Influence of Distributed Generation on Congestion and LMP in Competitive Electricity Market

Durga Gautam and Mithulananthan Nadarajah

Abstract—This paper presents the influence of distributed generation (DG) on congestion and locational marginal price (LMP) in an optimal power flow (OPF) based wholesale electricity market. The problem of optimal placement to manage congestion and reduce LMP is formulated for the objective of social welfare maximization. From competitive electricity market standpoint, DGs have great value when they reduce load in particular locations and at particular times when feeders are heavily loaded. The paper lies on the groundwork that solution to optimal mix of generation and transmission resources can be achieved by addressing congestion and corresponding shadow prices. The influence of DG on congestion and locational marginal prices has been demonstrated in a modified IEEE 14 bus test system.

Keywords—Congestion management, distributed generation, electricity market, locational marginal price, optimal power flow, social welfare.

I. INTRODUCTION

Distributed generators (DGs) are small-scale generation that provides incremental capacity to generation system of power systems. These small-scale generators are getting wide spread adoption as there are constrains in building new transmission lines, increased customer demand for highly reliable electricity, deregulated electricity markets, concerns about climate change and energy security [1],[2]. Technological advancements and unbundling of electric utilities along with real-time pricing mechanisms are two other major reasons for increase penetration of small-scale generation in power systems. From the competitive electricity market standpoint DG has great values, when they reduce load in particular locations and at particular times when feeders are heavily loaded. The condition is prevalent due to increased volume of power exchanges among the market participants leading to intensive use of transmission system and frequent congestion.

High congestion cost is essentially the signal for expanding transmission capacity. Explication of optimal mix of generation and transmission resources under such scenario can be achieved by addressing congestion and corresponding locational marginal price (LMP). Pricing of congestion using LMP provides the basis for efficient pricing of both energy and transmission, and valuing financial transmission rights [3]. If frequent congestion is observed, DG would be effective in relieving the congestion. DG becomes attractive because load served by the DG avoids paying the high price. It can fetch the locational price to the extent it sells excess power to the market [4].

Several researchers have also revealed that DG could be considered an attractive alternative to transmission system expansion. The placement, however, should be carried out with due consideration to its size and location. The placement should be optimal in order for the maximum benefit of DG implemented in the network. Improper placement in some situations can even jeopardize the system performance and operation, by increasing losses and creating congestion.

Several techniques have been proposed so far to manage with congestion in deregulated electricity market. However, most of the techniques are focused on application of FACTS devices. Methodologies for congestion management with placement of series FACTS devices have been proposed in [5]. An analysis of congestion management problem and associated issues in a power pool that incorporates nodal pricing is presented in [6]. A two-step approach for congestion management using FACTS devices has been proposed in [7]. A methodology to calculate transmission congestion cost and LMP for a given time period at any selected bus in the transmission system is proposed in [8].

Besides FACTS devices, use of DG for congestion management has also gained attention, recently. By installing close to loads, DGs can act as a fast and reliable energy solution to hedge congestion. The validity of DG for congestion management and the methods to locate and analyze the operation strategy of these resources have been discussion in [9]. In this paper, the influence of DG is studied for managing congestion in deregulated electricity market. The DG is expected to serve local load and relieve transmission bottlenecks, consequently, lowering the LMP and congestion.
The paper is organized in 6 sections. Section II discusses on basic attributes of congestion management and the features inherent in deregulated electricity market. Section III sets out the OPF formulation along with the physical and mathematical interpretation of associated shadow prices. Section IV presents the methodology adopted to observe the influence of DG on congestion management and LMP. The OPF results and inferences drawn from the same are covered in section V. Conclusions that can be drawn from the analysis are presented in section VI.

II. CONGESTION

Congestion in power system is a consequence of network constraints characterizing a finite network capacity that limits the simultaneous transfer of power from all required transactions. Transmission congestion exists in power systems as a natural consequence of supply and demand.

Whenever physical or operational constraints in a transmission network become active, power system, more specifically a portion of the system is said to be in a state of congestion. Electric grid congestion develops when one or more restrictions on transmission system prevent economic, or least expensive, supply of energy from serving the demand. This may be the transmission constraint, where transmission lines may not have enough capacity to carry all the power required to meet the demand at a specific location. These delivery constraints are likely to result in higher prices in regions that have a limited ability to import power and lower prices in regions with limited power export capability.

