Numerical Investigation on Latent Heat Storage Unit of Different Configurations
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Abstract—The storage of thermal energy as a latent heat of phase change material (PCM) has created considerable interest among researchers in recent times. Here, an attempt is made to carry out numerical investigations to analyze the performance of latent heat storage units (LHSU) employing phase change material. The mathematical model developed is based on an enthalpy formulation. Freezing time of PCM packed in three different shaped containers viz. rectangular, cylindrical and cylindrical shell is compared. The model is validated with the results available in the literature. Results show that for the same mass of PCM and surface area of heat transfer, cylindrical shell container takes the least time for freezing the PCM and this geometric effect is more pronounced with an increase in the thickness of the shell than that of length of the shell.

Keywords—Enthalpy Formulation, Latent heat storage unit (LHSU), Numerical Model, Phase change material (PCM)

I. INTRODUCTION

Storage of energy in suitable form and its conversion to the required form is a present day challenge to the technologists. In applications like solar thermal systems or waste heat recovery, an appropriate thermal storage system is essential. Latent heat storage unit (LHSU) using the phase change materials (PCMs) is highly attractive option in such applications. This is because of their inherent high storage capacity and their nearly isothermal performance during the charging and discharging processes. Practical difficulties associated with such latent heat storage units are the low thermal conductivity of the available phase change materials. An enhancement in thermal performance would make LHSU an economically viable proposition and would encourage wide spread use of such storage systems in applications involving solar appliances, waste heat recovery, heat pump system, conditioning of buildings such as ice storage, cooking of food and milk powders etc.

A detailed review of different low temperature latent heat storage materials has been carried out by Abhat [1], Hasnain [2], Zalba et al. [3] and Mahkamav and Murat [4]. Thermal analysis of phase change process in PCMs is a moving boundary problem. As moving boundary is non-linear in nature, the behavior of a phase change system is difficult to predict. Theoretical and numerical model for the transient thermal performance characteristics of a latent heat storage unit was developed by Voller and Cross [5]-[6], Lacroix [7], Esen et al. [8], Ismail et al. [9], Zivkovic and Fujii [10] and Adine and Qarnia [11].

Lacroix [7] developed a theoretical model to predict the transient thermal performance characteristics of a latent heat storage unit. It was concluded that for a given type of PCM, the thermal and geometric parameters must be selected carefully in order to optimize the performance of the storage unit. Esen et al. [8] developed a theoretical model for predicting the heat transfer between the HTF and the PCM of solar aided cylindrical latent heat storage tanks. It was noticed that calcium chloride hexahydrate (CCHH) stores heat energy much faster than the other PCM’s used. Ismail and Alves [9] numerically analyzed the shell and tube latent heat storage system assuming the process of solidification being mainly dominated by radial conduction. The energy equation for the PCM was solved employing the control volume method. Zivkovic and Fujii [10] numerically analyzed the isothermal phase change of PCM encapsulated in rectangular and cylindrical containers. A mathematical model based on a slightly modified enthalpy method, which enables decoupling of the temperature and liquid fraction fields, were obtained. The major emphasis of their work was to obtain a comparison between the melting time for rectangular and cylindrical containers of same volume and same heat transfer area between the heat transfer fluid (HTF) and the container wall. Saman et al. [12] numerically studied the transient performance of a thermal storage unit consisting of several layers of rectangular containers filled with PCM and air as the HTF flowing through the spaces in between the PCM layers.

It is concluded from the literature that for the maximum efficiency of LHSU, the design of the container containing PCM is very crucial. To determine the size and the shape of the container, the melting/freezing time of the encapsulated PCM is one of the essential parameters. In light of this, the present investigation deals with establishing the influence of the container dimensions and shape on freezing time of LHSU units. This work is quite similar to Zivkovic and Fujii [10] who compare the performance of LHSU having rectangular and cylindrical containers only. In this work, freezing time of PCM packed in three different shaped containers viz. rectangular, cylindrical and cylindrical shell is compared.

II. MATHEMATICAL MODEL

Fig. 1 illustrates the three different configurations considered for this investigation. The rectangular, cylindrical
and cylindrical shell containers are initially filled with PCM in liquid phase at an initial temperature, $T_0$, which is lower than its freezing temperature, $T_m$. At time $t = 0$, the lateral surface of the rectangular and cylindrical containers are exposed to a finite temperature, $T_\infty < T_0$.

