Modeling and Simulation of a Serial Production Line with Constant Work-In-Process

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Abstract—This paper presents a model for an unreliable production line, which is operated according to demand with constant work-in-process (CONWIP). A simulation model is developed based on the discrete model and several case problems are analyzed using the model. The model is utilized to optimize storage space capacities at intermediate stages and the number of kanbans at the last stage, which is used to trigger the production at the first stage. Furthermore, effects of several line parameters on production rate are analyzed using design of experiments.

Keywords—Production line simulator, Push-pull system, JIT system, Constant WIP, Machine failures.

I. INTRODUCTION

SIMULATION has been extensively used in modeling and analyzing production control systems. A particular type of production control, which has become a common trend in industry, is just in time (JIT) or “pull” system of production control. In a JIT system, production is initiated according to demand for finished products at each stage to produce what is needed at the right time and in the right quantity. Alternative to a purely pull system is the hybrid push-pull system, where the production at the first stage is scheduled according to the demand for the products in the last stage. Withdrawal of finished products from the last stage triggers the production at the first stage by an information signaling card, called a kanban. Intermediate operations are performed by a push system. Push-pull systems are commonly used in electronics assembly operations. Several studies have been carried out on the implementation and efficiency of JIT systems.

[1]-[6] have analyzed JIT systems from different perspectives using simulation as well as other meta modeling approaches, including neural network models. [7]-[10] studied a hybrid push-pull system and presented a control algorithm for multi-stage, multi-line production systems. [11] compared three pull control policies, namely the kanban, base stock, and generalized kanban.

The effects of kanbans and other factors on JIT system performance have been investigated mostly for pull types of production control strategy. Countless number of other JIT applications and related models can be seen in the literature. Most of the literature deals with the efficiency of JIT systems under different operational conditions. Either mathematical models are developed based on restrictive assumptions or simulation models are utilized in the analysis of JIT systems. In relation to the effects of intermediate buffer capacities on a push type of serial production line and optimum allocations of buffers on the line, several papers have been published. In particular, papers related to buffer allocations include [12]-[22].

In this paper, we developed a discrete mathematical model to analyze a push-pull production line of production with constant work-in-process (CONWIP). When a final product is withdrawn from the finished products inventory in the last stage, a kanban is signaled to the first stage to start the production. Fig. 1 illustrates operation of such a system.

![Fig. 1 A Push-pull production control system](image)

Successful operations are carried by completion of each product at each station (Mi) and its delivery to the succeeding station or its buffer store (βi), if the station is busy. It is assumed that the intermediate buffer sizes, which represent maximum work-in process at each stage, are limited in capacity. Thus, when the storage of finished units in the final products inventory reaches a specified maximum capacity, the last station stops its production. Similarly, when an intermediate buffer βi is filled up to its maximum capacity, the preceding station, Mi-1, stops its production or the completed part stays on the station until a part is removed from the succeeding buffer. The capacity of buffer i is zi.
numbers of kanbans and the sizes of in-process buffers, it is necessary to consider equipment reliability in model development. In the following section, we present a discrete mathematical model, which is based on the flow of discrete parts or batches from stage to stage. The model is used to analyze the behavior of the push-pull system under various operational conditions including equipment availabilities and randomness in demand.

II. MATHEMATICAL MODEL FOR THE PUSH-PULL SYSTEM

The basic principal of the discrete model is to determine the total time a batch of parts n spends in station i, the time instant at which batch n is completed in station i, and the time instant at which batch n leaves the station i. Storages \( S_{2}, \ldots, S_{m} \) are called intermediate buffer storages, having finite capacity \( z_{i} \), \( i=2,\ldots,m \). Initial input storage is assumed to have unlimited capacity for the raw material, while the final output storage has limited capacity for completed batches of products with attached kanbans. The final buffer \( m+1 \) is assumed to be the finished products storage with time between part departures being equal to time between demand. The following notations are used in the formulation:

