Abstract—In this paper, we show that the association of the PI regulators for the speed and stator currents with a control strategy using the linearization by state feedback for an induction machine without speed sensor, and with an adaptation of the rotor resistance. The rotor speed is estimated by using the model reference adaptive system approach (MRAS). This method consists of using two models: The first is the reference model and the second is an adjustable one in which two components of the stator flux, obtained from the measurement of the currents and stator voltages are estimated. The estimated rotor speed is then obtained by canceling the difference between stator-flux of the reference model and those of the adjustable one. Satisfactory results of simulation are obtained and discussed in this paper to highlight the proposed approach.

Keywords—Asynchronous actuator, PI Regulator, adaptive method with reference model, Vector control.

I. INTRODUCTION

With the advent of automated production, the control became the operation consisting in creating a strong dependency between an articulated system and its command. As usual, asynchronous actuators influence the articular variables of a robot; the position control is realized through a suitable action on their command.

The vector control allows having a decoupling between torque and flux of the machine and consequently dynamic performances similar to those of a DC machine are achieved [1], [2]. Its implementation based on a linear approximation of the machine dynamics around the operation point does not allow having a good compensation of the machine non-linear model, in particular, for the disturbed and the transitory modes. In order to solve some specific problems related to vector control and ensure a decoupling whatever the machine operation mode, the usage of nonlinear commands become unavoidable.

Among these nonlinear techniques, we can quote the control using the linearization strategy by state feedback. However, from literature studies, PI regulators strongly depend on the machine parameters in particular the rotor time-constant (\(T_r\)). A bad identification of these parameters or a bad estimation of the variables to be controlled will lead to a deterioration of the adjustment performances especially when the machine is loaded. Moreover, the presence of mechanical sensor increases the volume and overall system cost. In addition, the installation of this sensor is difficult and reduces the reliability of the system especially for small machines.

Considering this major inconvenience, a solution is obtained by the association of the PI regulators for the speed and stator currents with a control strategy using the linearization by state feedback for an induction machine without speed sensor, and with an adaptation of the rotor resistance [3], [4], [5], [6]. The rotor speed is estimated by using the model reference adaptive system approach (MRAS). This method consists of using two models. The first one is the reference model and the second is an adjustable one in which two components of the stator flux, obtained from the measurement of the currents and stator voltages are estimated [7] [8].

The estimated rotor speed is then obtained by canceling the difference between stator-flux of the model of reference and those of the adjustable model. The adaptation mechanism is obtained from the hyperstability theory. Satisfactory results of simulation are presented at the end of this paper to highlight the proposed approach.

II. PRINCIPLE OF THE LINEARIZATION BY STATE FEEDBACK

The control of the system is governed by the following state equation: [3]

\[
\frac{dx}{dt} = f(x) + g(x) u,
\]

Where:
- \(f(x)\) and \(g(x)\) are nonlinear functions of the state variable \(x\),
- \(u\) represents the command vector applicable to the system.

Each output \(x_i\) is derived several times until one of the components of the control vector appears. Thus, the following equation is obtained:

\[
\frac{d^n x_1}{dt^n},
\]

\[
\frac{d^n x_2}{dt^n} = B(x) + A(x) u,
\]

\[
\frac{d^n x_n}{dt^n}.
\]
If the matrix $A(x)$ is invertible, it becomes possible to adopt any dynamic behaviour on the outputs of the system:

$$\frac{dx_1}{dt} = v_1,$$
$$\frac{dx_2}{dt} = v_2,$$
$$\frac{dx_n}{dt} = v_n. \quad (3)$$

By giving the following command:

$$u = A^{-1}(x) [v - B(x)]. \quad (4)$$

The flowchart of figure (1) is obtained by the linearization by state feedback, which transforms the original system into a linear and decoupled equivalent system with the condition that the decoupling matrix is non-singular.