On the other hand, for the cylindrical shell container the lateral surface of the shell is well insulated and a finite temperature $T_\infty$ is maintained at the tube side. Because of the temperature difference at the exposed surface of the containers, heat is transferred from the exposed surface of the containers, to the PCM, lowering its temperature due to sensible heat storage. Once the temperature of the PCM anywhere in the container reaches $T_m$, phase change takes place.

![Fig. 1 Physical models](image)

The mathematical model for phase change of the encapsulated PCM is derived under the following assumptions.

- The PCM is homogeneous and isotropic.
- The thermo-physical properties of the PCM are considered to be independent of temperature.
- Thermal resistance across the walls of the container is considered to be negligibly small and the heat loss from the container to its surroundings is neglected.
- The heat transfer process in the PCM is assumed to be dominated by conduction.
- The heat conduction in directions other than normal to the HTF flow is considered to be negligible.

The governing equations are reformulated in terms of the enthalpy, $H$ (i.e. the sum of the sensible and latent heats) which eliminates problem to trace the position of the moving boundary. The numerical problems associated with the discontinuity of the temperature gradient are also avoided.

The conservation of energy for the conduction dominated phase change in the PCM earlier can be expressed in terms of total enthalpy and temperature as:

$$\rho \frac{\partial H}{\partial t} = \nabla \cdot (k \nabla T) \tag{1}$$

where the conductivity ($k$) and density ($\rho$) are independent of temperature, ($T$).

The temperature and enthalpy are related via (2)-(3),

$$H(T) = \begin{cases} 
C_p T & \text{for } T < T_m \\
C_p T + L & \text{for } T \geq T_m
\end{cases} \tag{2}$$

Equivalently equation (2) can be written as

$$H(T) = \begin{cases} 
C_p T & \text{if } T < T_m \\
C_p T + L & \text{if } T \geq T_m
\end{cases} \tag{3}$$

where $T_m$ is the melting temperature of the phase change material. For the problem of isothermal phase change, the local fraction ($f$) is defined as:

- $f(T) = 1$ if $T > T_m$
- $f(T) = 0$ if $T < T_m$ \hspace{1cm} \tag{4}

Considering the geometry of the containers shown in fig. 1, the initial and boundary conditions associated with the phase change problem can be written as follows:

(i) For rectangular container

$$T(x=0) = T_m \quad \text{at } x = 0 \quad \frac{\partial T}{\partial x} = 0 \quad \text{at } x = w \tag{5}$$

(ii) For cylinder

$$T(r=r_o) = T_m \quad \text{at } r = r_o \quad \frac{\partial T}{\partial r} = 0 \quad \text{at } r = 0 \tag{6}$$

(iii) For cylindrical shell

$$T(r=r_i) = T_m \quad \text{at } r = r_i \quad \frac{\partial T}{\partial r} = 0 \quad \text{at } r = r_o \tag{7}$$

III. NUMERICAL SOLUTION

The governing equations are discretised using finite volume method. The region of interest is subdivided into a set of discrete volumes such that values of variables at the node are representative of the values in the volume. After selection of space and time steps and appropriate treatment of boundary conditions, the numerical solution is marched in time. At each time step, the governing equation is explicitly solved for the nodal enthalpy field, $H$. Calculations in a time step are completed on updating the nodal temperature field ($T$), via (2). In addition the nodal liquid fraction is calculated from...
following expression.

\[
0; \quad T_p < T_m
\]

\[
f = \left( \frac{H_x - c T_m}{L} \right); \quad T_m = 0
\]

\[
1; \quad T_p > T_m
\]

The solution is then proceeds to the next time step.

IV. VALIDATION

The temperature distribution and the freezing time for the PCM encapsulated in containers are obtained by the computer program developed in MATLAB 6.5.1. The code is validated with results available in literature for rectangular as well as cylindrical geometry.

A. Validation for Rectangular Container

Voller [5] carried out an experiment using a pure liquid initially at 2°C occupying a semi-infinite space \( x \geq 0 \). At time \( t = 0 \) the surface at \( x = 0 \) is fixed at the temperature of \(-10^\circ\text{C}\), which is below the freezing point of the substance \( T_m = 0^\circ\text{C}\). The results of present simulations are compared with experimental results obtained by Voller [5]. The position of the phase front after 25 days as obtained by the present calculation is \( x = 0.8486 \text{m} \) as compared to a value of \( x = 0.8415 \text{m} \) obtained by Voller [5].