- \( \tau_{in} \) = Time duration that nth batch stays on the ith station not considering imposed stoppages due to equipment failures; \( i=1,2,\ldots,m \).
- \( \tau_{o} \) = Number of stations on the line.
- \( \tau_{p} \) = Processing time of batch n on station i (this may be a random variable with certain distribution).
- \( \eta_{n} \) = Repair time of the ith station required for correction of a failure during processing of the nth batch. Time to failures and the repair times are assumed to follow certain distributions which are generated and incorporated into the model when the model is solved iteratively.
- \( \varphi_{in} \) = Instant of time at which processing of the nth batch is completed on the ith station.
- \( \Delta_{0n} \) = Instant of time at which nth batch departs from the ith station.
- \( \Delta_{in} \) = Instant of time at which nth batch enters the first station.
- \( \eta_{n} \) = Instant of time at which ith station is ready to process the nth batch.
- \( \Omega_{in,i-1,n} \) = Instant of time at which nth batch departs from the final buffer \( m+1 \).
- \( \delta_{n} \) = Mean time between demand for batches n-1 and n from the final buffer. This time may also be a random variable with certain distribution.

A part stays in a station for three reasons: (i) The part is being machined; (ii) The machine has failed during machining of the part and a repair is taking place; (iii) The successive buffer is full and the part can not be transferred to the next station due to an imposed stoppage. The residence time of the nth part on the ith station, \( \tau_{in} \), without considering imposed stoppages is given as follows:

\[ \tau_{in} = \tau_{o} + \mu_{in} \]  

The discrete mathematical model of the push-pull system consists of calculating part completion times, \( \varphi_{in} \), and part departure times, \( \Delta_{in} \), in an iterative fashion. The following formulation is developed for \( \varphi_{in} \) and \( \Delta_{in} \) to be used in iterative calculations.

Processing of batch n cannot be started on station i until the previous batch, n-1 leaves station i. Therefore the time instant at which ith station is ready to begin the nth batch, denoted by \( \eta_{in} \), is given by \( \eta_{in} = \Delta_{o,n-1} \). If, \( \Delta_{o,n-1} \geq \eta_{in} \), then the nth batch must wait in buffer \( \beta_{n} \), since it has left station i-1 before station i is ready to accept it. Therefore, processing of the nth batch in the ith station will start at the instant \( \eta_{in} \). If however, \( \Delta_{o,n-1} < \eta_{in} \), then processing of the nth batch in the ith station can start immediately at the time instant \( \eta_{in} \). Considering both cases above, one gets the relation for the ready time of the nth batch to be processed in the ith station as follows:

\[ \eta_{in} = \max\{\Delta_{o,n-1}, \Delta_{i,n-1}\} \]  

Since the nth batch will stay in station i for a period of \( \pi_{in} \) time units, its processing will be completed by the instant \( \varphi_{in} \) given by:

\[ \varphi_{in} = \max\{\Delta_{i,n-1}, \Omega_{in-1,i} + \pi_{in}\} = \eta_{in} + \pi_{in} \]  

where \( i=2,\ldots,m \).

In case of the first station, a kanban must arrive before the batch can be processed. The arrival of a kanban from storage \( m+1 \) is modeled as follows:

Let \( p=n-L_{2} \) where, \( L_{2} \) = Total number of batches initially in stations \( S_{2} \) and storages \( \beta_{2},\ldots,\beta_{m} \). Then,

\[ \varphi_{in} = \max\{\Delta_{i,n-1}, \Omega_{in-1,i} + \pi_{in}\} \]  

Time instant at which nth batch is ready to enter the first station is assumed to be \( \Delta_{0n} < \Delta_{i,n-1} \) since we assumed that there are always batches of parts available in front of the first station. However, a kanban must arrive from the buffer \( m+1 \) to start the process at station 1. Now, it remains to determine the time instant at which nth batch departs from the ith station, \( \Delta_{in} \). It is found by considering two cases:

Let \( k=n-z_{i-1}-1 \)

In the first case, \( \varphi_{in} < \Delta_{i,n-1,k} \)

which indicates that the nth batch has been completed on the ith station before processing of the \( (n-z_{i-1})^{th} \) batch has started on the \( (i+1)^{th} \) station. Since buffer i-1, which is between station i and i+1 with capacity \( z_{i+1} \), is full and station i has completed the nth batch, the \( n^{th} \) batch may leave the ith station only at the instant of time at which the \( (n-z_{i+1})^{th} \) batch of the \( (i+1)^{th} \) station has started processing.

