is found the following information:

For each of these characteristics, it is found the following information shown in Table IV:

<table>
<thead>
<tr>
<th>AVAILABLE CANDIDATES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
</tr>
<tr>
<td>$P_1$</td>
</tr>
<tr>
<td>$P_2$</td>
</tr>
<tr>
<td>$P_3$</td>
</tr>
</tbody>
</table>

**Step 4:** Comparison between the ideal worker and the different candidates considered, and determination of the level of removal using the OWA operators. We will consider the normalized Hamming distance, the weighted Hamming distance, the OWAD operator and the AOWAD operator. In this example, we assume that the company decides to use the following weighting vector: $W = (0.1, 0.1, 0.2, 0.3, 0.3)$. With this weighting vector, we can calculate the degree of optimism of the decision as: $\alpha(W) = 0.35 \Rightarrow 35\%$, and the degree of dispersion as: $H(W) = 1.504$.

If we elaborate the selection process with the Hamming distance, we will get the following. First, we have to calculate the individual distances of each characteristic to the ideal value of the corresponding characteristic forming the fuzzy subset of individual distances for each candidate. Once obtained all the distances, we will go for the aggregation. Then, we will reorder the different values of each fuzzy subset using (2) and considering the type of aggregation we are developing. The results are shown in Table V.

<table>
<thead>
<tr>
<th>AGGREGATED RESULTS – HAMMING DISTANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NHD$</td>
</tr>
<tr>
<td>$P_1$</td>
</tr>
<tr>
<td>$P_2$</td>
</tr>
<tr>
<td>$P_3$</td>
</tr>
</tbody>
</table>

In this case, our decision will consist in selecting the candidate with the smallest distance. Then, we will select $P_2$ as it gives us the lowest distance in the four cases.

If we develop the selection process with the adequacy coefficient, we will get the following. First, we have to calculate how close the characteristics are to the ideal worker. Once calculated all the different individual values, we will construct the aggregation. In this case, the arguments will be ordered using (3). The results are shown in Table VI.

<table>
<thead>
<tr>
<th>AGGREGATED RESULTS – ADEQUACY COEFFICIENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NAC$</td>
</tr>
<tr>
<td>$P_1$</td>
</tr>
<tr>
<td>$P_2$</td>
</tr>
<tr>
<td>$P_3$</td>
</tr>
</tbody>
</table>

The decision will consist in selecting the candidate with the highest result because this will mean a higher approximation to the ideal worker. Then, we will select $P_3$ because it gives us the highest result for all the cases.

Analogously to this index, we can obtain its equivalent removal index. That is: $Q(P_k \rightarrow P) = 1 - K(P_k \rightarrow P)$. The results are shown in Table VII.

<table>
<thead>
<tr>
<th>AGGREGATED RESULTS – DUAL ADEQUACY COEFFICIENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NDAC$</td>
</tr>
<tr>
<td>$P_1$</td>
</tr>
<tr>
<td>$P_2$</td>
</tr>
<tr>
<td>$P_3$</td>
</tr>
</tbody>
</table>

Finally, if we use the index of maximum and minimum level in the selection process as a combination of the normalized Hamming distance and the normalized adequacy coefficient,
we will get the following. In this example we will assume that the characteristics $C_1$ and $C_2$ have to be treated with the adequacy coefficient and the other three characteristics have to be treated with the Hamming distance. Its resolution will consist in the following. First, we will calculate the individual removal of each characteristic to the ideal, independently that the instrument used is the Hamming distance or the adequacy index. Once calculated all the values for the individual removal, we will construct the aggregation using (6). Here, we note that in the reordering step, it will be only considered the individual value obtained for each characteristic, independently that the value has been obtained with the adequacy coefficient or with the Hamming distance. The results are shown in Table VIII.

**TABLE VIII**

<table>
<thead>
<tr>
<th></th>
<th>NIMAM</th>
<th>WIMAM</th>
<th>OWAIMAM</th>
<th>AOWAIMAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>0.28</td>
<td>0.35</td>
<td>0.2</td>
<td>0.36</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.12</td>
<td>0.16</td>
<td>0.06</td>
<td>0.18</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.18</td>
<td>0.19</td>
<td>0.13</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Then, our decision will consist in select $P_2$ because it is the candidate with the smallest removal to the ideal.

Analogously to this index, we can obtain its equivalent approximation index. In an abbreviated form, this index can be obtained by using $R(P_k \rightarrow P) = 1 - S(P_k \rightarrow P)$. The results are shown in Table IX.

**TABLE IX**

<table>
<thead>
<tr>
<th></th>
<th>NDIMAM</th>
<th>WDIMAM</th>
<th>OWADIMAM</th>
<th>AOWADIMAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>0.72</td>
<td>0.65</td>
<td>0.8</td>
<td>0.64</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.88</td>
<td>0.84</td>
<td>0.94</td>
<td>0.82</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.82</td>
<td>0.81</td>
<td>0.87</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Again, we see that the optimal choice is $P_2$ because it is the candidate with the highest results.

V. CONCLUSION

In this paper, we have studied a large number of instruments for the selection of human resources. Due to the neutrality in the attitudinal character of the old methods, we have suggested the possibility of change this neutrality with the introduction of the OWG operator in the selection process. As we have seen, the OWG operator permits under estimate or over estimate the selection process, which has allowed us to manipulate the initial neutrality. With this information, we have developed three new instruments for the selection of human resources, consisting in combining the old selection indexes with the OWG operator. Then, we have obtained three new methods that permits reflect the attitude of the decision makers in the selection process of human resources. Moreover, these methods have generalized a wide range of aggregation operators in the selection process such as the geometric distance or the weighted geometric distance.

This work represents an extension about the possibility of combining the OWG operator with different selection indexes. In this paper, we have focused in the selection of human resources but it is important to note that these new methods can also be applied to other selection processes such as the selection of assets, investments, strategies, etc. In future research, we will analyze how these methods can be applied to other selection processes and combined with other selection indexes.

REFERENCES


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