Incorporation of Long-Term Redundancy in ECG Time Domain Compression Methods through Curve Simplification and Block-Sorting

Bachir Boucheham, Youcef Ferdi, and Mohamed Chaouki Batouche

Abstract—We suggest a novel method to incorporate long-term redundancy (LTR) in signal time domain compression methods. The proposition is based on block-sorting and curve simplification. The proposition is illustrated on the ECG signal as a post-processor for the FAN method. Test applications on the new so-obtained FAN+ method using the MIT-BIH database show substantial improvement of the compression ratio-distortion behavior for a higher quality reconstructed signal.

Keywords—ECG compression, Long-term redundancy, Block-sorting, Curve Simplification.

I. INTRODUCTION

Digital signals compression is a requirement in modern applications for efficient storage and or transmission over the network. Basically, compression consists in the principle of eliminating redundancy. Ideally, we desire the smallest representation of the original data as a result of compression, i.e. a fully decorrelated form where ‘no structure in the data is discernible’ [1]. This objective is yet to be attained since all existing compression methods employ partially the existing redundancy in the data. For instance in the case of ECG signal, Jalaleddine et al [2] classify compression methods in three categories: Direct data compression methods, like AZTEC [3] and FAN [4]; transform methods, e.g. [5] and parameter extraction methods, e.g. [6]. From another point of view, signals compression can be achieved in the time domain or in the frequency domain. This study concentrates particularly on a specific class of time domain compression methods known as piecewise linear approximation (PLA). This approach is widely used in many computer areas including time series representation for analysis, compression and data mining and, although no assumption is made on the nature of the signal, we concentrate specifically on the ECG signal.

Time domain compression methods (PLA) have in common the principle of reducing redundancy through selection of a set of characteristic points (CP) on the signal trace. These points are selected based on wisely predetermined rules so as the most significant points are selected. The so-obtained set of CPs stand for the reduced form of representation of the original signal. The reconstruction process is achieved through interpolation between successive CPs in this set. This type of lossy compression is acceptable in many biological signals, including the ECG.

It can be noted that in PLA methods, the CPs are selected on the basis that all samples between two successive CPs are correlated. Therefore, in this type of compression, short-term redundancy (STR) only is considered. Yet, in biological signals (especially the quasi-periodic signals like ECG), in addition to short-term redundancy (within a period), there exists also long-term redundancy (LTR, between periods). This is the main reason for existing PLA methods low compression performances. But, since these methods consider few points only at a time, they have the advantage of possessing linear temporal complexity allowing effective real-time implementation.

The objective of the proposed method is to incorporate LTR in this category of methods. Our objective is clearly justified since the so-enhanced methods would have competing compression performances and at the same time would allow real-time implementation. Our proposition is based on two main tools: block-sorting through a variant of the quite recent Burrows-Wheeler algorithm (BWA) [7] and curve simplification through a variant of the Douglas-Peucker algorithm [8]. Our algorithm is coupled as a post-processing step to the FAN method in the specific case of ECG signal, as an illustration.

The rest of this work is organized as follows. In section 2, tools and methods used in the proposed approach are presented. In section 3 experiments on the novel method are illustrated on selected ECG records from the MIT-BIH database.
A. Curve Simplification

Given a discrete curve, formally expressed by polyline \( P=(p_i), i=1..N \), where \( p_i=(x_i,y_i) \), with \( x_i \) the horizontal coordinate and \( y_i \) the vertical coordinate of \( p_i \), the simplification of \( P \) consists in the computation of another polyline \( Q=(q_j), j=1..M, \) satisfying the following conditions:

a. \( M<N \);

b. \( q_1=p_1 \) and \( q_M=p_N \);

c. Let \( \|P\|, \|Q\| \) be a distance between \( P \) and any PLA of \( Q \), then \( \|P\|, \|Q\| < \varepsilon \), with \( \varepsilon>0 \), a preset threshold on the tolerance of the simplification error.