Therefore, \( \Delta_{i,n} = \Delta_{i,n-1,k} \)  

(7)

In the second case, \( \varphi_{in} > \Delta_{i,n-1,k} \)

which indicates that, at the instant \( \varphi_{in} \) there are free spaces in buffer \( i+1 \) and therefore part n can leave machine i immediately after it is completed; that is, \( \Delta_{in} = \varphi_{in} \) holds under this case.

Considering both cases above, we have the following relations for \( \Delta_{in} \):

\[ \Delta_{i,n} = \varphi_{in} \text{ if } n \leq z_{i+1} + 1 \]  

(9)

\[ \Delta_{i,n} = \max\{\varphi_{in}, \Delta_{i,n-1}\} \]  

(10)
Therefore, it is important to start the iterations with some batches of parts that are often left from a previous shift or day.

When iterations are started, one can assume that, at the initial time instant \( t=0 \), parts \( 1, 2, \ldots, L_{i+1} \) are already processed on station \( S_i \), since these batches are initially in the line right after station \( S_i \). Therefore, given the initial values of \( h_{i1}, \ldots, h_{im} \), the initial values of \( y_{m} \) and \( \Lambda_{mn} \) for \( n=1, 2, \ldots, L_{i+1} \) are expressed by:

\[
\Psi_{m} = \Lambda_{mn} = 0; \quad i = 1, 2, \ldots, m. \tag{15}
\]

In order to carry out iterative computations, a simulation procedure is developed and implemented on the computer to determine several production line performance measures, which include average number of batches completed by the line during a specified period, average number of batches completed by each station during the same time, percentage of time for which each station is up and down due to imposed or inherent stoppages.

### III. Computations of the Model

The discrete model is coded into a simulation program and implemented on the computer to calculate system performance measures. In addition to the variables described for the discrete model, the simulation allows several distributions, including: exponential, uniform, Weibull, normal, log normal, Erlang, gamma, beta and constant values to be specified for failure and repair times of the equipment in each station. Iterative simulation model basically calculates the time instant at which each part enters a station, duration of its stay, and the total loss factor of station \( S_i \) respectively. After the first simulation, calculations of \( \bar{\psi}_{mn} \) and \( \bar{\omega}_{mn} \) are performed iteratively with the consideration given to equipment failures and repairs as the parts flow through the system.

Reliable results cannot always be obtained from a single simulation realization. Therefore, additional runs have to be performed and the results tested statistically until the error in the line production rate is less than an \( \varepsilon \) value with a probability, both of which are predefined. This is accomplished by comparing the average production output rate from simulation \( \bar{Q} \) to the expected value \( \bar{Q} \) using the confidence interval calculation given below. Here,

\[
\bar{Q} = \frac{N}{T_{\text{sim}}} \quad \text{and} \quad N = \sum_{i=1}^{n} (N_i)/n \tag{16}
\]

where \( N_i \) = production output obtained from simulation run \( i \) and \( n \) is the number of simulation runs.

\[
\Pr \left[ 1 - Z_{\alpha/2} \frac{\sqrt{V(\bar{Q})}}{\bar{Q}} < \bar{Q} < 1 + Z_{\alpha/2} \frac{\sqrt{V(\bar{Q})}}{\bar{Q}} \right] = 1 - \alpha \tag{17}
\]

The aim is to have an estimated output rate, \( \bar{Q} \), as close to the actual mean output rate \( \bar{Q} \) as possible. To achieve this, \( Z_{\alpha/2} \sqrt{V(\bar{Q})}/\bar{Q} \) is minimized by obtaining more runs. As this value gets closer to 0, \( \bar{Q} \rightarrow \bar{Q} \) with probability 1 - \( \alpha \). An \( \varepsilon \) value is entered by the user; the simulation program calculates \( Z_{\alpha/2} \sqrt{V(\bar{Q})}/\bar{Q} \) after each iteration; compares this quantity with \( \varepsilon \) and terminates the program if it is less than \( \varepsilon \). If it is not less than \( \varepsilon \) after a maximum number of runs, the program is still terminated to avoid excessive computation. The iterative simulation model allows one to determine various parameters and dependent variables with significant effects on productivity and other performance measures. Estimation indices are obtained for such variables as the total, inherent, and imposed time losses due to failures and stoppages for each station as follows:

