Abstract—This paper presents an analytical model to estimate the cost of an optimized design of reinforced concrete isolated footing base on structural safety. Flexural and optimized formulas for square and rectangular footing are derived based on ACI building code of design, material cost and optimization. The optimization constraints consist of upper and lower limits of depth and area of steel. Footing depth and area of reinforcing steel are to be minimized to yield the optimal footing dimensions. Optimized footing materials cost of concrete, reinforcing steel and formwork of the designed sections are computed. Total cost factor TCF and other cost factors are developed to generalize and simplify the calculations of footing material cost. Numerical examples are presented to illustrate the model capability of estimating the material cost of the footing for a desired axial load.

Keywords—Footing, Depth, Concrete, Steel, Formwork, Optimization, Material cost, Cost Factors.

I. INTRODUCTION

SAFETY and reliability were used in the flexural design of reinforced concrete footings using ultimate-strength design method used under the provisions of ACI building code of design [1, 2, 3, 4]. Footings are very important structure members and the most common shape of reinforced concrete footing is square footing. Footing sizes are mostly governed by the axial load P, allowable soil pressure qa, unit weight of concrete γc, soil unit weight γs, and the depth of the footing base below the final grade. The optimized dimensions of reinforced concrete footing could be achieved by minimizing the optimization function of slab depth and reinforcing steel area, fig. 1, [5, 6, 7].

This paper presents an analytical model to estimate the cost of an optimized design of reinforced concrete footings with yield strength of nonprestressed reinforcing steel bars fy and compression strength of concrete f'c based on shear and flexural capacities of the footing. The optimization of footings is formulated to achieve the best footing dimension that will give the most economical section to resist the external axial loads P that is made of summation of dead loads DL and live loads LL. The optimization is subjected to the design constraints of the building code of design ACI such as maximum and minimum reinforcing steel area, footing depth, developmental length in tension and compression, [8, 9, 10].

The total cost of the footing materials is equal to the summation of the cost of concrete, steel and formwork. Total cost factor TCF, cost factor of concrete CFC, Cost Factor of steel CFS, and cost factor of timber CFT are developed to generalize and simplify the estimation of footing material cost. The required footing area FA is computed based on the axial load P and the effective soil pressure Qe:

\[
F_A = \frac{P}{Q_e} = \frac{DL + LL}{Q_e}
\]  (1)

\[
Q_e = Qa - Wc - Ws
\]  (1a)

\[
Wc = \gamma c * h
\]  (1b)

\[
Ws = \gamma s * D_f
\]  (1c)

where

\[
F_A = \text{Footing Area}
\]

\[
Qa = \text{Allowable Soil Pressure}
\]

\[
Qe = \text{Effective Soil Pressure}
\]

\[
Wc = \text{Concrete Weight}
\]

\[
Wc = \text{Soil Weight}
\]

\[
h = \text{Total Footing}
\]

A. Shear and Flexural Design Formulas

Both one-way shear and two-way shear are considered for estimating the footing effective depth, Fig. 2.

The ACI design code 2008 formulas for one way shear depth:

\[
d_{\text{one way}} = \frac{V_u}{\varphi_s \gamma_c b_w} \text{(ACI Eq. 11 – 3)}
\]

where

\[
d = \text{Effective depth}
\]

\[
V_u = \text{Factored shear force}
\]

\[
\varphi_s = \text{Shear reduction factor}
\]

\[
\gamma_c = \text{Shear stress carried by the concrete}
\]

\[
b_w = \text{Footing width}
\]

The ACI design code 2008 formulas for two way shear depth, is the largest value obtained from the following equations:

\[
d_{\text{two way}} = \frac{6 V_u}{\varphi_s \left(1 + \frac{8}{f'c}b_w\right)} \sqrt{\gamma c} \text{(ACI Eq. 11 – 31)}
\]
Fig. 1 Isolated Footing Dimensions and Reinforcement Detailing

Fig. 2 Shear in footing

\[ P = DL + LL \]