If \( Q \) satisfies conditions a-c, it is said to be an \( \varepsilon \)-Simplification of \( P \). Fig. 1 illustrates a polyline \( P=(p_1, p_2, ..., p_m) \), simplified with eight points \( Q=(q_1, q_2, ..., q_8) \). It is clear that simplification is a compression tool too, with the additional condition that the essential characteristics of the initial curve be reconstructed with precision. These characteristics depend on the nature of the signal. For instance, in the case of ECG, the main features P, QRS and T are of clinical importance. It can be shown also that for a given curve, there exist more than one \( \varepsilon \)-Simplification. Therefore, a minimal set of rules must be established so as to ensure selection of the \( \varepsilon \)-Simplification with the minimal number of CPs, according to these rules. In this paper, we propose a method, using the variant of a Douglas-Peucker line simplification algorithm (DPA)[8]. This algorithm uses a recursive selection strategy, reducing gradually the distance between \( P \) and \( Q \) by the maximal possible amount under norm \( \|\cdot\| \) at each selection. Our choice for the recursive approach is motivated by the excellent performance of this strategy at selection of most perceptually important points on the initial curve. By contrast, the classical PLA methods use a sequential strategy leading to selection of locally only significant points. The DPA main steps are as follows. The initial curve endpoints are first selected \((Q=[p_1, p_N])\). The next selected point, \( q_i=(x_i, y_i) \), is s.t.:

\[
\max_{q_j \in [p_2, p_{N-1}]} \| q_j - \hat{q} \|
\]

(1)

where \( \hat{q} \) is the vertical projection of point \( q_i \) on polyline \( Q \).

Therefore, point \( q_i \) is the most perceptible CP in the interval \( [y_{j-1}, y_j] \). The process is then recursively repeated for the resulting sub-curves \( [q_j, q_i] \) and \( [q_i, q_{i+1}] \) until the precision of simplification \( \varepsilon \) is attained. In this study, we reduce the norm given by equation 2:

\[
\|P', Q'\| = \sqrt{\sum_{j=1}^{\min(N,M)} (y_j - \hat{y}_j)^2}
\]

(2)

In the general case, for segment \([p_{n-1}, p_n]\) under process, the selected CP, say \( q_\text{opt} \), is s.t.:

\[
q_\text{opt} = \operatorname{Arg} \max_{q_k \in [p_{n-1}, p_n]} \left\| q_k - \hat{q} \right\|
\]

with

\[
\|q_k - \hat{q}\| = |y_k - \hat{y}_k|
\]

(4)

In Eqs. (2) and (4), \( y_i \) is the magnitude of point \( q_i \) and \( \hat{y}_i \) that of \( \hat{q} \). Note that the so computed CPs are selected according to a binary tree of segmentation where the most perceptible points are selected in the upper levels. Fig. 2 illustrates this property in the case reported in Fig. 1. It is easy to derive from the binary tree of segmentation that the process temporal complexity is of \( O(N \cdot \log_2(N)) \) order. The simplification algorithm is formally described in Fig. 3.

B. Compression Through Block-Sorting

Block sorting is quite a recent trend as fare as compression is concerned. The Burrows and Wheeler Algorithm (BWA) [7] is one of the first compression algorithms using this technique. The original BWA is a lossless compression method, reported to yield excellent results on images, text and sound [1]. The main idea behind the BWA is computation of a reversible permutation of the original data that creates concentrations. These concentrations of data are successively coded by RLE (run length encoding), MTF (move to front) techniques and finally, Huffman coding is applied. The decoder proceeds in reverse order, which allows reconstruction of the initial permutation. The permutation in question is the last column of the \( NxN \) matrix obtained by cyclic shifting of the initial data, \( N \)-times. The reconstruction of the original data from the permutation is achieved through a well-established process. Our idea is that, since PLA algorithms reduce STR only, introduction of a block-sorting algorithm as a post-processing step to these methods would take into account LTR too. Therefore, the output \((x_i, y_i), i=1..M\) of a specific STR method is sorted on the \( y_i \) coordinate. This yields a novel curve \((x_i, y_i'), i=1..M\). This last curve is then simplified with the Douglas-Peucker algorithm of section II.A to reduce LTR. This yields another curve \((x_i, y_k'), k=1..L, L<M\). The compression ratio associated with the STR reduction (first simplification), as expressed in terms of number of samples reduction is given by Eq. 5:

\[
CR_{\text{STR}} = \frac{N}{2(L \cdot M)}
\]