\[
Q_{ui} = 60 / \tau_i \quad \text{is the nominal productivity of station \( i \), where} \quad \tau_i \quad \text{is the cycle time for station \( i \);} \quad \bar{Q}_{ui} = 60 \bar{N}_{ui} / T_{\text{sim}} \quad \text{is the relative productivity of station \( i \);} \quad K_{\text{loss}}(i) = 1 - Q_{ui} / Q_{ui} \quad \text{is the total loss factor of station \( i \);} \quad K_{\text{in}}(i) = 1 - T_{\beta} / (T_{\beta} + \bar{T}_{\beta}) \quad \text{is the inherent loss factor of station \( i \); and} \quad K_{\text{im}}(i) = K_{\text{loss}}(i) - K_{\text{in}}(i) \quad \text{is the imposed loss factor for station \( i \).} \tag{18}
\]

The terms \( T_{\beta} \) and \( \bar{T}_{\beta} \) are mean times to failure and to repairs, of station \( i \) respectively. After determining these loss factors, they are compared for all stations. The station with the highest total loss factor is then
chosen for improvement. If K_{imp}(i)>K_{inh}(i), stoppages are mainly due to blocking and starvation; therefore it is necessary to increase the capacity of buffers immediately preceding and mainly due to blocking and starvation; therefore it is necessary to increase the capacity of buffers immediately preceding and mainly due to blocking and starvation; therefore it is necessary to increase the capacity of buffers immediately preceding and mainly due to blocking and starvation; therefore it is necessary to increase the capacity of buffers immediately preceding and mainly due to blocking and starvation; therefore it is necessary to increase the capacity of buffers immediately preceding and mainly due to blocking and starvation; therefore it is necessary to increase the capacity of buffers immediately preceding

The problem can be stated as follows: Given a total amount of acceptable buffer space of Z units, allocate this total space to individual buffers S_2, S_3, ..., S_{m+1}, the quantities z_2, z_3, ..., z_{m+1}, respectively such that the total production output rate of the line, Q(z), is maximized. The problem is stated as follows:

Choose z_2, z_3, ..., z_{m+1} so as to

Maximize Q(z)

Subject to: \[ \sum_{i=2}^{m+1} z_i \leq Z \] (18)

This problem has been discussed by [7] for production lines with exponential processing times in all stations. The optimization model is a linearly constrained integer nonlinear programming problem that is difficult to solve due to the fact that Q(z) has to be evaluated by either continuous time Markov chains or by some other stochastic processes approximation. [7] evaluated Q(z) for the serial line using Markov chains approach and indicated that the number of states are too large and exceeds well over 20,000 equations to be solved to obtain the value of Q(z) for a given buffer size combination. Even if it was practical to solve the problem, it would not be still applicable to the cases with equipment failures and non-exponential process times. The buffer allocation model is applied to the push-pull system in this paper. However, we obtain the solution for Q(z) using the iterative solution procedure presented above. This procedure is not restricted to exponential process times and all reliable equipment, since it is based on simulation. A fixed number of buffers are specified and the iterative computations are performed to determine optimum combinations by evaluating all buffer combinations. The optimum corresponds to the maximum production output rate. For small size problems, such as lines with up to 8 stations and up to 10 buffer capacities, computational time is in the order of minutes, depending on the accuracy required. However, for larger problems, such as more than m=10 stations and more than Z=15 buffer capacities, computational time is relatively large since number of possibilities evaluated is large. If a small accuracy with 5% error is acceptable, large problems can also be solved in a reasonable time.

V. SIMULATED CASE PROBLEMS

The model is illustrated by several case problems. Table I and Table II are the input data and the output results obtained for a 5-station line with all stations available 85% of the time. Processing times, failure distributions, their parameters, repair distributions and their parameters are shown in Table I.