One-way shear

Two-way shear
The ACI design code 2008 formulas for one way shear depth:

\[ d_{\text{one way}} = \frac{Vu}{\varphi_s \, \psi_c \, b_w} \text{(ACI Eq. 11 - 3)} \]

where
- \( d \) = Effective depth
- \( Vu \) = Factored shear force
- \( \varphi_s \) = Shear reduction factor
- \( \psi_c \) = Shear stress carried by the concrete
- \( b_w \) = Footing width

The ACI design code 2008 formulas for two way shear depth, is the largest value obtained from the following equations:

\[ d_{2\text{uti}} = \frac{6 Vu}{\varphi_s \left(1 + \frac{8}{\beta_c} \sqrt{f_c b_o}\right)} \text{(ACI Eq. 11 - 31)} \]

\[ d_{2\text{ulti}} = \frac{12 Vu}{\varphi_s \left(2 + \frac{2 \alpha_c}{b_o} \sqrt{f_c b_o}\right)} \text{(ACI Eq. 11 - 32)} \]

\[ d_{2\text{ulti}} = \frac{3 Vu}{\varphi_s \sqrt{f_c b_o}} \text{(ACI Eq. 11 - 33)} \]

where
- \( \beta_c \) = Ratio of long side of the column to the short of the column
- \( f_c \) = Specified compression strength of concrete
- \( b_o \) = Perimeter around the punching area
- \( \alpha_c \) = Ratio equals to 40, 30 and 20 for interior column, edge column and corner column respectively

The bending moments \( Mu \) in both axes are considered at the face of the column Fig. 3.

\[ Mu = \frac{L_p^2}{2} \, q_u \, b_w \]  

\[ q_u = \frac{P_u}{F_A} = \frac{DLF \ast DLF + LL \ast LLF}{F_A} \]  

where
- \( Mu \) = Fully factored bending moment
- \( L_p \) = Projected length
- \( q_u \) = Bearing pressure for strength design
- \( DLF \) = Dead Load Factor equal 1.2
- \( LLF \) = Live Load Factor equal 1.6

The reinforcement area \( As \) of the footing

\[ As = \frac{Mu}{\varphi_b \, f_y \left(d - \frac{a}{2}\right)} \]  

where
- \( \varphi_b \) = Bending reduction factor
- \( f_y \) = Specified yield strength of nonprestressed reinforcing
- \( As \) = Area of tension steel
- \( d \) = Effective depth
- \( a \) = Depth of the compression block

B. Footing Optimization

The optimization of isolated footing is formulated to achieve the best footing dimensions that will give the most economical footing size and steel reinforcement to resist the ultimate bending moment \( Mu \). The optimization is subjected to the constraints of the building code of design ACI for depth, reinforcement and footing size. The optimization function of the footing

Minimize

\[ F(As,d) = \varphi_b \, As \, f_y \left(d - \frac{a}{2}\right) - Mu \]  

Fig. 3 Moment in footing about section 1-1
Must satisfy the following constraints:

\[ d_S^L \leq d \leq d_S^U \]  
(4a)

\[ A_{S_{Min}} \leq A_s \leq A_{S_{Max}} \]  
(4b)

\[ A_{S_{Max}} = 0.75 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \left( \frac{600}{600 + f_y} \right)bd \]  
(4c)

\[ A_{S_{Min}} = \left( \frac{1.4}{f_y} \right)bd \]  
(4d)

\[ \beta_1 = 0.85 \text{ for } f' \leq 30 \text{ MPa} \]  
(4e)

where \( d_S^L \) and \( d_S^U \) are footing depth lower and upper bounds, and \( A_{S_{Min}} \) and \( A_{S_{Max}} \) are slab steel reinforcement area lower and upper bounds. The reinforcing bars must have the required length to provide sufficient strength. In other words, the bars must extend developmental length \( L_d \) from the face of the column.

\[ L_d < Ld_{AVAILABLE} \]  
(4g)