A. Application to the Asynchronous Actuator

The equations of the electric part of the asynchronous actuator can be written in the Park $d$-$q$ configuration as follows:

$$\frac{di_{ds}}{dt} = \frac{1}{\sigma L_s} (V_{ds} - R_s i_{ds} - M \frac{d\psi_{dr}}{dt}) + \omega i_{qs}, \quad (5.a)$$
$$\frac{di_{qs}}{dt} = \frac{1}{\sigma L_s} (V_{qs} - R_s i_{qs} - \frac{M \omega}{L_r} \psi_{dr} - \omega i_{ds}), \quad (5.b)$$
$$\frac{d\psi_{dr}}{dt} = \frac{R_r}{L_r} (M i_{ds} - \psi_{dr}). \quad (5.c)$$

The position of the reference frame is selected in command to obtain $\psi_{qr} = 0$, rotor flux is then entirely represented by $\psi_{dr}$. By introducing the equation (5.c) in (5.a) we obtain the following equations:

$$\sigma L_s \frac{di_{ds}}{dt} + R_s i_{ds} = V_{ds} + \sigma L_s \omega i_{qs} - \frac{L_s (1-\sigma)}{L_r} (i_{ds} \frac{\omega}{M} \phi_r)$$
$$\sigma L_s \frac{di_{qs}}{dt} + R_s i_{qs} = V_{qs} - \omega (\sigma L_s i_{ds} + \frac{L_s (1-\sigma)}{M} \phi_r) \quad (6)$$

The equilibrium equation between the actuator torque and the load torque generated by the mechanical part of the system has the following form:

$$J \frac{d\omega_m}{dt} = C_{em} - f \omega_m - C_r \quad (7)$$
$$\omega_m = \omega_r / p$$

Where:

- $i_{ds}$ and $i_{qs}$, $v_{ds}$ and $v_{qs}$ are respectively the $d$ and $q$ axes stator currents and voltages;
- $\psi_{dr}$ is the rotor flux;
- $\omega$ is the angular speed of the $d$-$q$ reference frame with respect to the actuator stator and $\theta$ its position;
- $\omega_r$ is the difference between the angular speed of the reference frame and the electrical speed $\omega_r$ of the rotor (the electrical speed of the rotor is equal to $p$ times its mechanical speed $\omega_m$, $p$ being the pole pairs number) $\omega_{qs} = \omega - \omega_r$;
- $T_s$ and $T_r$ are respectively the stator and rotor $d$ and $q$ windings time constants;
- $\sigma$ is the leakage coefficient of the machine;
- $M$ is the mutual inductance between rotor and stator $d$-$q$ windings;
- $L_r$ is the rotor $d$-$q$ windings inductance;
- $R_s$ and $R_r$ are respectively the stator and rotor $d$-$q$ windings resistances;
- $C_{em}$ is the electromagnetic torque;
- $J$ is inertia of actuator;
- $f$ is Viscous friction coefficient.

In our case we take:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix}, \quad [U] = \begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix}$$

Therefore:

$$B(x) = \begin{bmatrix} -\frac{1}{\sigma L_s} \left( (R_s + M^2 \frac{R_r}{L_r^2}) i_{ds} - \frac{MR_r}{L_r^2} \psi_{dr} \right) + \omega i_{qs} \\ -\frac{1}{\sigma L_s} \left( R_s i_{qs} + \omega M \psi_{dr} \right) - \omega i_{ds} \end{bmatrix}$$

$$A(x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Let us remind that $A(x)$ is an invertible matrix.
B. Nonlinear Law of Control

Since the system is linearisable, it is possible to impose on it the desired dynamics. In our case we considered the control law given by equation 8.

\[
\begin{align*}
V_{ds} & = A^{-1} \left[ -B(x) + \left( v_1, v_2 \right) \right] \\
V_{qs} & = A^{-1} \left[ -B(x) + \left( v_1, v_2 \right) \right]
\end{align*}
\]

Where \( v_1 \) et \( v_2 \) are the outputs of the PI regulators of the stator currents appearing in the flowchart of the asynchronous machine given by Fig. 3.

III. ADAPTATION OF THE ROTOR RESISTANCE

To estimate the resistance of the rotor we used a method which makes possible to calculate the latter in function to flux (real, reference) and of the electromagnetic torque (real, reference) [4].

To clarify the relation which binds the electromagnetic torque and rotor flux to the variations of the parameters of the machine, we let us proceed as follows:

\[ L_r = K_i L_r^* \]
\[ R_r = K_i R_r^* \]

\( L_r^*, R_r^* \) Values used in the command.

