A Method of Planar-Template-Based Camera Self-Calibration for Single-View

Yue Zhao and Chao Li

Abstract—Camera calibration is an important step in 3D reconstruction. Camera calibration may be classified into two major types: traditional calibration and self-calibration. However, a calibration method in using a checkerboard is intermediate between traditional calibration and self-calibration. A self-calibration algorithm is proposed based on a square in this paper. Only a square in the planar template, the camera self-calibration can be completed through the single view. The proposed algorithm is that the virtual circle and straight line are established by a square on planar template, and circular points, vanishing points in straight lines and the relationship between them are be used, in order to obtain the image of the absolute conic (IAC) and establish the camera intrinsic parameters. To make the calibration template is simpler, as compared with the Zhang Zhengyou’s method. Through real experiments and simulation experiments, the experimental results show that this algorithm is feasible and available, and has a certain precision and robustness.

Keywords—Absolute conic, camera calibration, circle point, vanishing point.

I. INTRODUCTION

In the process of acquiring three-dimensional (3D) information from two-dimensional (2D) image, the camera calibration is an indispensable step. Camera calibration usually is divided into two classes: traditional calibration and self-calibration. The traditional calibration approach needs to prepare a high precision geometric calibration object, and then uses the relationship between world coordinate and image coordinate of calibration object to complete the camera calibration. In 1992, Hartley and Faugeras first put forward the thought of camera self-calibration, which was called camera self-calibration by using corresponding relationships of images. Now, camera self-calibration has become one of hot spot in computer vision [1-7]. Zhang Zhengyou (2000) [8] put forward one kind more flexible and simple calibration method which was to use a dot matrix template instead of traditional calibration object to solve linearly camera intrinsic parameters via homography between dot matrix and image. On the basis of Zhang Zhengyou’s calibration method, Meng Xiaoqiao and Hu Zhanyi (2002) [9] put forward a kind of camera self-calibration method based on planar circle, which first introduced the concept of circular points into the technology of camera self-calibration. From then on, the technology of camera self-calibration enters a new field—the self-calibration by using circular points. Then, many self-calibration methods appear in succession based on Zhang Zhengyou and Meng Xiaoqiao. Wu Fuchao et al. (2003) [10], a kind of camera self-calibration method was proposed by using two non-parallel rectangles in space plane to solve circular points. In addition, Li Xinju et al. (2004) [11] analyzed the shortage of Zhang Zhengyou’s calibration method and put forward a kind of camera self-calibration method according to the similarity of plane scene image. Wang Guanghui (2008) [12] proposed a kind of camera self-calibration method using calibration object similar to Zhang Zhengyou’s checkerboard based on Kruppa equation. In recent years, a large number of self-calibration methods based on circular points or vanishing points have emerged [13-16]. However, in this paper, we use a square as calibration object to solve camera intrinsic parameters by circular points, vanishing point and projective invariance.

II. CAMERA MODEL

In this paper, we use perspective projection model which corresponds to a pinhole camera model (Fig.1).

![](image.png)

Fig. 1 Pinhole camera model

Let the world coordinate of a point \((X)\) in space be \((X, Y, Z)\) and the corresponding image coordinates be \((u, v, v_0)\). \((u_0, v_0)\) is the coordinates of principal point and \(s\) is skew factor. \(f_x, f_y\) represent the focal length of the lens. \((r_x, t_x, t_y)\) are rotation matrix and translation vector. So, the perspective projection matrix may be expressed

\[
\begin{bmatrix}
\alpha \\
\beta \\
1
\end{bmatrix} = K \begin{bmatrix} R & T \end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]

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where \( K \) is camera intrinsic parameters in Equation (1). In this paper, the calibration object is a square on a planar template, so the planar template containing square may be defined as \( X - Y \) plane of world coordinate system (WCS) and the coordinate \( Z \) of space points is always zero. Now, the simplified expressions of perspective projection model can be obtained

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} =
\begin{bmatrix}
f_x & s & u_0 & r_1 & r_2 & r_3 & t_1 \\
0 & f_y & v_0 & r_1 & r_2 & r_3 & t_2 \\
0 & 0 & 1 & r_1 & r_2 & r_3 & t_3
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix},
\]

where \( H \) is called homography matrix from 3D to 2D coordinates.