For the dowel bars under compression

\[ A_{S_{dowels}} \geq 0.005 A_{Column} \]  
(4h)

\[ Ld_{Comp} < Ld_{AVAILABLE} \]  
(4i)

\[ h > Ld_{Comp} + Cover + 2d_b \]  
(4j)

where

\( A_{S_{dowels}} \) = Steel area of the dowels

\( A_{Column} \) = Column area

\( Ld_{Comp} \) = Required bar developmental length in compression

\( Ld_{AVAILABLE} \) = Available length in compression

\( h \) = Total footing depth

\( Cover \) = Concrete cover thickness

\( d_b \) = Bar diameter

\textbf{C. Footing Formwork Materials}

The form work material is limited to footing four sides of 20 mm thickness, Fig. 4. The formwork area \( AF \) of the isolated footing

\[ AF_{SQUARE} = 4(0.2(B \times h)) \]  
(5)

\textbf{D. Footing Cost Analysis}

The total cost of the beam materials is equal to the summation of the cost of the concrete, steel and the formwork per square meter:

\[ \frac{Total Cost}{m^2} = \frac{Ag(m^2)}{m} \cdot Cc + \frac{As(m^2)}{m} \cdot Cs + \frac{Af(m^2)}{m} \cdot Cf \]  
(6)

where

\( Cc \) = Cost of 1 m$^3$ of ready mix reinforced concrete in dollars

\( Cs \) = Cost of 1 Ton of steel in dollars

\( Cf \) = Cost of 1 m$^3$ timber in dollars

\( \gamma_s \) = Steel density = 7.843 \text{ Ton}$^{-3}$

Total Cost Factor \( TCF \) and other cost factors are developed to generalize and simplify the calculations of footing material cost.

\[ CFC = \frac{(Concrete \ Cost)}{m^2} = \frac{Ag(m^2)}{m} \cdot Cc \]  
(7)

\[ CFS = \frac{Steel \ Cost}{m^2} = \frac{As(m^2)}{m} \cdot \gamma_s \cdot \frac{Ton}{m^3} \cdot Cs \]  
(8)

\[ CFT = \frac{Timber \ Cost}{m^2} = \frac{Af(m^2)}{m} \cdot Cf \]  
(9)

and

\[ TCF = CFC + CFS + CFT = \frac{Total \ Cost}{m^2} \]  
(10)

where

\( CFC \) = Cost Factor of Concrete

\( CFS \) = Cost Factor of Steel

\( CFT \) = Cost Factor of Timber

\( TCF \) = Total Cost Factor

Therefore the total cost of the footing is equal to the product of the total cost factor \( TCF \) and the footing area.
Fig. 4 Footing formwork material – Four sides

### TABLE I
FOOTING DIMENSIONS

<table>
<thead>
<tr>
<th>Service Load (kN)</th>
<th>Ultimate Load (kN)</th>
<th>Soil Pressure Qa (kN/m²)</th>
<th>Axial Design Loads (kN)</th>
<th>Ultimate Load (kN)</th>
<th>Axial Design Loads (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>700</td>
<td>200</td>
<td>340</td>
<td>1502</td>
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<tr>
<td>900</td>
<td>1260</td>
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<td>455</td>
<td>2913</td>
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<tr>
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<td>1960</td>
<td></td>
<td>550</td>
<td>4400</td>
<td>2.9</td>
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<tr>
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<td>6686</td>
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<tr>
<td>2500</td>
<td>3500</td>
<td></td>
<td>700</td>
<td>8125</td>
<td>3.9</td>
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<tr>
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<td>4480</td>
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<td>780</td>
<td>1034</td>
<td>4.4</td>
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</table>