(5)

The compression ratio related to the LTR reduction (second simplification) is given by Eq. 6:

\[
CR_{\text{LTR}} = \frac{N}{(M^2 + 2L)}
\]

(6)

The 2 factor in Eqs. 5 and 6 is due to the fact that in this type of compression, both the magnitude \( y_i \) and its temporal index \( x_i \) are stocked/sent. Then, for \( L<M/2 \), \( CR_{\text{LTR}} > CR_{\text{STR}} \), hence, gain in compression. Yet, \( CR \) is upper bounded by 2.\( CR_0 \). Finally, for \( L>M/2 \) : \( CR_0 < CR < 2CR_0 \).

The reconstruction is conducted as follows. Magnitude \( \hat{y} \) of the \( M \) CPs associated with the STR compression is first computed using the \( L \) CPs of the LTR compression through linear interpolation between successive CPs, using \((x_0, y_0), (x_1, y_1) \) and \((x_{L-1}, y_{L-1})\) according to Eq. 7.
The reconstructed magnitude approximation of the M CPs of the STR compression. Finally, tuples on the first coordinate in ascending order yields the M.

\[ DPA([\pi, \ldots, q_k], p_1 = q_{100}) = \{ [p_1, p_{45}], [p_{45}, p_{100}] \} \]

### Procedure

- **Step 0:** \( Q \leftarrow \{ p_1, p_{100} \} \)
- **DAI(p_1, p_{100}, \theta) ;**
- **Q ← Sort(Q) ;**
- **M ← ||Q|| ;**
- **Return(Q, M) ;**

**Procedure**

\[ DPA([p_{i-1}, p_i], p_{i+1}) \text{ if } i < j \] N

\[ q_k \leftarrow \text{ArgMax}_{q_j} \sum_{i=1}^{j} d(q_i, \hat{q}_j) \]

\[ Q \leftarrow Q \cup \{ q_k \} ; \]

**DAI([p_{i-1}, p_i], \theta) ;**

**DAI([p_{i-1}, p_i], \theta) ;**

**End.**

#### Fig. 1. An \( \epsilon \)-Simplification of polyline \( P=(p_1,\ldots,p_{100}) \) with polyline \( Q=(q_1,\ldots,q_8) \).

#### Fig. 2. CPs selection tree for the case in Fig. 1.

#### Fig. 3. Modified Douglas-Peucker Algorithm.

The result is a set of tuples \((x_i, \hat{y}_i)^{i=1..N} \) representing the mean original magnitude. The FAN temporal complexity is of linear order, which allows effective real-time implementation of this method. Our proposed algorithm for LTR reduction is coupled as a post-processing step to this method. The so-enhanced FAN method is denoted herein FAN+. Evaluation of the FAN and FAN+ methods is performed on carefully selected records from the Massachusetts Institute of Technology – Beth Israel Hospital (MIT-BIH) ECG database. The MIT-BIH database is a collection of 48 records sampled at 360 Hz. Each record is 30 minutes long and each sample is coded on 12 bits. This base serves as a cross-reference for researchers. The evaluation is performed on the numerical level through the compression ratios CR for FAN and CR for FAN+ where PRD is given by Eq. 9 and PRD, by Eq. 10 with \( \bar{y} \) representing the mean original magnitude.
The first application is a detailed illustration of the proposed method and is reported in Fig. 4. This figure shows in (a) a 1000 samples ECG from the beginning of rec. 105. Obtained results are as follows: M=71, then CR_{0}=1000/(2\times71)=7.04:1, with a reconstruction distortion of PRD_{0}=7.18%. The 71 CPs were processed with our proposed algorithm yielding L=10 CPs at the same precision. The new compression ratio is then CR_{1}=1000/(71+2\times10)=11:1 and a new distortion of PRD_{1}=7.90%. The gain in compression ratio is more than 56% for a small additional distortion. Note that the block size has great impact on LTR in the proposed method. In the specific case of the MIT-BIH database, the samples of which are coded on 12 bits, the maximal block size is 2^{12} samples, when expressing the compression ratio in terms of samples reduction. This constitutes no barrier, since the compression ratio can be expressed in terms of bits reduction (bit rate). The advantage of expressing the compression ratio in terms of samples reduction is to appreciate the true compression capabilities of a given method regardless of coding considerations. It is also of major importance to link the compression capability to a metric of distortion, for it is established that the highest compression ratios are obtained at higher distortion prices. We would then be interested in methods yielding higher compression ratios for lower distortions, ideally, in the operational rate-distortion sense (ORD). Fig. 5 clearly shows the enhancement of the RD behavior for all used records. For example, for a distortion of 10%, the compression ratios for the three records are as follows: 4.5:1, 7.75:1 and 10:1 for FAN and 8.5:1, 13.25:1 and 16.5:1 for FAN+, which represents gains in compression ratio for the same distortion respectively as follows: 88%, 71% and 65%. It can be checked that this gain varies for the different records as follows: Rec. 108 : 79%-87%, rec. 105 : 77%-79% and rec. 119 : 65%-71%. It can also be checked that the highest gains are obtained for the lowest distortions, i.e. for near-lossless compression. This is due to the fact that the more the FAN method goes to near-lossless compression, the more there are CPs to be post-processed, thus more LTR. In the case of higher compression ratios, it is all the way around. Nevertheless, the compression ratio is at most doubled. It is essential that the numerical evaluation be accompanied with visual inspection of the reconstructed signals in this type of applications. For this purpose, Fig. 6 shows the original signals (a), the reconstructed signals by FAN (b) and the reconstructed signals by FAN+ (c) for the three records. Note that in this case all plots are obtained around CR_{1}=10:1. The plots clearly show the enhancement of the reconstructed signals due to taking into account LTR in the reconstruction process. This is an important property for our method with regard to classically used coding techniques to take into account LTR where the reconstructed signal is unchanged.
V. Conclusion