The actual value of the rotor time-constant:

\[ T_r = \frac{K_i L_r^*}{K_r} = K T_r^* \]

The block of decoupling imposes on the command of the inverter the values \( V_{ds}, V_{qs} \) and \( \omega_{sr} \). In permanent mode we have:

\[ C_e = \frac{PM}{L_r^*} \varphi_r i_{qr} = \frac{PM^2}{L_r^*} i_{ds} i_{qs} \]
\[ \omega_{sr} = \frac{1}{T_r^*} \frac{i_{qr}}{i_{ds}} \]
\[ \varphi_r = M \ i_{ds} \]

From the equations of Park of the machine we determine the direct components and in squaring from rotor flux and the real torque of the machine with permanent regime:

\[ \varphi_{dr} = M \ i_{ds} + \omega_{sr} T_r i_{qr} \]
\[ \varphi_{qr} = M \ i_{qr} - \omega_{sr} T_r i_{ds} \]

\[ C_e = \frac{PM}{L_r^*} \varphi_r i_{qr} = \frac{PM^2}{L_r^*} i_{ds} i_{qs} \]
\[ \frac{1 + \left( \frac{i_{qr}}{i_{ds}} \right)^2}{1 + \left( \frac{i_{qr}}{i_{ds}} \right)^2} \]

\[ \frac{1 + \left( \frac{i_{qr}}{i_{ds}} \right)^2}{1 + \left( \frac{i_{qr}}{i_{ds}} \right)^2} \]

The rapport of the real torque and flux on the estimated values are:

\[ \frac{\varphi_r}{\varphi_r} = \frac{1 + \left( \frac{i_{qr}}{i_{ds}} \right)^2}{1 + \left( \frac{i_{qr}}{i_{ds}} \right)^2} \]

\[ \frac{C_e}{C_r} = \frac{1 + \left( \frac{i_{qr}}{i_{ds}} \right)^2}{1 + \left( \frac{i_{qr}}{i_{ds}} \right)^2} \]

\[ K_i = \frac{C_e}{C_r} \left( \frac{\varphi_r}{\varphi_r} \right)^2 \]

Therefore, the estimation of the rotor resistance is given by following relation:

\[ R_r = K_i R_r^* \]

IV. SPEED ESTIMATION BY THE MRAS TECHNIQUE

To estimate rotor speed, it is judicious to use a reference frame related to the stator (\( \alpha, \beta \)) given by the following equations. This transformation does not call upon the position of the rotor which we estimate by the model reference method [7],[8].

\[ V_{ca} = R_s i_{ca} + \frac{d \varphi_{ca}}{dt} \]
\[ V_{cb} = R_s i_{cb} + \frac{d \varphi_{cb}}{dt} \]
\[ 0 = R_s i_{ca} + \frac{d \varphi_{ca}}{dt} + \omega_r \varphi_{cb} \]
\[ 0 = R_s i_{cb} + \frac{d \varphi_{cb}}{dt} - \omega_r \varphi_{ca} \]

While using (15), (14) rewrites itself in the following form:

\[ 0 = L_s (R_r + p \omega_r) i_{ca} + \omega_r L_r \varphi_{ca} + (R_r + p \omega_r) \varphi_{ca} + \omega_r L_r \varphi_{cb} \]
\[ 0 = L_s (R_r + p \omega_r) i_{cb} - \omega_r L_r \varphi_{ca} + (R_r + p \omega_r) \varphi_{cb} \]
Basing on the dynamic model of the asynchronous
machine, formulated in a stator reference frame, and by using
measurements of the stator currents and tensions, we build
two estimators of stator flux. The first is based on (14) and
the second on (16), such as:

\[ \hat{\varphi}_{\text{stator}} = \frac{1}{2}(V_{\text{stator}} - R_s i_{\text{stator}}) dt \]

(17)

In the system (17) relating to the stator observations, it is
easy to notice that it does not depend on the rotor speed \( \omega_r \).
This model is retained as a reference. In the rotor estimators
relating to the system (18), we notice the existence of the rotor
speed \( \omega_r \). It is the adjustable model. The system (18) can be
written in the following form:

\[
\dot{\hat{\varphi}}_{ss} = \left[ \frac{1}{T_r} - \frac{1}{T_r} \right] \hat{\omega}_{ss} + \left[ i_{ss} \right]
\]