<table>
<thead>
<tr>
<th>Station</th>
<th>Process Time</th>
<th>No. of Failures</th>
<th>Failure Distrib.</th>
<th>Repair Distrib.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>1</td>
<td>Expo(85)</td>
<td>Normal (15, 2.25)</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>1</td>
<td>Expo(85)</td>
<td>Normal (15, 2.25)</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>1</td>
<td>Expo(85)</td>
<td>Normal (15, 2.25)</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>1</td>
<td>Expo(85)</td>
<td>Normal (15, 2.25)</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
<td>1</td>
<td>Expo(85)</td>
<td>Normal (15, 2.25)</td>
</tr>
</tbody>
</table>

The outputs for 2000 time units of simulation with \( \theta=0.05 \), \( \varphi=0.005 \), and maximum iterations=100 are shown in Table II. The results include relative production rate of each station and various loss factors due to equipment failures as discussed in section III. The output also includes suggestions for line improvement.

<table>
<thead>
<tr>
<th>Station</th>
<th>Relative Prod. Rate</th>
<th>Imposed Loss Factors</th>
<th>Inherent Loss Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.564</td>
<td>0.130</td>
<td>0.285</td>
</tr>
<tr>
<td>2</td>
<td>0.565</td>
<td>0.151</td>
<td>0.285</td>
</tr>
<tr>
<td>3</td>
<td>0.565</td>
<td>0.148</td>
<td>0.287</td>
</tr>
<tr>
<td>4</td>
<td>0.566</td>
<td>0.144</td>
<td>0.290</td>
</tr>
<tr>
<td>5</td>
<td>0.566</td>
<td>0.143</td>
<td>0.291</td>
</tr>
</tbody>
</table>

Suggestions: Station No. 1 has the Maximum Total Loss Factor. Down Time is Mainly Imposed. Increase the Capacity of Storage Adjacent to This Station. Also Increase Reliability and Productivity of Adjacent Stations and Try Simulation Again. Error<\( \text{Epsilon Is Reached at Iteration = 100} \)

Total Computation Time = 104.39 Seconds

In the second case problem, a push-pull production line with 5 serial stations is considered as before. All stations are assumed to be reliable, except one station which was placed at
the beginning (B), in the middle (M), or at the end (E) of the line to see the effects of unreliable station at different segments of the line. Failure and repairs for the unreliable station are as in Table I. Standard deviation of the repair times was taken as 15% of the mean. Since availability is \( A = \frac{MTBF}{MTBF + MTTR} \), selected parameters represented 85% equipment availability for the particular station, whose effect on the line was investigated. In other words, we wanted to see how the buffers would be allocated if the unreliable station was at the beginning, at the middle or at the end of the line. The system was analyzed by the simulation over a period of 2000 time units. All intermediate buffer combinations, which add up to less than or equal to total buffer capacity \( Z \) (\( Z=0,1,2,3,5,10 \)) are evaluated in order to determine line productivity, as shown in Fig. 2, and optimum buffer combinations \( (Z_2, Z_3, Z_4, Z_5) \), which resulted in maximum production rate, as shown in Table III, for three different locations of the unreliable equipment at the beginning (B), in the middle (M), and at the end (E) of the line.

The corresponding production output rates are given as the percentage of nominal rate, which would be 100 if the line was all reliable, balanced with constant processing times. An important observation related to buffer size allocation can be seen in Table III. If the line has an unreliable station at the start or at the end, the buffer capacity available should be located immediately after the last station, except in the case of 5 and 10 buffer sizes, in which case one unit is allocated to buffer 4, after station 3 in the center of the line. A similar observation is seen in the case if the center station is unreliable. In this case if there is one buffer space to be allocated on the line, it is preferred to be at buffer 3, immediately after station 2.

Additional buffer sizes are mostly allocated to the end of the line after station 5, except in the case of 2 and 10 buffer sizes, in which case one space is allocated after the center station. The main reason that the buffer spaces are mostly allocated to the end of the line could be due to the fact that the line is operated as a push-pull system and therefore the first station can not start processing a part unless a part or batch is withdrawn from the last station. Buffer spaces after the last station helps increasing part availability during demand. The same three cases were evaluated for a purely push type of production line, where the last station does not have a limit on outputting its product and the first station can start without waiting for a part withdrawal from the last station. The results, which are not shown here, are almost opposite of what is obtained for the push-pull system and the buffer allocation is preferred to be immediately after the first station near the start of the line in all cases.