B. Validation for Cylindrical Container

Voller [6] suggested approximate dimensionless solidification time \( t^* \) for the circular cylinder by the following expression, viz.

\[
t^* = \left( 0.14 + 0.085 T_m^* \right) + \left( 0.252 + 0.0025 T_m^* \right) L^* \tag{9}
\]

Where,

\[
t^* = \frac{kt}{\pi C_p r}; \quad r^* = \frac{r}{R}; \quad L^* = \frac{L}{C_p (T_m - T_0)}
\]

\[
H^* = \frac{H}{C_p (T_m - T_0)} \quad T^* = \frac{T - T_0}{T_m - T_0}
\]

The total solidification time for the cylindrical container obtained by the present model is 5268s as compared to the value of 5572s obtained by (9). This deviation in the solidification time can be attributed to the approximations made by Voller [6] for obtaining (9).

V. RESULTS AND DISCUSSION

A. Comparison of Freezing Time of PCM for Different Shape of LHSU Unit

For the comparison of the freezing time of the PCM in different configurations, the PCM encapsulated in the containers is chosen to be water with initial temperature of \( T_0 = 2^\circ\text{C} \). The wall temperature is kept constant at \( T_{\infty} = -10^\circ\text{C} \). At time \( t = 0 \), the lateral surface of the rectangular and cylindrical containers are exposed to a finite temperature, \( T_\infty < T_0 \). The lateral surface of the cylindrical shell container is well insulated and a finite temperature \( T_\infty \) is maintained at the tube side. In order to compare the freezing time of PCM with different shape of LHSU, the dimensions of the containers were chosen in such a manner that the volume as well as the heat transfer area for all the rectangular, cylindrical and cylindrical shell containers is equal. The length of the geometrically different containers is assumed to be equal. The width of the rectangular container is kept \( \pi \) time radius of cylinder and other relevant dimensions of the containers are computed on the basis of equal volume of the PCM encapsulated and equal surface area of heat transfer.

Fig. 2 shows the variation of the PCM temperature with time in the rectangular and cylindrical containers for the different dimensions of the containers. Analysis is carried out for three different thickness of the rectangular container viz. 20mm, 40mm and 60mm. Correspondingly these values represent the radius of the cylindrical container in terms of equivalent volume and surface area of heat transfer. Fig. 2 shows considerably lesser freezing time for the cylindrical container as compared to the rectangular container packed with the same mass of PCM and having equal surface area of heat transfer. It can also be noticed from this figure that the difference in freezing time of PCM between the rectangular and the cylindrical containers increases with increase in the size of the containers. Fig. 3 shows the variation of liquid fraction at the centers of both the rectangular and cylindrical containers for the different dimensions of the containers. It can be noted that the difference in the total freezing time of the PCM between the rectangular and cylindrical containers becomes almost triple for a two fold increase in their volume. A three fold increase in the volume of the containers results in almost six times decrease in total freezing time of the PCM packed in the cylindrical container as compared to that of the rectangular container.

Fig. 4 illustrates a comparison of the temperature time history in general and freezing time in particular at the center of the PCM packed in cylinder of three different sizes with those of cylindrical shell containers. The radius of the cylindrical containers for this comparison are chosen as 20mm, 40mm and 60mm, the equivalent thickness of the cylindrical shell containers are calculated as 8.28mm, 16.56mm and 24.58mm respectively, on the basis of equivalent volume and surface area of heat transfer. It is evident from fig. 4 that freezing time of the PCM corresponding to all three cylindrical shell containers are considerably less than those of the cylindrical containers having equivalent volume and surface area of heat transfer. However, it is important to emphasize here that the difference in freezing time of the PCM corresponding to cylindrical and cylindrical shell containers increases with increase in their sizes. The same results can also be reflected from the fig. 5 which shows the variation of liquid fraction at the centers of both the cylindrical and cylindrical shell for the different dimensions of the containers.

It can be noted from fig. 5 that a two fold increase in the volume of the cylinder and cylindrical shell containers results in almost five time increase in the difference in freezing time of the PCM, while approximately ten times increase in the difference in freezing time of the PCM is observed for three
fold increase in the volume of the two containers.

On the basis of the detailed discussion of these results, it is concluded that for the same mass of the PCM i.e., same volume of the container and surface area of heat transfer, cylindrical shell containers take the least time for freezing process. It is also concluded that this geometric effect is more pronounced with an increase in the mass of the PCM or the volume of the container.

