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Reduction of Linear Time-Invariant Systems Using Routh-Approximation and PSO

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Abstract**—**Order reduction of linear-time invariant systems employing two methods; one using the advantages of Routh approximation and other by an evolutionary technique is presented in this paper. In Routh approximation method the denominator of the reduced order model is obtained using Routh approximation while the numerator of the reduced order model is determined using the indirect approach of retaining the time moments and/or Markov parameters of original system. By this method the reduced order model guarantees stability if the original high order model is stable. In the second method Particle Swarm Optimization (PSO) is employed to reduce the higher order model. PSO method is based on the minimization of the Integral Squared Error (ISE) between the transient responses of original higher order model and the reduced order model pertaining to a unit step input. Both the methods are illustrated through numerical examples.

Keywords**—**Model Order Reduction, Markov Parameters, Routh Approximation, Particle Swarm Optimization, Integral Squared Error, Steady State Stability.

I. Introduction

THE exact analysis of high order systems is both tedious and costly. The problem of reducing a high order system to its lower order system is considered important in analysis, synthesis and simulation of practical systems. Bosley and Lees [1] and others have proposed a method of reduction based on the fitting of the time moments of the system and its reduced model, but these methods have a serious disadvantage that the reduced order model may be unstable even though the original high order system is stable.

To overcome the stability problem, Hutton and Friedland [2], Appiah [3] and Chen *et. al.* [4] gave different methods, called stability based reduction methods which make use of some stability criterion. Other approaches in this direction include the methods such as Shamash [5] and Gutman *et. al.* [6]. These methods do not make use of any stability criterion but always lead to the stable reduced order models for stable systems.

Some combined methods are also given for example Shamash [7], Chen *et. al.* [8] and Wan [9]. In these methods the denominator of the reduced order model is derived by some stability criterion method while the numerator of the reduced model is obtained by some other methods [6, 8, 10].

In recent years, one of the most promising research fields has been “Evolutionary Techniques”, an area utilizing analogies with nature or social systems. Evolutionary techniques are finding popularity within research community as design tools and problem solvers because of their versatility and ability to optimize in complex multimodal search spaces applied to non-differentiable objective functions. Recently, the particle swarm optimization (PSO) technique appeared as a promising algorithm for handling the optimization problems. PSO is a population-based stochastic optimization technique, inspired by social behavior of bird flocking or fish schooling [11]. PSO shares many similarities with the genetic algorithm (GA), such as initialization of population of random solutions and search for the optimal by updating generations. However, unlike GA, PSO has no evolution operators, such as crossover and mutation. One of the most promising advantages of PSO over the GA is its algorithmic simplicity: it uses a few parameters and is easy to implement [12].

In the present paper, two methods for order reduction of linear-time invariant systems are presented. In the first method, the denominator of the reduced order model is obtained using advantages of Routh approximation method of Hutton and Friedland [2, 13]. The numerator of the reduced model is then determined using the Indirect approach of retaining the time moments and/or Markov parameters of original system [14]. In the second method, PSO is employed for the order reduction where both the numerator and denominator coefficients of LOS are determined by minimizing an ISE error criterion.

The reminder of the paper is organized in five major sections. In Section II statement of the problem is given. Order reduction by Routh approximation method is presented in Section III. In Section IV, order reduction by PSO has been presented. In Section V, two numerical examples are taken and both the proposed methods are applied to obtain the reduced order models for higher order models and results are shown. A comparison of both the proposed method with other well known order reduction techniques is presented in Section VI. Finally, in Section VII conclusions are given.

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II. Statement of The Problem

The Let the n^{th} order system and its reduced model ($r < n$) be given by the transfer functions:

$$G(s) = \frac{\sum_{i=0}^{n-1} d_i s^i}{\sum_{j=0}^{n} e_j s^i} \tag{1}$$

$$R(s) = \frac{\sum_{i=0}^{r-1} a_i s^i}{\sum_{j=0}^{r} b_j s^i} \tag{2}$$

where a_i, b_j, d_i, e_j, are scalar constants.