(18)

The derivative of the components of error (20) is defined by:

\[
p \begin{bmatrix} e_{ss} \\ e_{ss} \end{bmatrix} = \begin{bmatrix} \frac{1}{T_r} - \frac{1}{T_r} \\ \frac{1}{T_r} - \frac{1}{T_r} \end{bmatrix} e_{ss} + \begin{bmatrix} \hat{\omega}_{ss} - \sigma L_s \omega_r \\ \hat{\omega}_{ss} - \sigma L_s \omega_r \end{bmatrix} + \begin{bmatrix} \hat{\varphi}_{ss} - \sigma L_s \omega_r \\ \hat{\varphi}_{ss} - \sigma L_s \omega_r \end{bmatrix} (\omega_r - \hat{\omega}_r)
\]

The derivative of the components of error (20) is defined by:

\[
p \begin{bmatrix} e_{ss} \\ e_{ss} \end{bmatrix} = \begin{bmatrix} \frac{1}{T_r} - \frac{1}{T_r} \\ \frac{1}{T_r} - \frac{1}{T_r} \end{bmatrix} e_{ss} + \begin{bmatrix} \hat{\omega}_{ss} - \sigma L_s \omega_r \\ \hat{\omega}_{ss} - \sigma L_s \omega_r \end{bmatrix} (\omega_r - \hat{\omega}_r)
\]

In the form of state

\[
p \begin{bmatrix} e_{ss} \\ e_{ss} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} e_{ss} \\ e_{ss} \end{bmatrix} + \begin{bmatrix} W \end{bmatrix}
\]

(21)

According to the general structure of the adaptive
mechanism, the speed estimation is a function of the error. It is
given by:

\[
p \dot{\hat{\omega}}_r = A_1 + P A_2
\]

(22)

Functions \( A_1 \) and \( A_2 \), are calculated starting from the
inequality of Popov. Thus, we obtain:

\[
A_1 = K_2 \left[ \hat{\varphi}_{ss} \hat{\Phi}_{ss} - \varphi_{ss} \hat{\Phi}_{ss} - (i_{ss} e_\beta - i_{ss} e_\alpha) \sigma L_s \right]
\]

(23)

\[
A_2 = K_1 \left[ \hat{\varphi}_{ss} \hat{\Phi}_{ss} - \varphi_{ss} \hat{\Phi}_{ss} - (i_{ss} e_\beta - i_{ss} e_\alpha) \sigma L_s \right]
\]

(24)

Where \( K_1 \) and \( K_2 \) positive constants are called profits of
adaptation.

V. DIGITAL SIMULATION

The flowchart of vector control using the linearization by
state feedback without speed sensor of the induction machine
fed by PWM inverter, by using PI regulators for the speed and
stator currents is given by the Fig. 3.
Fig. 3 Flowchart of control with a linearization by state feedback of the asynchronous machine

Figs. 4 (a), (b), (c), shows the dynamic responses of the adjustment using the linearization by state feedback of the asynchronous machine supplied by PWM inverter. The stator-currents are controlled by PI regulators.

These figures represent the dynamic responses of the real speed, estimated and of reference, the torque and rotor flux at a step speed of 100 rd/s followed by an application of a load equal to the nominal torque between 1 and 1.5 sec, then of an inversion speed from 2 sec in the following cases:

- Without variation of the rotor time-constant $T_r$
- Reduction of 50% of the rotor time-constant $T_r$

VI. CONCLUSION

The results obtained carry out us to conclude that the control strategy used offers good performances like its insensitivity with respect to the disturbances internal and external, such as the level speed, the load torque, in the presence of a variation of the rotor time-constant $T_r$, which attests robustness of the command used.

This association could be employed for the robotics or the numerical control of the machine tools where the parametric variations frequently occur.

APPENDIX

Machine parameters:

0.25 kW, 110 V, 1800 rpm

The parameters are:

$R_r = 1.739 \, \Omega$, $R_s = 1.923 \, \Omega$, $L_r = 0, 1154 \, H$
$M = 0, 1126 \, H$, $L_s = 0, 1157 \, H$
$J = 0.004 \, kgm^2$, $f = 2, 5 \, Nm/rd/s$
REFERENCES