**B. Two Dimensional Temperature Distribution in Cylindrical Shell Container**

The variation of temperature distribution of PCM in cylindrical shell and the movement of the phase front in radial as well as in axial direction is also established with respect to time. The boundary conditions considered for the two dimensional phase change problem in cylindrical shell are:

\[
T = T_0 \quad \text{at} \quad r = r_e \quad \frac{\partial T}{\partial r} = 0 \quad \text{at} \quad r = r_i \quad \frac{\partial T}{\partial x} = 0 \quad \text{at} \quad x = L \quad (11)
\]

Fig. 6 illustrates the movement of the phase front with time in cylindrical shell of inner radius \(r_i = 0.06\)m, outer radius \(r_o = 0.085\)m. The length of the shell is \(L = 0.5\)m. As expected, the position of the phase front increases with time. It is important to notice that the rate of freezing decreases with time.

**C. Parametric Analysis for Cylindrical Shell Container**

As it is concluded that cylindrical shell takes least time for freezing than other mentioned containers, a parametric analysis for establishing the optimum performance of LHSU is carried out for the cylindrical shell container. The effect of heat source given at lateral surface of cylindrical shell and the same at tube side on freezing time required and the temperature distribution in the PCM is established by considering two different boundary conditions as follows:

(a) The lateral surface of the cylindrical shell container is well insulated, while a finite temperature \(T_{\infty}\) is maintained at the tube.

(b) The tube surface of the cylindrical shell container is well insulated, while a finite temperature \(T_{\infty}\) is maintained at
outer surface of the cylindrical shell.

Fig. 6 Movement of the phase front in cylindrical shell with time

Fig. 7 illustrates a comparison of the temperature time history in general and freezing time in particular at the center of the PCM packed in cylindrical shell for both the type of boundary conditions. It is observed from the fig. 7 that freezing time of the PCM corresponding to case (a) is considerably less than that of case (b). It can also be concluded that a cylindrical shell configuration of latent heat storage unit stores/releases much more energy in a given time when a finite temperature is applied at the inner radius of the shell as compared to the configuration where finite temperature is applied at outer side of the shell.

Fig. 8 shows the temperature-time history of the PCM encapsulated in a cylindrical shell container for the case when the constant wall temperature is applied at the inner radius of the shell. It is observed from the figure 8 that as the temperature at the wall decreases, the total freezing time of the PCM decreases considerably.

The influence of geometric parameters of cylindrical shell on freezing process is also investigated. Keeping the volume of the PCM same, the inner radius of the shell is varied. As the inner radius of the shell is increased \((r_i = 40, 50\, \text{and}\, 60\, \text{mm})\), the annulus thickness of the shell is reduced for the same mass of the PCM. Fig. 9 shows the movement of the phase front in the cylindrical shell for different annular thickness at a fixed time of 2000 s. From the figure, it is observed that lesser the annulus thickness, quicker is the heat transfer rate. This is only valid for the constant temperature wall condition applied to the inner radius for the present simulations. However, if freezing of the PCM takes place using the cold fluid passing through the shell, the temperature potential at the inner surface of the shell will be reduced due to increase in inner radius and more mass flow through the tube. Hence for efficient utilization of energy storage unit, an optimized inner radius of the shell should be chosen for the same mass of the PCM.
VI. CONCLUDING REMARKS

Complete freezing of the PCM is a necessary condition for long-term (seasonal) thermal energy storage. In light of this, the effect of the container dimensions and shape on freezing time of PCM is numerically investigated. On the basis of the detailed discussion of the results, it is concluded that freezing time of the PCM corresponding to cylindrical shell containers are considerably less as compared to that of the cylindrical and rectangular containers having equivalent volume and surface area of heat transfer. The effect is pronounced as the mass of PCM is increased. The freezing time of the PCM, corresponding to the case when the constant wall temperature is at inner radius of the shell, is considerably less than that when constant wall temperature is applied at outer radius. The effect of constant wall temperature on the freezing time of PCM packed in cylindrical shell container is also analyzed. As the temperature at the wall decreases, the total freezing time of the PCM decreases considerably. It can be summarized from the present investigation that the operating and geometric parameters must be chosen carefully in order to obtain optimum thermal performance of the latent heat storage unit.

REFERENCES