Creating Streamtubes Based on Mass Conservative Streamlines

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Abstract—Streamtube is used to visualize expansion, contraction and various properties of the fluid flow. These are useful in fluid mechanics, engineering and geophysics. The streamtube constructed in this paper only reveals the flow expansion rate along streamline. Based on the mass conservative streamline, we will show how to construct the streamtube.

Keywords—Flow visualization, mass conservative, streamline, streamtube.

I. INTRODUCTION

STREAMLINES, streamtube and streamribbon are three of the most fundamental techniques for visualizing steady flow fields. Streamlines are paths of massless particles that are released in a steady flow. Streamtube and streamribbons show the expansion and rotation of the flow.

Formally a streamtube is defined as the surface formed by all streamlines passing through a given closed curve in the flow [1]. Such a streamtube can show expansion, contraction and deformation of flow fields. These are useful in fluid mechanics, engineering and geophysics. This paper follows the streamtube construction in [2, 3] where a streamtube is created by connecting the circular cross flow sections along a streamline. The radius of a cross flow section is determined by the local cross flow expansion rate. The process of creating a streamtube is: (1) generating a streamline; (2) connecting the circular cross flow sections along the streamline.

The accuracy of created streamtube is dependent on both the accuracies of the streamline and the circular cross flow sections. Li [4] introduced an adaptive streamline tracking method for three-dimensional CFD velocity fields based on the law of mass conservation. This method suits to the CFD velocity fields and various properties of the fluid flow. These are useful in fluid mechanics, engineering and geophysics. The streamtube constructed in this paper only reveals the flow expansion rate along streamline. Based on the mass conservative streamline, we will show how to construct the streamtube.

The conditions (CSFD) for seeking more data of the velocity fields in a hexahedron are: for a hexahedron, subdividing it into five tetrahedra, and then calculating the Jacobean forms of \( f \) for different Jacobean forms of matrix \( A \) and then describe the conditions about when more data of the velocity fields are needed in a hexahedron. The expressions of \( f \) for all possible cases of a non-conservative 3D linear field are listed in Table I in [4].

A. Conditions for Seeking More Data in a Hexahedron

The conditions (CSFD) for seeking more data of the velocity fields in a hexahedron are: for a hexahedron, subdividing it into five tetrahedra, and then calculating the Jacobean forms of \( A \) in the linear interpolation of values of the velocity field and the coordinates at the vertices of each of the five tetrahedra respectively, if there exist at least one of the five expressions of \( f \) corresponding to the particular Jacobean of \( A \) in Table I in [4] equaling zero or infinity, after taking \( C \) as non-zero constant, at some points on the corresponding tetrahedra, further data are needed to be found inside the hexahedron. The locations of the more data are the...
We will create a streamline by generating mass flow sections along the streamline [1]. We are able to track the streamline by the method described in last section. Thus the radius of a streamline is governed by the following ordinary differential equation:

\[
\frac{dr}{d\xi} \equiv \frac{1}{\sqrt{1 + \left( \frac{du}{d\xi} \right)^2}}
\]

where \( V \) is the local cross-flow divergence and is defined as

\[
\nabla \cdot V = \nabla \cdot (u_A X B)
\]

Theoretical streamlines can be created by drawing circles with radii in (3) and centre at the points on the streamlines constructed by the method in [4]. A physical streamtube can be created by drawing cylinders with the two ends of circles at two instants with the radii in (3) and centre at the points on the streamlines constructed by the method in [4]. A physical streamtube can be created by drawing cylinders with the two ends of circles at two instants with the radii in (3) and centre at the points on the streamlines constructed by the method in [4].

III. RADIUS OF STREAMTUBE

The change of velocity magnitude along the streamline is as follows.

\[
\frac{du}{d\xi} = \frac{1}{r} \left( \frac{du}{d\xi} \right)
\]

The magnitude of the velocity field represents the change of the velocity magnitude along the streamline. For the velocity field given in Section 2, the formula for the radius of streamtube is derived in this section. The values of the velocity field at the vertices of the eight smaller hexahedra by connecting the midpoints of opposite sides on each of the six faces (see Fig. 3). We are able to track the vector field that satisfies the law of mass conservation subject to the preset tolerance, i.e., even though the linear vector field is very accurate, the more accurate the calculation for the change of velocity magnitude gives the change of velocity magnitude along the streamline.

\[
\frac{d}{dt} (u_A + v + w) = \nabla \cdot V
\]

The change of velocity magnitude along the streamline is as follows.

\[
T = \frac{1}{\sqrt{1 + \left( \frac{du}{d\xi} \right)^2}}
\]

where \( T \) is the threshold number. The bigger the threshold number is, the more accurate the streamline. Some of the figures are not high quality due to the current facilities available. The examples in [4] include closed streamlines and streamlines that are very accurate as shown by the examples in [4]. The streamlines constructed by the method reviewed above are very accurate as shown by the examples in [4]. The accuracy is shown by comparing the tracked streamlines with the exact streamlines. The accuracy is shown by comparing the tracked streamlines with the exact streamlines.

1. Set \( r = 0 \).
2. Find the seed point and go to Step 4.
3. Subtract hexahedron in (2) and go to Step 2.
4. Take the elements (smaller hexahedra) in the subdivided hexahedron as new elements of the mesh by replacing the boundaries of the subdivided hexahedron with the vertices of the smaller hexahedra and then let \( r = r + 1 \).
tracked streamline in red shown in Fig. 1 is for the threshold numbers $T=6$.

**Example 1** Saddle-spiral flow

$$\mathbf{V} = (xz - y, yz + x, -z^2)$$

with seed point (-0.8, 0.8, 1).

Fig. 1 shows the streamtube in three dimensions. Fig. 2 shows the projection of the streamtube in Fig. 1 on $yz$-plane. Fig. 3 shows the projection of the streamtube in Fig. 1 on $xy$-plane. The variation of the expansion rate for the saddle-spiral flow in Example 1 is very small in the time period shown in the figures.

**Example 2** Toroidal flow velocity field

$$\mathbf{V} = \left(\frac{2x(z-1)}{r^2} - \frac{2y}{5r}, \frac{2y(z-1)}{r^2} + \frac{2x}{5r}, \frac{2(r-9)}{r}\right)$$

with seed point (6.0006, 6.7076, 1), where $r = \sqrt{x^2 + y^2}$.

Fig. 5 shows the exact streamline in blue and tracked streamline in red used the method in [4]. This figure indicates that the tracked streamline is very accurate comparing with the exact streamline. When we drew the streamtube for this example, we were not allowed to draw circles like Example 1 due to the limit of the facilities available. We drew two dots instead of one circle.

Fig. 6 shows the streamtube in three-dimensions or more precisely, cross-section of the streamtube. The expansion rate for this example varies significantly from Fig. 6, and the projections of Fig. 6 on $yz$-plane in Fig. 7, on $xy$-plane in Fig. 8, and on $xz$-plane in Fig. 9.
VI. DISCUSSION

This paper has introduced a streamtube construction by connecting the circular cross flow sections along a streamline. The streamtube created can show the local cross flow expansion rate. We still have some of following issues relating to the appearance of the streamtubes.

1. We used MATLAB in drawing the streamtubes. The streamtubes are not really tubes. We may need to write a function that draws a tube when the circles at the two ends are given.
2. The colors in the streamtubes may indicate the expansion rate.
3. Reduce the usage of the visual memory. For the computer we are using, the information of "low visual memory" was shown when we drew the figures. These issues will be our future research topics.
REFERENCES