III. CAMERA SELF-CALIBRATION

![Fig. 2(a) and (b) are calibration template in literature [12] and this paper, respectively](image1)

![Fig. 3 Virtual calibration template in this paper](image2)

Zhang Zhengyou [8] and Wang Guanghui's [12] self-calibration template is in Fig. 2 (a). The calibration template of this paper is in Fig. 2 (b). A virtual calibration template based on Fig. 2(a) is in Fig. 3. Without loss of generality, assume the planar template containing square (Fig. 2 (b)) is \( X - Y \) plane of WCS, in which the origin is \( a \), and \( X \)-axis, \( Y \)-axis are \( ab, ad \), respectively. We can get the following conclusion. Several conclusions are drawn.

**Lemma 1.** If in Fig. 3, let the side length of square be \( a \) on the planar template, the equations of virtual circle and virtual lines can be determined, and the virtual lines cut the virtual circle at \( e, f, g, h \), lines \( ac, bd \) cut the virtual circle at \( a, b, c, d \) are known.

**Proof.** Suppose the center of the virtual circle is \( O \) where the two diagonals of a square \( abcd \) cross and a length of the diagonal is its diameter, as shown in Fig. 3, the coordinates of the center of a circle is \((a/2, a/2, 0)\) in the WCS, and the algebraic equation of the virtual circle in \( X - Y \) plane is

\[
\left( x - \frac{a}{2} \right)^2 + \left( y - \frac{a}{2} \right)^2 = a^2 .
\]

So the virtual lines are arbitrary lines through the center \( O \) of the virtual circle, without loss of generality, assuming that slopes of the virtual lines \( ge, hf \) are \( k_i, k_j \), the equations of \( ge, hf \) are

\[
y_i = k_ix_i + b_i ,
\]

\[
y_j = k_jx_j + b_j .
\]

Take line \( ge \) as an example to seek the intersection of virtual circle and virtual line. As the virtual line passes through the center \( O(a/2, a/2) \) of the virtual circle, replacing Equation (4), we can get

\[
a^2 = k_i a^2 + b_i ,
\]

arrange

\[
b_i = a^2 (1 - k_i) ,
\]

and finally determine line \( ge \). Combining Equations (3), (4) and (5), we can get

\[
(1 + k_i^2)x_i^2 + (2k_ib_i - ak_i - a)x_i + \left( a^2 + b_i^2 - a^2 \right) = 0 .(6)
\]

Solve the monic quadratic equation (Equation (6)) to get two intersections, and then the coordinates of the virtual lines and circle can be obtained through Equation (4).

**Lemma 2.** Assuming that homography matrix \( H \) from 3D to 2D coordinates is known, image coordinates of any 3D points of the planar template can be obtained in image coordinate system (ICS), particularly, circular points also can be gotten in ICS.

**Proof.** From Equation (2), Lemma 2 is obviously true. Particularly, \( [1 \ 0 \ i \ 0]^T \) are two circular points of the calibration plane in the WCS, so the image coordinates of circular points are unique

\[
\begin{bmatrix}
m_i \\
m_j
\end{bmatrix} \approx H \begin{bmatrix}
1+i \\
1-i
\end{bmatrix}^T
\]

Obviously, homography matrix \( H \) can be estimated by at least four matching points. In Fig. 3, the image coordinates of four vertices \( a, b, c, d \) in the square can be obtained by Harris Corner Detection [17] and the world coordinates of \( a, b, c, d \) are obviously known, so \( H \) is estimated by four pairs of matching points. Combining Lemma 1 and Lemma 2, the image coordinates \( e, f, g, h, O \) and the circular points \( m_i, m_j \) can be gotten.
Lemma 3. Suppose $v_1, v_2$ are vanishing points on mutually orthogonal directions, $v_1' o v_2' = 0$ ($\omega = K^{-1} K^{-1}$ is IAC).

Proof: Let $p_{\infty}$, $p_2$ be points at infinity on mutually orthogonal directions, of which image coordinates are $v_1, v_2$. According to the camera model with imaging principles, we have

$$\lambda v_1 = K [ R \ T ] p_{\infty}, \lambda v_2 = K [ R \ T ] p_2,$$

and Equation (7) can be rewritten as

$$[ R \ T ] p_{\infty} = \lambda K^{-1} v_1, [ R \ T ] p_2 = \lambda K^{-1} v_2.$$