### TABLE II
FOOTING TOTAL COST

<table>
<thead>
<tr>
<th>Service Load DL + LL (kN)</th>
<th>Ultimate Load DL(1.2) + LL(1.6) (kN)</th>
<th>Footing Area (m²)</th>
<th>Total Cost (Qatar)</th>
<th>Total Cost (USA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>1120</td>
<td>4.84</td>
<td>396</td>
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<td>1500</td>
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<td>9</td>
<td>958</td>
<td>990</td>
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<td>2000</td>
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<td>2600</td>
<td>3640</td>
<td>16</td>
<td>2100</td>
<td>2177</td>
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<td>3500</td>
<td>4620</td>
<td>20.25</td>
<td>3037</td>
<td>3135</td>
</tr>
<tr>
<td>4000</td>
<td>5600</td>
<td>25</td>
<td>4044</td>
<td>4174</td>
</tr>
</tbody>
</table>
Fig. 5 The Process of Computing Cost Factors

1. Start
2. $i = 1 \ldots 2000$ Axial Load Range
3. $j = 50 \ldots 200$ Effective soil bearing pressure Range
4. $P_i = i$ Axial Load
5. $Q_{ej} = j$ Effective soil bearing pressure
6. $A_{gij} = \frac{p_i}{q_{ej}}$ Footing Area, $Q_{uij} = \frac{P_u}{A_{gij}}$ Ultimate bearing pressure, $As$ (Equation #3)
7. Initial Design Parameters ($As, d$)
8. Optimization
9. Constraints
   a. No
   b. Yes
10. Material Quantities: Steel $As$, Concrete $Ag$, Timber $AF$
11. Footing Cost Factors: Equations 7-10
12. $j > 1$
   a. No
   b. Yes
13. $i > 2000$
   a. No
   b. Yes
14. End
II. RESULT AND DISCUSSION

The footing were analyzed and designed optimally to ACI code of design in order to minimize the total cost of the footing that includes cost of concrete, cost of steel, and formwork cost. The footings were subjected to different dead load DL, live load LL, and effective soil pressure Qe. In order to optimize the footing size, thickness, and steel area, a list of constraints (equations 4–4J) that contain shear, bending and developmental length have to be met. Areas of concrete, reinforcing steel and timber area AF (equations 5–5A) are computed based on optimum footing dimensions. The formwork area AF of the slab cross section is made of four vertical sides of 20mm thickness and height of footing total depth. The total cost of footing material is calculated using equation 6 base on Qatar and USA prices respectively of $100,$131 for 1 m3 of ready mix concrete, $1070,$1100 for 1 ton of reinforcing steel bars, and $531,$565 for 1 m3 of timber. Total Cost Factor TCF, Cost Factor of concrete CFC, Cost Factor of steel CFS, and Cost Factor of Timber CFT, are developed in equations 7 - 10 to generalize and simplify the calculation of footing material cost. To determine the cost factors that are to be used for estimating the footing material cost, an iterative cost safety procedure of estimating the footing material cost base on optimal criteria is applied to external axial load range of 10kN to 2000kN as the maximum axial load, Fig. 5. The design parameter used in iterative cost procedure are 400 MPa, 30 MPa, 200KPa, 25kN/m3, 15kN/m3 and 1meter for fy, fc, Qa, γc, γs and Df respectively. In order to verify the design results made by the analytical method a comparison of footing designed by the software STAAD Foundation is presented in Table I [10].
Table II shows the total cost of different footing areas due to different axial loads and allowable soil bearing pressure $Q_a$. As an example, an axial load of 1500kN and allowable soil pressure of 200kN/m² will require a footing cost $1457 and $1505 in Qatar and USA prices respectively, Fig. 6.

III. CONCLUSION

Footing optimum design analytical model is developed to estimate the cost of footing materials based on various design constraints. Total cost factor $TCF$, cost factor of concrete $C_{FC}$, Cost Factor of steel $C_{FS}$, and cost factor of timber $C_{FT}$ are developed and presented as formulas to approximate material cost estimation of optimized reinforced concrete footing base on ACI code of design. Cost factors were used to produce footing cost charts that relate the required footing area $F_A$ due to axial load, soil bearing pressure, and other design parameters to the slab material cost. The model is flexible and could be used for other codes of design by modifying equations of shear, flexural and optimization to check cost estimates for isolated footings.

REFERENCES