A. Consequence of Congestion

The determination of market clearing price follows the modern macro economic theory. Accordingly, maximum welfare to consumer as well as to producers (society on a whole) is achieved for single market equilibrium point. Congestion in the system, however, creates a different scenario leading to different market equilibrium point for producers and consumers. The price at consumer locations is generally higher than that in the generator locations. The sum of payments from the consumers is greater than the sum of revenues raised by the producers or generators. This difference in payment and revenue is referred to as merchandise surplus [10]. Besides, the merchandise surplus, also known as congestion rent, society would have to bear the loss of social welfare termed as market efficiency loss or the dead weight loss. This can be illustrated with a supply demand curve as shown in Fig. 1. If there is no congestion, the amount of power that can be transacted is determined by the intersection point of supply and demand curves.

B. Congestion Management

The task of congestion management in deregulated market requires ISO to identify and relieve such situations through the deployment of various physical and financial mechanisms [11]. The dual objective of congestion management schemes is to minimize the interference of transmission network in the market for electrical energy and to simultaneously ensure secure operation of power system. The rules must be robust, because there will be many aggressive entities seeking to exploit congestion to create market power and increase profits for themselves at the expense of market efficiency.

When congestion occurs, transmission users are constrained from delivering power. Normally, generators on the import side of the constraint have to increase production while the generators on export side are “constrained off”, relative to production schedules they would otherwise prefer, to serve load [12].

Some of the prevalent schemes for mitigating congestion involved, (i) Real power rescheduling by generators only, (ii) Real power rescheduling by generators and curtailment of load, and (iii) Real and reactive power rescheduling by generators and curtailment of load. Recently, the congestion management schemes involving the deployment of FACTS devices and DG have been envisaged as promising options. Traditional concept of load curtailment and price area congestion management is considered and a modified approach to congestion management based on locating DG in an OPF based market has been proposed in the present paper. Hence, concern of present study is to manage congestion by alleviating the root of congestion, that is, to supply the demand locally instead of bringing power from far away through long transmission lines.

III. PROBLEM FORMULATION

The objective of social welfare maximization is widely accepted as basis of problem formulation in competitive electricity market. Objective function is formulated as quadratic benefit curve submitted by buyers (DISCO) minus quadratic bid curve supplied by sellers (GENCO) minus quadratic cost function supplied by the DG owner.

\[
\max \sum_{i=1}^{N} \left( B_i(p_{Di}) - C_i(p_{Di}) - C(p_{Di}) \right) 
\]

(1)

Most non-linear optimization solver is based on minimization of the objective function. So, to formulate a
minimization type of problem, above equation is multiplied by -1.

\[ \min \sum_{i=1}^{N} \left[ C_i \left( P_{Di} \right) - B_i \left( P_{Di} \right) \right] + C(P_{DG}) \]  

(2)

It is subjected to following constraints:

**Equality Constraints**

Network for transmission of electric energy is modeled via power balance of each node in the network. The sum of power flows, active and reactive, injected into a node minus the power flows extracted from the node has to be zero as given in (3) and (4), respectively. Notice that in (3) only real power injection by the DG is considered as the DG type considered here injects real power only. If other types of DG is considered both (3) and (4) should be modified accordingly.

\[ P_i = P_{Di} + P_{DG} - P_{Gi} = \sum_{j=1}^{N} \left[ V_i \left( G_{ji} \cos(\delta_i - \delta_j) + B_{ji} \sin(\delta_i - \delta_j) \right) \right] \]  

(3)

\[ Q_i = Q_{Gi} - Q_{Di} = \sum_{j=1}^{N} \left[ V_i \left( G_{ji} \sin(\delta_i - \delta_j) - B_{ji} \cos(\delta_i - \delta_j) \right) \right] \]  

(4)

**Inequality Constraints**

Generation limits:

The generating plants have maximum and minimum generating capacities and out those limits, it is not feasible to generate due to technical or economic reasons. Generating limits are specified as upper and lower limits for real and reactive power outputs given for real and reactive power separately as follows:

- Real power generation limits:
  \[ P_{Gi}^{\text{min}} \leq P_i \leq P_{Gi}^{\text{max}} \]