The objective is to find a reduced r^{th} order reduced model $R(s)$ such that it retains the important properties of $G(s)$ for the same types of inputs.

III. Reduction by Routh Approximation Method

The transfer function of the control system is expressed as:

$$G(s) = \frac{N(s)}{D(s)} = \frac{d_0 + d_1 s + \ldots + d_{n-1} s^{n-1}}{e_0 + e_1 s + \ldots + e_{n-1} s^{n-1} + e_n s^n} \tag{3}$$

Where, $N(s)$ and $D(s)$ are numerator and denominator polynomials of original higher order model $G(s)$ respectively. Let the order of $D(s)$ be even. Following Hutton and Friedland, the reduced denominator can be obtained by Routh approximation method [13] using the following steps:

Step-1

Construct the Routh array for the denominator polynomial of the given transfer function starting with the first entry as the constant term. To obtain a reduced model of order 'r' a new Routh array is formed, where the first $(r+1)$ terms of the above array forms the first column. The remaining entries of the array are now easily filled. Once the array is complete, it will be noted that the last element in the first column move two places up and one to the right at each step. The denominator of the reduced system $D_r(s)$ can be directly written from the first two rows of the array thus formed as:

$$D_r(s) = b_0 + b_1 s + b_2 s^2 + \ldots + s^r \tag{4}$$

which is the r^{th} order reduced normalized denominator and can be expressed as:

$$D_r(s) = \sum_{j=0}^{r} b_j s_j \tag{5}$$

$$b_r = 1 \tag{6}$$

Step-2

The transfer function in equation (3) can be expanded into a power series about $s = 0$ as:

$$G(s) = c_0 + c_1 s + c_2 s^2 + \ldots. \tag{7}$$

where,

$$c_0 = \frac{d_0}{e_0} \tag{8}$$

$$c_i = \frac{1}{e_0}\left(d_i - \sum_{j=1}^{i} e_j c_{i-j}\right),\ i > 0 \tag{9}$$

with $d_i = 0$ for $i > n$-1

It should be noted that the time moments of $G(s)$ are directly proportional to the c_i's. The relation between them is given by Shamash. [7]:

$$c_i = \frac{1}{i!}\ (i^{th} \text{ time moment of the system}) \tag{10}$$

The transfer function in equation (3) can also be expanded into a power series about $s = \infty$ as:

$$G(s) = M_1 s^{-1} + M_2 s^{-2} + M_3 s^{-3} + \ldots \tag{11}$$

Where

$$M_1 = \frac{d_{n-1}}{e_n} \tag{12}$$

$$M_i = \frac{1}{e_n}\left(d_{n-1} - \sum_{j=1}^{i-1} e_{n-j} M_{i-j}\right), \text{ for } i > 0 \tag{13}$$

with $d_i = 0$ for $i > n$-1

where M_i's are called the Markov parameters of the system.

Step-3

The reduced model $R_1(s)$ of order 'r' obtained by matching initial time moments is given by:

$$R_1(s) = \frac{\sum_{j=0}^{r} b_j s^j}{\sum_{j=0}^{r} b_j s^j}(c_0 + c_1 s + \ldots) \tag{14}$$

Collecting terms up to (r -1) powers of s in numerator, we get

$$R_1^*(s) = \frac{\sum_{i=0}^{r-1} a_i s^i}{\sum_{j=0}^{r} b_j s^j} \tag{15}$$

Alternatively the reduced model $R_2(s)$ of order 'r' may also be obtained by matching initial Markov parameters and it is given by:

$$R_2(s) = \frac{\sum_{j=0}^{r} b_j s^j}{\sum_{j=0}^{r} b_j s^j}(M_1 s^{-1} + M_2 s^{-2} + ...) \tag{16}$$

Neglecting terms with negative powers of s and collecting terms up to (r -1) powers of s in numerator, we get

$$R_2^*(s) = \frac{\sum_{i=0}^{r-1} a_i^* s^i}{\sum_{j=0}^{r} b_j s^j} \tag{17}$$

Step-4

The steady state is always matching if the time moments are matched however if the Markov parameters are matched, there is a steady state error between the outputs of original and reduced systems. To avoid steady state error we match the steady state responses by:

$$\frac{d_0}{e_0} = k \frac{a_0^*}{b_0} \tag{18}$$

The final reduced order model is obtained by multiplying gain correction factor 'k' with the numerator of $R_2^*(s)$.