A novel method for incorporation of long-term redundancy (LTR) in signal time domain compression methods has been proposed. The novel method is based on curve simplification and block sorting. The method is quite a general-purpose one-dimensional signals compression method, with more efficiency for quasi-periodic signals. It was implemented as a post-processing step for the FAN method in the specific case of the ECG signal. Results of the enhanced FAN+ method

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Fig. 5. RD behavior of FAN (triangles) and FAN+ (stars).

Fig. 6. Original (a) and reconstructed signals by FAN (b) and by FAN+ (c) of MIT-BIH records 108, 105 and 119.
confirm the substantial improvement of the compression ratio-distortion behavior with respect to that of the FAN method for a better reconstructed quality signal. Note that the proposed method takes into account LTR on the magnitude axis only. It also needs no specific segmentation like R wave detection in the ECG signal. These two properties grant our method to be a general-purpose signal compression tool. Yet, for quasi-periodic signals, like ECG for instance, we believe that LTR reduction can be incorporated on the temporal axis too. Note also that the authors used the Douglas-Peucker algorithm as a unified tool for ECG baseline correction, features detection and STR compression in previous works [9]-[12].

REFERENCES


Bachir Boucheham was born in Constantine, Algeria in June, 1960. He received the Diploma of Ingénieur d’Etat in computer science in June 1984 from the University of Constantine, Algeria and the Master’s degree in computer science from the University of Minnesota, Minneapolis, Minnesota, USA, in December 1987.

Yousef Ferdi was born in Constantine, Algeria, on November 06, 1960. He received the Diplôme d’Ingénieur in Electronics, Magister and Doctorat d’Etat degrees in Automatics and Signal Processing, from the University of Constantine, Algeria, in 1988, 1992 and 2001, respectively. He was a Professor at Lycée Technique EL-Khroub, Constantine, from 1988 to 1992. He was an assistant Professor from 1992 to 2001, and a Professor since 2001 at The Department of Electrotechnics, University of Skikda, Algeria. His research interests are in the area of Signal Processing with applications of Fractional Order Differentiation.

Mohamed Batouche received his M.Sc. and Ph.D. degrees from the Institut National Polytechnique de Lorraine, France, in 1989 and 1993, respectively. Currently he is a full Professor at the University of Constantine, Algeria. His research areas include artificial intelligence and pattern recognition.

Mr. Boucheham is a member of the Laboratory de Recherche en Electronique de Skikda” (LRES). His main research interests include pattern recognition, image and signal processing and compression.