- Reactive power generation limits:
  \[ Q_{Gi}^{\text{min}} \leq Q_i \leq Q_{Gi}^{\text{max}} \]

Line flow limit:

Line flow limit specifies the maximum power that a given transmission line is capable of transmitting under given conditions. These limits can be based on thermal or various stability considerations. Thermal limits usually dominate shorter lines power transfer. The following constraint checks for the absolute power flow both at sending end and receiving end of particular line to be within the upper limit of the line.

\[ S_{ij} = S_{ij}^{\text{max}} \]

Bus voltage Limit:

Voltage limits refer to the requirement for system bus voltage to remain within a narrow range of levels.

\[ v_i^{\text{min}} \leq v_i \leq v_i^{\text{max}} \]

where,

- \( N \) denotes the total number of buses.
- \( P_{Di} \) denotes real power generated at bus i.
- \( P_{Gi} \) denotes real power demand at bus i.
- \( B_i(P_{Gi}) = a_{Di} + b_{Di} P_{Di} - c_{Di} (P_{Di})^2 \) denotes purchaser benefit functions at bus i

\[ C_i(P_{Gi}) = a_{Gi} + b_{Gi} P_{Gi} + c_{Gi} (P_{Gi})^2 \] denote the producer offer (bid) price at bus i

\[ C(P_{DG}) = a_{DG} + b_{DG} P_{DG} + c_{DG} (P_{DG})^2 \] denote the cost characteristic of DG at bus i

\[ v_i \] denotes the voltage at bus i

\[ \delta_i \] denotes the power angle at bus i

\[ B_{ij} \] denotes the susceptance of line ij

\[ G_{ij} \] denotes the conductance of line ij

\[ Q_{Gi} \] denotes reactive power generated at bus i.

\[ P_{DG} \] and \( P_{DG}^{\text{min}} \) denotes upper and lower real power generation limits of generator at bus i.

\[ Q_{Gi}^{\text{min}} \] and \( Q_{Gi}^{\text{max}} \) denotes upper and lower reactive power generation limits of generator at bus i.

\[ v_i^{\text{min}} \] and \( v_i^{\text{max}} \) denote upper and lower limits of voltage at bus i.

\[ S_{ij} \] denotes the complex power transfer from bus i to bus j

\[ S_{ij}^{\text{max}} \] and \( S_{ij}^{\text{min}} \) denote the complex power flow limit for line ij and line ji

For base case OPF,

\[ P_{DG} = 0 \]

For load bus,

\[ P_{Di} = 0 \]

For generator bus,

\[ P_{Gi} = 0 \]

The lagrangian for social welfare model can be obtained as:

\[ L = \sum_{i=1}^{N} \left( C_i(P_{Di}) - B_i(P_{Di}) \right) + C(P_{DG}) + \sum_{i=1}^{N} \lambda_{Di}(P_{Di} - P_{DG} + P_{Di}) \]

\[ + \sum_{i=1}^{N} \lambda_{Gi}(Q_{Gi} - Q_{Di}) + Q_{Di}) + \sum_{j=1}^{N} \mu_{ij}(P_{ij}^{\text{min}} - P_{ij}^{\text{max}}) \]

\[ + \sum_{j=1}^{N} \mu_{ij}(Q_{ij}^{\text{min}} - Q_{ij}^{\text{max}}) \]

\[ + \sum_{j=1}^{N} \mu_{ij}(v_{ij}^{\text{min}} - v_{ij}^{\text{max}}) \]

(5)

where,

\( \lambda_{Di} \) and \( \lambda_{Gi} \) represent the Lagrange multipliers associated with equality constraints (power balance equations) and \( \mu_{ij}^{\text{min}}, \mu_{ij}^{\text{max}}, \mu_{ij}^{\text{max}}, \mu_{ij}^{\text{min}} \) are the multipliers associated with inequality constraints namely, line flow limit, generator real and reactive power limits and bus voltage limits, respectively. They also represent shadow price of respective constraints. The shadow prices derived from the constrained optimization more accurately reflect opportunity costs of available resources.
A. Interpretation of LMP

LMP is generally composed of three components, a marginal energy component (same for all buses), a marginal loss component and a congestion component. Considering the case of real power spot price, LMP at bus $i$ is given by:

$$L_i = \lambda_i + \sum_{m}^{N} \frac{\partial P_i}{\partial P_m} \mu_i + \sum_{i}^{N} \frac{\partial P_i}{\partial P_j} \lambda_{C,ij}$$

$$\lambda_L = \lambda + \lambda_{L,i} + \lambda_{C,i}$$

where,

- $\lambda$ is the marginal energy component at the reference bus which is same for all buses,
- $\lambda_{L,i}$ is the marginal loss component and
- $\lambda_{C,i}$ is the congestion component.