IV. Particle Swarm Optimization Method

In conventional mathematical optimization techniques, problem formulation must satisfy mathematical restrictions with advanced computer algorithm requirement, and may suffer from numerical problems. Further, in a complex system consisting of number of controllers, the optimization of several controller parameters using the conventional optimization is very complicated process and sometimes gets struck at local minima resulting in sub-optimal controller parameters. In recent years, one of the most promising research field has been "Heuristics from Nature", an area utilizing analogies with nature or social systems. Application of these heuristic optimization methods a) may find a global optimum, b) can produce a number of alternative solutions, c) no mathematical restrictions on the problem formulation, d) relatively easy to implement and e) numerically robust. Several modern heuristic tools have evolved in the last two decades that facilitates solving optimization problems that were previously difficult or impossible to solve. These tools include evolutionary computation, simulated annealing, tabu search, genetic algorithm, particle swarm optimization, etc. Among these heuristic techniques, Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) techniques appeared as promising algorithms for handling the optimization problems. These techniques are finding popularity within research community as design tools and problem solvers because of their versatility and ability to optimize in complex multimodal search spaces applied to non-differentiable objective functions.

The PSO method is a member of wide category of swarm intelligence methods for solving the optimization problems. It is a population based search algorithm where each individual is referred to as particle and represents a candidate solution. Each particle in PSO flies through the search space with an adaptable velocity that is dynamically modified according to its own flying experience and also to the flying experience of the other particles. In PSO each particles strive to improve themselves by imitating traits from their successful peers. Further, each particle has a memory and hence it is capable of remembering the best position in the search space ever visited by it. The position corresponding to the best fitness is known as *pbest* and the overall best out of all the particles in the population is called *gbest* [11].

The modified velocity and position of each particle can be calculated using the current velocity and the distances from the $pbest_{j,g}$ to $gbest_g$ as shown in the following formulas [12,15, 16]:

$$v_{j,g}^{(t+1)} = w * v_{j,g}^{(t)} + c_1 * r_1(\ ) * (pbest_{j,g} - x_{j,g}^{(t)}) + c_2 * r_2(\ ) * (gbest_g - x_{j,g}^{(t)}) \tag{19}$$

$$x_{j,g}^{(t+1)} = x_{j,g}^{(t)} + v_{j,g}^{(t+1)} \tag{20}$$

With $j = 1,2,...,n$ and $g = 1,2,...,m$

Where,

n = number of particles in the swarm

m = number of components for the vectors v_j and x_j

t = number of iterations (generations)

$v_{j,g}^{(t)}$ = the g-th component of the velocity of particle j at iteration t, $v_g^{\min} \le v_{j,g}^{(t)} \le v_g^{\max}$;

w = inertia weight factor

c_1, c_2 = cognitive and social acceleration factors respectively

r_1, r_2 = random numbers uniformly distributed in the range (0, 1)

$x_{j,g}^{(t)}$ = the g-th component of the position of particle j at iteration t