### Table III

<table>
<thead>
<tr>
<th>Buffer Capacity Distribution to Stations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beginning</strong></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>( Z )</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
</tbody>
</table>

| 
| **Line Productivity** (%) | 84.80 | 85.20 | 85.40 | 85.60 | 85.80 | 86.00 |
| **Total Buffer Capacity (Z)** | 0 | 1 | 2 | 3 | 5 | 10 |

Fig. 2 Line productivity as a function of \( Z \)

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### VI. Effects of Line Configurations, Maintenance Policies and Line Parameters on Line Performance

In order to see effects of various production related parameters and factors on line performance measure, such as the production rate, several experiments were set up and results were obtained. In particular, the following production line factors were taken into consideration:

1. Production line length (3, 5, and 9 stations);
2. Buffer capacities between stations (0, 2, 4, 6, 8);
3. Process time variability measured by its coefficient of variation (\( CV_{\text{pt}} = 0, 0.2, 0.5, 0.7 \));
4. Demand interval variability (\( CV_{\text{dm}} = 0, 0.2, 0.5, 0.7 \));
5. Type of maintenance applied (Design out maintenance resulting in full reliability [REL], reliability centered maintenance [CM-PM], corrective maintenance [CM])

Process time at each station was assumed to be normally distributed with mean of 3.0 time units and varied according to the coefficient of variation (\( CV_{\text{pt}} \)) selected. Similarly, time interval between the demands for withdrawal of products from the finished products storage was assumed to be normally distributed with mean of 3.0 time units and also varied according to the coefficient of variation (\( CV_{\text{dm}} \)) selected. The production lines are simulated over 2400 time units. 10 runs are carried out for each combination and average values are recorded. Figures 3-5 illustrates the production output rate as a function of various line configuration and factors mentioned above. CM-i, CM-PM-i, and REL-i represent two levels of maintenance and full reliability case for each station i.

In order to compare effects of corrective maintenance (CM) only to the CM with preventive maintenance (PM), reliability centered maintenance (RMC) concept was incorporated into the model. Under RMC, equipment is subjected to PM just before a failure is expected. Mean time between failures (MTBF) must be determined in advance. In this case, it is assumed that failures due to wear outs are eliminated and only
random failures remain. This idea can be implemented analytically if time between failures are uniformly distributed. This concept has been explained in detail by Savsar[23].

Following is a mathematical procedure to separate random failures from the wear-out failures. This separation is needed in order to be able to see the effects of maintenance on the productivity and availability of a line when simulating the system.

Let \( f(t) \) = Probability distribution function (pdf) of time between failures.
\[ F(t) = \text{Cumulative probability distribution function (cdf) of time between failures}, \]
\[ R(t) = \text{Reliability function (Probability that the equipment survives by time } t\text{)}, \]
\[ h(t) = \text{Hazard rate (or instantaneous failure rate)}. \]

Hazard rate \( h(t) \) consists of two components, the first due to random failures and the second due to wear-out failures as:
\[ h(t) = h_1(t) + h_2(t) \]  
(19)

Since the equipment failures are either due to chance causes or wear-outs, reliability of the equipment, which is the probability that equipment survives by time \( t \), can be expressed as follows:
\[ R(t) = R_1(t) R_2(t) \]  
(20)

It is known that
\[ h(t) = f(t)/[1-F(t)] = \lambda + h_2(t) \]  
(23)

The reliability function for each component would be is as follows:
\[ R_1(t) = e^{-\lambda t}, \text{ and } R_2(t) = e^{-h_2(t) t} \]  
(21)

\[ h(t) = \frac{f(t)}{e^{-\lambda t} - \lambda} = \frac{f(t)}{e^{h_2(t) t}} \]  
(24)

The corresponding time to failure probability density functions for each type of failure rate is:
\[ f_1(t) = \alpha \times e^{-\alpha t}, \quad 0 < t < \mu \]  
(31)
\[ f_2(t) = \frac{\alpha \times t \times e^{\alpha t}}{e^{\alpha t} - 1}, \quad 0 < t < \mu \]  
(32)

When the preventive maintenance (PM) is introduced, failures due to wearouts are eliminated and thus the machinery fails only due to random causes, which are exponentially distributed as given by \( f_1(t) \). Sampling for the time to failures in simulations is thus based on exponential distribution with mean \( \mu \) and a constant failure rate of \( \alpha = 1/\mu \). In case of CM without PM, in addition to the random failures, wear-out failures are also present and thus the time between equipment failures is uniformly distributed between 0 and \( \mu \) as given by \( f(t) \).