LMP is a market-pricing approach used to manage the use of transmission system in efficient way when congestion occurs on the bulk power grids. It defines the true and full opportunity cost in the short run. Also known as spot pricing or nodal pricing, it is to obtain the short run marginal cost (SRMC) of electricity [13]. LMP sends price signals that show where the delivery of energy is constrained by the transmission system and provides market participants a clear and accurate signal of the price of electricity at every location on the grid [3].

B. Interpretation of $\mu$

The Lagrange multiplier $\mu$ is the shadow price for the power flow restrictions of optimization problem and depicts the impact that each congested line has on power system. The multiplier denotes the increase in social welfare which could be achieved by slightly increasing the power limit of the corresponding line [14]. Hence, a higher multiplier means in general a higher impact of the corresponding congested line to the group of the nodal prices.

C. Congestion Rent

In a congested network, the locational price deviates from the marginal costs. The deviation in supply and demand bid curves from the corresponding cost and willingness-to-pay functions so as to capture the congestion rent is shown in Fig. 2.

Price differential at the nodes of constrained branch is used to calculate congestion rent. Mathematically, congestion rent for the branch $ij$ can be calculated as follows:

$$\text{congestion rent} = (\lambda_{n} - \lambda_{i}) \times P_{i}$$

IV. METHODOLOGY

For a specific combination of supplier and demand bid curves, base case OPF calculates electricity prices at different nodes of power system. The prices are obtained as lagrangian multipliers of non-linear equality constraints. The difference in prices will result from active line constraints and losses in the transmission system. The increasing functions for the supplier bids and decreasing functions for the consumer bids are treated as marginal cost or benefits of the bidder.

The base case OPF is performed with supply and demand bids excluding the DG component. To proceed with the placement, the nodes are ranked in descending order of their LMP, the node with highest LMP as the first candidate for DG placement. The candidate nodes are iteratively selected for the placement. The placement is carried out with several cost characteristics assumed for DG. The DG placement is intended to supply the demand which is otherwise pressed due to transmission bottleneck. Cost characteristic of DG along with the bids submitted by the supplier is included in OPF. As the placement is intended to bring down LMP, DG with operating cost higher than LMP will find no incentive for placement. The DG with operating cost lower than those bid by supplier is expected to have higher penetration while the one with higher cost is expected to have smaller penetration.

As consumers may not change their usage patterns especially over a short period of time, demand is assumed to be same as the one obtained from base case. The OPF is then performed with DG so that social welfare is maximized and prices reduced. The placement will then result a node with maximum net social welfare. The optimal DG size corresponding to the maximum net social welfare is then identified. Consequently, the influence of DG on congestion is also investigated. The proposed methodology is coded in MATLAB environment using sequential quadratic programming (SQP) method for the OPF formulation.
V. RESULTS AND DISCUSSION

This section presents the simulation results of influence of DG on congestion and LMP along with the proposed placement technique in modified IEEE 14 bus test system [15]. Single line diagram of the test system is shown in Fig. 3. Assumptions are made for the line limits as these have not been included in the extracted test case.

![Fig. 3 Single line diagram of IEEE 14 bus test system.](image)

A. Base Case

The generator and customer bids are taken as inputs to OPF. The base case OPF based on social welfare maximizing algorithm evaluates the generation dispatch, demands and prices at each of the nodes. The detailed analysis of optimal penetration of DG with varying cost characteristics and their effect on social welfare has been dealt in [16]. The optimal generation dispatch together with price elastic demand obtained for maximum social welfare is shown in Table I.

The shadow prices corresponding to each of the line flow is shown in Table II. Notice that lines 7-8, 1-2 and 7-9 are having higher power flow in the base case.