$pbest_j$ = *pbest* of particle j

$gbest$ = *gbest* of the group

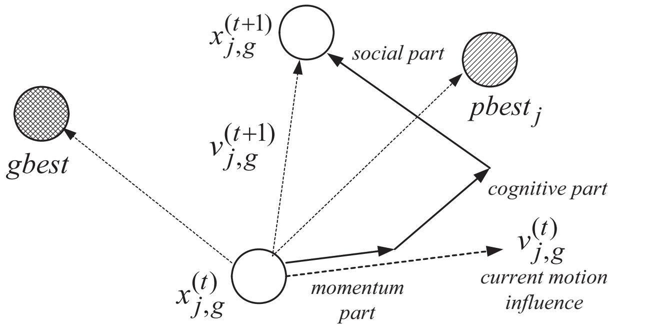

Fig. 1. Description of velocity and position updates in particle swarm optimization for a two dimensional parameter space

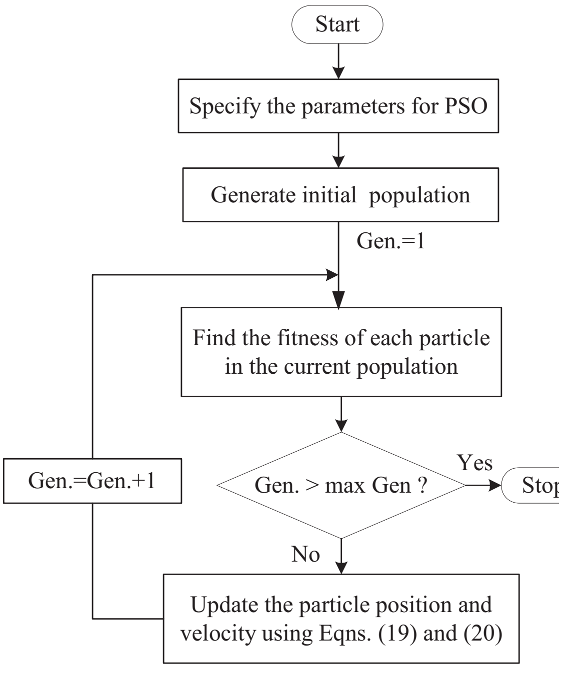

Fig. 2. Flowchart of PSO for order reduction

The j-th particle in the swarm is represented by a d-dimensional vector $x_j = (x_{j,1}, x_{j,2}, \ldots\ldots, x_{j,d})$ and its rate of position change (velocity) is denoted by another d-dimensional vector $v_j = (v_{j,1}, v_{j,2}, \ldots\ldots, v_{j,d})$. The best previous position of the j-th particle is represented as $pbest_j = (pbest_{j,1}, pbest_{j,2}, \ldots\ldots, pbest_{j,d})$. The index of best particle among all of the particles in the swarm is represented by the $gbest_g$. In PSO, each particle moves in the search space with a velocity according to its own previous best solution and its group's previous best solution. The velocity update in a PSO consists of three parts; namely momentum, cognitive and social parts. The balance among these parts determines the performance of a PSO algorithm. The parameters c_1 and c_2 determine the relative pull of *pbest* and *gbest* and the parameters r_1 and r_2 help in stochastically varying these pulls. In the above equations, superscripts denote the iteration number. Fig.1. shows the velocity and position updates of a particle for a two-dimensional parameter space. The computational flow chart of PSO algorithm employed in the present study for the model reduction is shown in Fig. 2.

V. Numerical Examples

Let us consider the system described by the transfer function due to Shamash [7]:

$$G(s) = \frac{14s^3 + 248s^2 + 900s + 1200}{s^4 + 18s^3 + 102s^2 + 180s + 120} \quad (21)$$

For which a second order reduced model $R_2(s)$ is desired.

A. Routh Approximation Method

Example 1:

Step-1

The denominator of this system is given by:

$$D(s) = s^4 + 18s^3 + 102s^2 + 180s + 120 \quad (22)$$

Applying first step to construct Routh table for the denominator as:

120	102	1
180	18	
90	1	
16		
1		

Using the first three entries as the first column, form a new array for the reduced system as:

120	90
180	
90	

Now using the first two rows

$$D_2^*(s) = 120 + 180s + 90s^2 \quad (22)$$

Normalizing $D_2^*(s)$ yields:

$$D_2^*(s) = s^2 + 2s + 1.334 \quad (23)$$

Step-2

The power series expansion of $G(s)$ about $s = 0$ gives time moments as:

$$G(s) = 10 - \frac{15}{2}s + ... \quad (24)$$

The power series expansion of $G(s)$ about $s = \infty$ gives Markov parameters:

$$G(s) = 14s^{-1} - 4s^{-2} + ... \quad (25)$$

Step-3

The reduced order model obtained by matching initial time moments is given by:

$$R_1(s) = \frac{s^2 + 2s + 1.334}{s^2 + 2s + 1.334}\left(10 - \frac{15}{2}s + ...\right) \quad (26)$$

Collecting terms up to $(r-1)$ powers of 's' in numerator, we get 'reduced system' whose transfer function is given by;

$$R_1(s) = \frac{2s + 13.34}{s^2 + 2s + 1.334} \quad (27)$$

The reduced order model obtained by matching initial Markov parameters is given by:

$$R_2(s) = \frac{s^2 + 2s + 1.334}{s^2 + 2s + 1.334}\left(14s^{-1} - 4s^{-2} + ...\right) \quad (28)$$

Neglecting terms with negative powers of s and collecting terms up to $(r-1)$ powers of 's' in numerator, we get:

$$R_2^*(s) = \frac{14s + 24}{s^2 + 2s + 1.334} \quad (29)$$

To avoid the steady state error, we multiply the numerator of $R_2^*(s)$ by gain correction factor k = 0.556 and get second order reduced system $R_2(s)$ whose transfer function is given by:

$$R_2(s) = \frac{7.784s + 13.344}{s^2 + 2s + 1.334} \quad (30)$$

Example 2:

Let us consider the system described by the transfer function due to Shamash [7]:

$$G(s) = \frac{N(s)}{D(s)} \quad (31)$$

Where,

$$N(s) = 18s^7 + 51s^6 + 5982s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320$$

and

$$D(s) = s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 185760s + 40320$$

For which a second order reduced model $R_2(s)$ is desired.

Step-1

Applying first step to construct Routh table for the denominator as:

40320	118124	22449	546	1
109584	67284	4536	36	
93367.7	20780	532.8	1	
42894.9	3910.7	34.8		
12267.5	457.1	1		
2312.4	31.3			
291.1	1			
23.6				
1				

Using the first three entries as the first column, form a new array for the reduced system as:

$$\begin{array}{ll} 40320 & 93367.7 \\ 109584 & \\ 93367.7 & \end{array}$$

Now using the first two rows:

$$D_2^*(s) = 93367.7s^2 + 109584s + 40320 \quad (32)$$

Normalizing $D_2^*(s)$ yields:

$$D_2^*(s) = s^2 + 1.17368s + 0.43184 \quad (33)$$

Step-2

The power series expansion of $G(s)$ about $s = 0$ is given by:

$$G(s) = 1 + 1.889286s - 2.55633s^2 + 2.786299s^3 - 2.890795s^4 + ... \quad (34)$$

The power series expansion of $G(s)$ about $s = \infty$ gives:

$$G(s) = 18s^{-1} - 134s^{-2} + 978s^{-3} - 7312s^{-4} + 55650s^{-5} + ... \quad (35)$$

Step-3

The reduced order model obtained by matching initial time moments is given by:

$$R_1(s) = \frac{s^2 + 1.17368s + 0.4318}{s^2 + 1.17368s + 0.43184}\left(1 + 1.889286s + \ldots\right) \tag{36}$$

Collecting terms up to ($r-1$) powers of 's' in numerator, we get 'reduced system'. Neglecting terms with negative powers of 's' and collecting terms up to ($r-1$) powers of 's' in numerator, we get:

$$R_2^*(s) = \frac{1.989544s + 0.43184}{s^2 + 1.17368s + 0.43184} \tag{37}$$

Step-4

After matching initial time moments the steady state is always matched so gain correction factor is k = 1, therefore the final reduced transfer function remains unchanged.