These derivations show that, total time between failures, \( f(t) \) can be separated into two distributions, time between failures due to random causes \( f_1(t) \) and time between failures due to wear-outs \( f_2(t) \). Since the failures due to random causes could not be eliminated, we must concentrate on the failures due to wear-outs in order to eliminate them by appropriate maintenance policies. By the procedure described above, it is possible to separate the two types of failures and develop the best maintenance policy to eliminate the wear-out failures. This separation is analytically possible for uniform distribution. However, it is not possible analytically for other distributions. It is assumed that when a preventive maintenance policy is implemented, failures due to wear-outs are eliminated and only failures due to random causes remain. These random failures are assumed to follow exponential distribution with constant hazard rate since they are completely random with unknown causes and the memoryless property of exponential is applicable.

For uniformly distributed time between failures, \( t \) in the interval \( 0 < t < \mu \), probability distribution function of time between failures without introduction of PM is given by:
\[ f(t) = 1/\mu. \]  
(28)

If we let \( \alpha = 1/\mu \), then, reliability is given as \( 1-\alpha t \) and the total failure rate is given as:
\[ h(t) = f(t)/R(t) = \alpha/(1-\alpha t). \]  
(29)

Let us assume that hazard rate due to random failures is a constant given by \( h_1(t) = \alpha \), then the hazard rate due to wear-out failures could be determined by:
\[ h_2(t) = h(t) - h_1(t) = (\alpha/(1-\alpha t) - \alpha) \times 1/(1-\alpha t) \]  
(30)

This concept has been explained in detail by Savsar[23].

In the simulation experiments considered, time to failure is assumed uniformly distributed between 0 and 200 time units with a mean of 100 time units for all stations for the case of CM only. In the case of PM, wearouts are eliminated and time to failure extends; it becomes
exponentially distributed with a mean of 200 time units. Time to repair was assumed normally distributed with mean of 15 time units and standard deviation of 3 time units.

Fig. 3 illustrates simulation results for the case when CV=0.0 for process time (CV_{pt}) and the demand (CV_{dm}). As it is seen in fig. 3, 800 units (2400/3.0) are produced on any length of line if the line is fully reliable and there is no other source of variability. However, if the line is under failures with CM only, production rate is significantly reduced when line length is increased. When PM is introduced in addition to CM, production rate is between the CM and REL cases. It can be seen from figure 3 that between the cases of unreliable lines, the lowest production rate is for a 9-station line with CM only, while the highest rate is for a 3-station line with CM and PM together.

Fig. 4 shows the results for CV_{pt}=0.0 and CV_{dm}=2.1; fig. 5 shows the results for CV_{pt}=2.1 and CV_{dm}=0.0; fig. 6 shows the results for CV_{pt}=2.1 and CV_{dm}=2.1. As it can be seen from figs. 3-6, as the process time and demand variability increase, production rate decreases. It is also clear from figs. 5 and 6 that as the process time becomes variable, the production rate can no longer reach to the maximum level of 800 units even for the reliable line. In all cases however, as the line length increases and the buffer capacities decrease, production rate decrease. Also, in all cases CM only results in lower production rate.