<table>
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<tr>
<th>Bus No.</th>
<th>Pg (MW)</th>
<th>Qg (MVAR)</th>
<th>Pd (MW)</th>
<th>Qd (MVAR)</th>
<th>LMP ($/MWh)</th>
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**Total** 330.53 133.46 323.49 147.39 -

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<th>From Bus</th>
<th>To Bus</th>
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<th>LMP\textsubscript{i} ($/MWh)</th>
<th>\mu\textsubscript{i} ($/MVA-hr)</th>
<th>P\textsubscript{j} (MW)</th>
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B. Placement of DG

Assumptions are made for the cost characteristic of DG. The DG considered in present analysis is assumed to inject only real power into the network and is approximated with the following cost characteristic:

\[ C(DG) = 0.01(P_{DG})^2 + 40P_{DG} \]  

(9)

The placement is carried out at each of the load bus. It is revealed that social welfare is maximized for the placement of DG1 at node 14 for which the LMP is the highest in the base case. The results corresponding to placement of DG1 encompasses the placement at node 14 which gives the optimal DG size of 42.84 MW for the maximum social welfare of 4577.18$/hr.

The dispatch scenario after placing DG at bus 14 is shown in Table III. Results reveal that, for the same demand, base case LMP in the higher range has reduced, accordingly, the LMP difference between the nodes has also reduced. It is noted that any deviation of size from the above optimal size tend to increase LMPs and congestion.

Injected power at each of the nodes and corresponding shadow prices for line flow and injected power after placing DG at bus 14 is shown in Table IV. Results reveal that shadow prices are reduced from the base case values tabulated in Table II.

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<th>Bus No.</th>
<th>( P_g ) (MW)</th>
<th>( Q_g ) (MVAR)</th>
<th>( P_d ) (MW)</th>
<th>( Q_d ) (MVAR)</th>
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As DG injects real power, its placement has direct effect of reducing LMP at the node where it is placed. Fig. 4 shows the LMP values corresponding to the placement of DG at load bus 14. The placement of DG has the global impact on power flow of the transmission system. The placement will thus influence the LMPs at other nodes besides the node where it is placed. This scenario is found to prevail for the present case as shown in Fig. 4 where the penetration of DG has positive influence on all the nodes. All customers are benefited as the amount they need to pay for the same quantity of electricity is reduced compared to the case without DG. However, it should be noted that any deviation of size other than the optimal size would increase the LMP from minimum value and reduce the net social welfare.

![Fig. 4 LMP at each load bus corresponding to DG placement at load bus 14.](image)

Congestion rent is evaluated at each branch of the test system as per equation 8. The impact of DG placement on congestion rent is shown graphically in Fig. 5. Results reveal that congestion rent is reduced when the placement is carried out at node 14.

### A. Comparison between the Placement of two DG types

In order to compare the results obtained between various types of DG, another DG (DG2) with cost characteristic as shown below is used.

\[ C(P_{DG2}) = 0.002(P_{DG2})^2 + 15P_{DG2} \]  

![Fig. 5 Comparison of Congestion Rent with and without DG placement.](image)

The DG2 is found to have lower incremental cost compared to DG1 which results in its higher real power injection into the network. Fig. 6 shows the LMP values corresponding to base case and with DGs placement scenarios.

![Fig. 6 Load bus LMP corresponding to three scenarios.](image)

As is revealed in Fig. 6, LMP has been reduced considerably from base case to the case with DG. Moreover, the LMP is found to reduce more with the placement of DG2. It is also observed that DG2 has improved the line congestion and has reduced LMP compared to DG1 having higher incremental cost. Apparently, the cheaper the unit, the higher the penetration and so is the net social welfare.
VI. CONCLUSION

The paper has presented the influence of DG on congestion and LMP in an OPF-based wholesale electricity market. LMP that consists of a congestion component, along with fixed and loss components, is conveniently used for placement of DG. The sizing of DG is formulated from social welfare point of view by maximizing it. The grid locations are examined to study the influence of DG penetration on LMP and to address the transmission bottlenecks. Locally, the effect of DG is reduction in LMP. The power injected by DG is found to reduce the congestion component, which is revealed from the reduced shadow prices associated with the constrained line flow.

The nodal price differential of the constrained branch is used to calculate the congestion rent for each branch of the network. The optimal dispatch from DG is thus found to reduce the congestion rent and shadow prices associated with the line flow. Moreover, DG with lower incremental cost is found to have better performance in terms of alleviating congestion in the network.

REFERENCES


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