From orthogonality in projective geometry, we have

$$\lambda v_1' K^{-1} K^{-1} v_2' = p_{\infty}' R [ R \ T ] [ R \ T ] p_2 = 0,$$

and therefore

$$v_1' K^{-1} K^{-1} v_2' = 0.$$

In Fig. 3, $ab$ and $cd$ are the intersection of lines $ab$ and $cd$ is a point at infinity of which image is called vanishing point, denoted as $v_1$. Similarly, vanishing points of lines $ad$ and $bc$, line $ac$ and line $bd$ are denoted as $v_2$, $v_3$, and $v_4$, respectively. Obviously, $v_1 \perp v_2, v_1 \perp v_3, \ldots$, with Lemma 3, two constraint equations about IAC $\omega$ may be gotten

$$v_1' o v_1 = 0 \quad v_2' o v_2 = 0,$$

where $v_1, v_2$ can be obtained by solving image intersections of lines $ab, cd$ and lines $bc, ad$, respectively, and $v_3, v_4$ can also be obtain from following lemma.

Lemma 4. Suppose $P_i \in l(P)$ ($i = 1, 2, 3, 4$) and $a_{12} = k (k \neq 0, 1, \infty)$, where three of $P_1, P_2, P_3, P_4$ and $k$ are known, the fourth point can be determined. ($l(P)$ is point-set of line $l$.)

Proof: Supposing that homogeneous coordinates of $P_1, P_2, P_3, P_4$ are $a, b, a + \lambda b, a + \lambda b$, have

$$(P_1 P_2 P_3 P_4) = \lambda_1 \lambda_2 = k.$$

Without loss of generality, assuming that known $P_1, P_3, P_4$ are $a, b, a + \lambda b$, now the solution of the fourth point is transformed to determine $\lambda_2$. From Equation (9), we have

$$\lambda_2 = \frac{\lambda_1}{k}.$$

Supposing that homogeneous coordinates of $P_1, P_2, P_3$ are equal to $(u_1, v_1, 1)$, $(u_2, v_2, 1)$, $(u_3, v_3, 1)$, combining with $P_1 = a + \lambda b$, we have

$$u_1 = \frac{u_1 + \lambda u_2}{1 + \lambda}.$$

Equation (11) can be rewritten as:

$$\lambda_1 = \frac{u_3 - u_1}{u_3 - u_2}.$$

Combining with Equation (10) and (12), $\lambda_2$ can be obtained, and then $P_4$ can be determined.

As shown in Fig. 3, take line $ac$ as an example to discuss how to solve a vanishing point of diameter of the virtual circle. In Lemma 4. If let $P_1 = a, P_2 = o, P_3 = c, P_4 = P_\infty$, it is reasonable assumption that the length of $PP_\infty$ and $P_\infty P_2$ are equal, and $\frac{PP_\infty}{P_2 P_\infty} = 1$. Based on perspective projection, have

$$(P_1 P_2 P_3 P_\infty) = \frac{P_1 P_3}{P_1 P_2} = (P_2 P_1 P_3 P_\infty),$$

where $P_1$ is the diameter and $P_2$ is the centre of the virtual circle, respectively, and $\frac{P_1 P_3}{P_2 P_\infty} = 1$. Additionally, under the projective transformation group, some graphics have projective properties and projective invariants. Combining with Lemma 4, we have

$$\left\{ \begin{array}{l} (P_1 P_2 P_3 P_\infty) = -1 \\ P_1 \times P_2 = 0 \end{array} \right.$$

Supposing the image coordinate of $P_\infty$ is $(u_\infty, v_\infty, 1)$, the matrix expression of Equation (13) is

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_\infty \\ v_\infty \end{pmatrix} = \frac{u_\infty (u_1 + u_2) - 2u_\infty u_2}{2u_\infty - u_1 - u_2} = \begin{pmatrix} u_1 \\ v_1 - v_2 \end{pmatrix}.$$

The image coordinate of $P_\infty$ can be obtained by the method of least squares to solve Equation (14). Similarly, points at infinity of any diameter of the virtual circle are gotten.

Lemma 5 [19, 20]. The sufficient and necessary conditions of which two non-isotropic lines are mutually orthogonal are that points at infinity of the two non-isotropic lines are conjugated with two circular points of the plane where the two non-isotropic lines are.

Fig. 4 An image of a plane at infinity

Fig. 4 is an image of a plane at infinity, where $\omega$ is absolute conic, $m_1, m_2$ are two circular points of the plane $\pi$ with the line at infinity $pq - P$ is the point at infinity of one non-isotropic line in the plane $\pi$. From Lemma 5, $q$ corresponds to the point at infinity, at which the line on the plane $\pi$ is perpendicular to
by solving image intersections of parallel sides.