VII. EXPERIMENTAL DESIGN
In order to see significance of the effects of significant factors on line production rate, a general factorial design was set up with five factors each at three levels. Thus, line lengths of 3, 5, and 7; buffer capacities of 0, 2, and 6; process time CV of 0, 0.2, and 0.7; demand CV of 0, 0.2, and 0.7; and maintenance policies of REL, CM, and CM-PM cases were
Fig. 6 Line production rate under different factors (CVₚ=2.1; CVₐₙ=2.1)

![Graph of production rate vs. buffers for different factors]

Considered. The ANOVA results shown in Table IV indicate that following factors are significant: A: Line length; B: Buffers; D: Demand CV; E: Maintenance policy; and three interactions AE, BD, and DE. 97.56% of the variation is explained by these significant terms. It is interesting that process time variation was not a significant factor for this model. In the general factorial design model, the factors are considered as qualitative and therefore the model is hierarchical. The production rate is given as function of significant factors and their interactions.

**Fig. 7, the normal probability plot for the residuals shows that the normality assumption is valid.**

![Normal probability plot of residuals]

**Table IV: ANOVA for Selected Factorial Model Response: Production Rate**

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
<th>Prob. &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>4.3E+6</td>
<td>20</td>
<td>2.1E+5</td>
<td>443.5</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>A: Line length</td>
<td>4.2E+5</td>
<td>2</td>
<td>2.1E+4</td>
<td>544.0</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>B: Buffers</td>
<td>7.9E+5</td>
<td>2</td>
<td>3.95E+4</td>
<td>822.2</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>D: Demand CV</td>
<td>8.4E+5</td>
<td>2</td>
<td>4.2E+3</td>
<td>870.26</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>E: Maintenance</td>
<td>1.94E+6</td>
<td>2</td>
<td>9.7E+2</td>
<td>2020.21</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>AE</td>
<td>6941.89</td>
<td>4</td>
<td>17354.7</td>
<td>36.09</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>BD</td>
<td>113.15</td>
<td>4</td>
<td>28248.6</td>
<td>58.75</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>DE</td>
<td>88468.0</td>
<td>4</td>
<td>22117.0</td>
<td>46.00</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>Residual</td>
<td>1.07E+5</td>
<td>222</td>
<td>480.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Tot.</td>
<td>4.4E+6</td>
<td>Total DF: 242</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The Model F-value of 443.5 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise. Values of "Prob > F" less than 0.05 indicate model terms are significant. In this case A, B, D, E, AE, BD, DE are significant model terms.

Values greater than 0.1 indicate the model terms are not significant.

Other ANOVA related statistics are as follows:

- Std. Dev.:21.93; R-Squared=0.9756; Mean=591.67; Adj R-Squared=0.9734; C.V.:3.71; Pred R-Squared=0.9707; PRESS=1.279E+5; Adeq Precision=91.49.

The "Pred-R-Squared" of 0.9707 is in reasonable agreement with the "Adj R-Squared" of 0.9734. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. The ratio of 91.49 indicates an adequate signal. Final equation, which relates the production rate to the coded values of the significant factors, is given as follows:

\[
\]

**VII. CONCLUDING REMARKS**

This paper has presented an iterative mathematical model and a computer simulation procedure for a multi-stage production flow line operated according to demand at the last station, while using a push system at the intermediate stations. Based on the discrete mathematical model, simulation process incorporates a three-stage procedure which allows the user to enter a set of data describing the system under study, simulate the system iteratively until selected statistical criteria are satisfied, obtain the output, and apply specific recommendations for productivity improvement until satisfied production output is achieved. The simulation model is very useful in estimating production line productivity for realistic systems. It allows the line designer or managers to evaluate effects of storage capacity and repair/maintenance policies on productivity of a system.

The model was utilized to see the optimum allocations of storage unit capacities along the line if the equipment were subject to random failures. If all the equipment had similar
failure rates, it was observed that the optimum allocation of buffer storages followed a bowl shape, meaning that more buffer spaces were allocated to the center stations. If only one station was subject to failures, most of the buffers were allocated to the final storage to achieve maximum production output irrespective to the location of the unreliable station being either at the start, at the middle, or at the end of the line. As a future study, the suggested iterative model can be incorporated into interactive computer software to be effectively utilized by engineers and managers.

Simulation model was utilized to investigate the effects of line configurations, maintenance policies, buffer capacities, process time variability, and demand variability on production rate of the line. A factorial design was set up to investigate the significant factors that affect the production rate. It was found that line length, buffer capacities, maintenance policies, and demand variability had significant effects on production rate.

REFERENCES

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