The reduced order model obtained by matching initial Markov parameters is given by

$$R_2(s) = \frac{s^2 + 1.17368s + 0.43184}{s^2 + 1.17368s + 0.43184}\left(18s^{-1} - 134s^{-2} + ..\right) \tag{38}$$

or

$$R_2(s) = \frac{18s - 112.8}{s^2 + 1.17368s + 0.43184} \tag{39}$$

B. Particle Swarm Optimization Method

For the implementation of PSO, several parameters are required to be specified, such as c1 and c2 (cognitive and social acceleration factors, respectively), initial inertia weights, swarm size, and stopping criteria. These parameters should be selected carefully for efficient performance of PSO. The constants c_1 and c_2 represent the weighting of the stochastic acceleration terms that pull each particle toward *pbest* and *gbest* positions. Low values allow particles to roam far from the target regions before being tugged back. On the other hand, high values result in abrupt movement toward, or past, target regions. Hence, the acceleration constants were often set to be 2.0 according to past experiences. Suitable selection of inertia weight, w, provides a balance between global and local explorations, thus requiring less iteration on average to find a sufficiently optimal solution. As originally developed, w often decreases linearly from about 0.9 to 0.4 during a run [17, 18]. One more important point that more or less affects the optimal solution is the range for unknowns. For the very first execution of the program, wider solution space can be given, and after getting the solution, one can shorten the solution space nearer to the values obtained in the previous iterations.

The objective function J is defined as an integral squared error of difference between the responses given by the expression:

$$J = \int_0^{t_\infty} [y(t) - y_r(t)]^2 \, dt \tag{40}$$

Where

$y(t)$ and $y_r(t)$ are the unit step responses of original and reduced order systems.

For Example-1:

The reduced 2nd order model for example-1 by using PSO technique is obtained as follows:

$$R_2(s) = \frac{12.0166s + 12.0226}{1.016s^2 + 2.1155s + 1.2022} \tag{41}$$

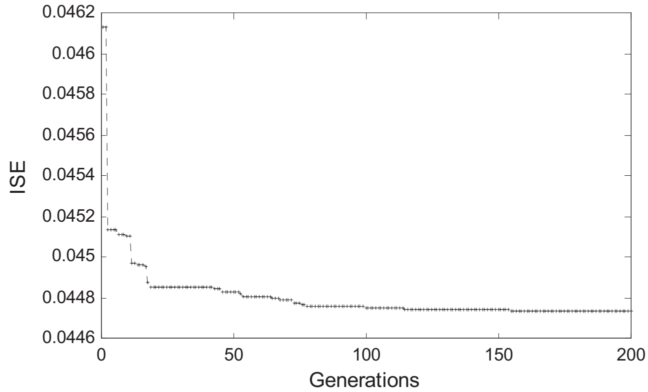

Fig. 3. Convergence of objective function for example-1

The convergence of objective function with the number of generations is shown in Fig. 3. The unit step responses of original and reduced systems by both the methods are shown in Fig. 4. For comparison Fig. 4 also shows the step response of reduced model by Gutman [6]. It can be seen that the steady state responses of all the reduced order models are exactly matching with that of the original model. However, compared to other methods of reduced models, the transient response of proposed reduced model by PSO is very close to that of original model.

For Example-2:

The reduced 2nd order model for example-2 by using PSO technique is obtained as follows:

$$R_2(s) = \frac{88.0369s + 26.4768}{4.0214^2 + 28.5882s + 2.6476} \tag{42}$$

The unit step responses of original and reduced systems by both the methods are shown in Fig. 5. It can be seen that the steady state responses of all the reduced order models are exactly matching with that of the original model. However, compared to other methods of reduced models, the transient response of proposed reduced model by PSO is very close to that of original model. The convergence of ISE with the number of generations for example-2 is shown in Fig. 6.

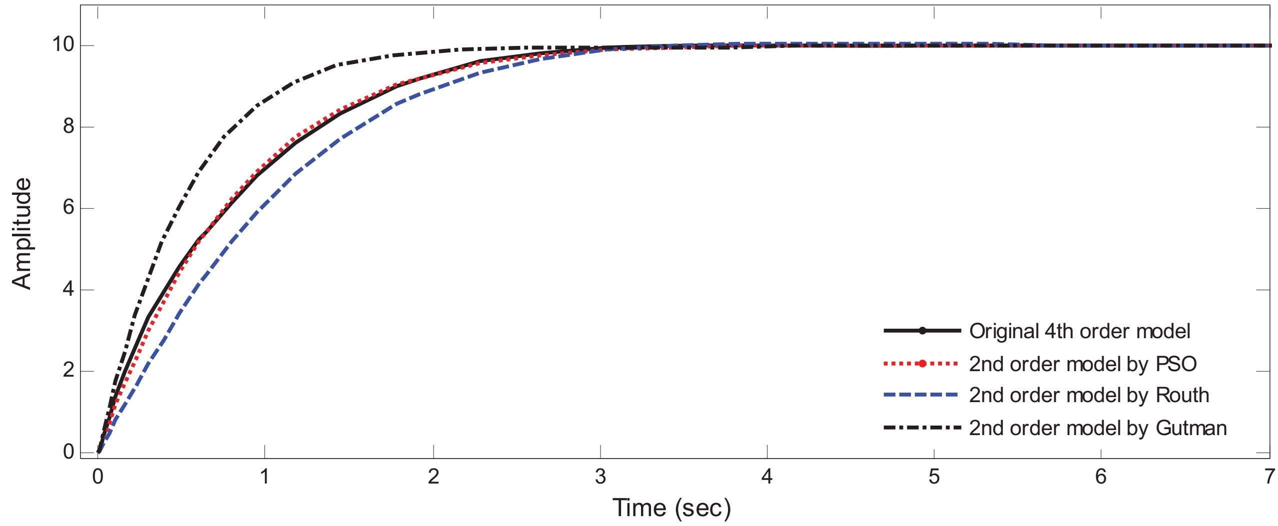

Fig. 4. Step Responses of original system and reduced model of example-1

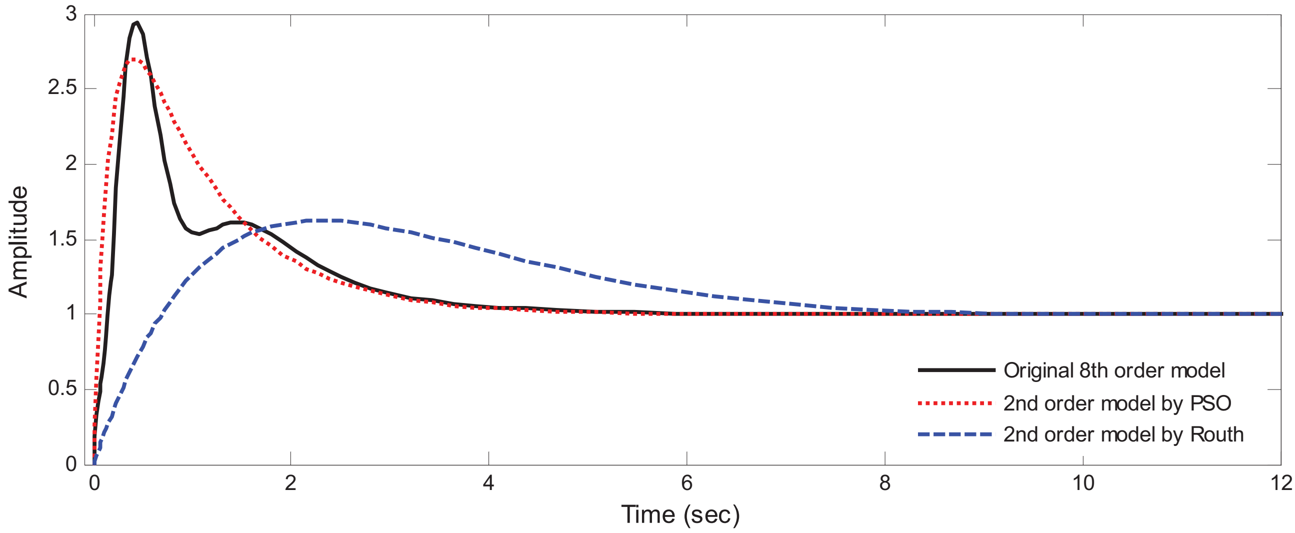

Fig. 5. Step Responses of original system and reduced model of example-2

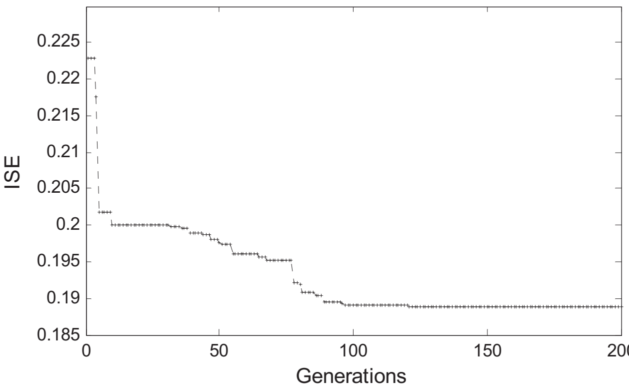

Fig. 6. Convergence of objective function for example-2

VI. Comparison of Methods

The performance comparison of both the proposed algorithm for order reduction techniques with other well known order reduction techniques is given in Table I. The comparison is made by computing the error index known as integral square error ISE [16] in between the transient parts of the original and reduced order model, is calculated to measure the goodness/quality of the [i.e. the smaller the ISE, the closer is $R(s)$ to $G(s)$, which is given by:

$$ISE = \int_0^{t_\infty} [y(t) - y_r(t)]^2 \, dt \tag{43}$$

Where $y(t)$ and $y_r(t)$ are the unit step responses of original and reduced order systems for a second- order reduced respectively. This error index is calculated for various reduced order models which are obtained by us and compared with the other well known order reduction methods available in the literature.

TABLE I COMPARISON OF METHODS FOR EXAMPLE 1

Method	Reduced model	ISE
Proposed PSO	$\frac{12.0166s+12.0226}{1.016s^2+2.1155s+1.2022}$	0.0447
Proposed Routh approximation	$\frac{7.784s+13.344}{s^2+2s+1.334}$	1.3667
Gutman et. al. [6]	$\frac{17.64706s+70.58824}{s^2+5.2491s+7.05582}$	3.4661
Shamash [7]	$\frac{8.83s+11.76}{s^2+1.765s+1.176}$	0.5763
Chen et al [4]	$\frac{8.8927s+11.9036}{s^2+1.78554s+1.19036}$	0.5418

VI. CONCLUSION

In this paper, two methods for reducing a high order system into a lower order system have been proposed. In the first method, a conventional technique has been proposed where the denominator of the reduced order model is obtained by the method of Routh approximation while the numerator of the reduced model is determined using the indirect approach of retaining the initial time moments and/or alternative approach for fitting the initial time moments and/or Markov parameters. In the second method, an evolutionary swarm intelligence based method known as Particle Swarm Optimization (PSO) is employed to reduce the higher order model. PSO method is based on the minimization of the Integral Squared Error (ISE) between the transient responses of original higher order model and the reduced order model pertaining to a unit step input. Both the methods are illustrated through numerical examples. Also, a comparison of both the proposed methods with other well known exciting methods has been presented. It is observed that both the proposed methods are comparable in quality with the other existing techniques. Further, the proposed methods preserve steady state value and stability in the reduced models and the error between the initial or final values of the responses of original and reduced order models is very less. However, PSO method seems to achieve better results in view of its simplicity, easy implementation and better response.

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