Step 5. According to Lemma 4, obtain vanishing points of two diagonals of the square.

Step 6. According to Lemma 4 and Lemma 5, obtain any of virtual lines and its vanishing point of orthogonal lines.

Step 7. Obtain \( \omega \) by the method of least squares, and then obtain \( K \) by Zhang’s method [8].

IV. Experiments

A. Experiments with Simulated Data

The values of camera intrinsic parameters are set to \( f_x = 1000, f_y = 1100, u_0 = v_0 = 0, s = 0 \) in simulation experiments. Simulations adopt three images which correspond to extrinsic parameters

\[
R = \begin{bmatrix}
0.8080 & 0.4106 & -0.4225 \\
0.0695 & -0.7785 & -0.6238 \\
-0.5850 & 0.4747 & 0.6576
\end{bmatrix};
T_1 = [-142.56 -34.891 521.456]^T;
\]

\[
R_2 = \begin{bmatrix}
-0.9993 & -0.0065 & 0.0361 \\
-0.0115 & -0.8780 & -0.4785 \\
0.0348 & -0.4785 & 0.8776
\end{bmatrix};
T_2 = [-142.56 -34.891 67.6]^T;
\]

\[
R_3 = \begin{bmatrix}
-0.0002 & -0.9244 & -0.3815 \\
0.0004 & -0.3815 & 0.9244 \\
-1.0000 & 0.0000 & 0.0000
\end{bmatrix};
T_3 = [-100 50 260]^T;
\]

respectively. The origin of WCS is the intersection of two diagonals of the square. We adopted three virtual lines, of which the corresponding slopes are \( \frac{\pi}{12}, \frac{\pi}{6}, \frac{\pi}{3} \). In order to test the robustness of our proposed method, add the Gaussian noise where the mean value is zero and the mean square error is from 0.1 pixels to 5 pixels on the position of pixels. 200 independent experiments can be executed for each noise level, and solve their averages.

In the same camera parameters setting, carry out a comparative study between ours and Wang Guanghui’s method [12] with skew factor \( s=0 \). The algorithm is denoted by Square in this paper, and Wang Guanghui’s algorithm is denoted by Hinf. Comparisons of two algorithms are as follows (Fig. 4):
B. Experiments with Real Images

The factual data can be used for comparing and testing two proposed algorithms. The kind of camera is a CCD digital camera where the size of the image is \(320 \times 240\). Fig. 5 (a) and (b) are two photos from different calibration templates as follows.

Fig. 4 Accuracy of two methods concerning the noise level

We carried out a comparative test with Wang Guanghui’s calibration algorithm. The results are as follows (Table I):

<table>
<thead>
<tr>
<th>Camera Intrinsic Parameters</th>
<th>Hinf</th>
<th>Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_x)</td>
<td>51.389731</td>
<td>50.982492</td>
</tr>
<tr>
<td>(f_y)</td>
<td>51.389731</td>
<td>50.982492</td>
</tr>
<tr>
<td>(s)</td>
<td>51.389731</td>
<td>50.982492</td>
</tr>
<tr>
<td>(u_0)</td>
<td>51.389731</td>
<td>50.982492</td>
</tr>
<tr>
<td>(v_0)</td>
<td>51.389731</td>
<td>50.982492</td>
</tr>
</tbody>
</table>

In addition, we executed the 3D reconstruction by the calibration results as follows
In this paper, use a square as calibration template. In the square, two diagonals as the diameter, the intersection of two diagonals as the center of a circle creates a virtual circle, and the intersection of diagonals are used as the beam-center of beams of virtual lines. First, the homography matrix $H$ is estimated from the four vertexes of the square to obtain image coordinates of the circular points and the intersections of the virtual circle and virtual lines. In addition, based on the cross-ratio invariance of projective transformation and conjugation between orthogonal vanishing points and circular points, the vanishing points of any of lines and the points of them can be solved on the beam of virtual lines. So, IAC can be obtained by solving the constraint equations which are made up of mutually orthogonal vanishing points. Further, intrinsic parameters are determined through Zhang’s decomposing method. Experimental data show that the algorithm is feasible and available, have a certain precision and robustness. In this paper, the homography matrix $H$ is determined only by four vertices of a square, so the accuracy of four vertices affect the results of calibration. Therefore, in order to effectively improve the accuracy and the robustness of the algorithm, we can consider checkerboard as calibration plane.

**ACKNOWLEDGEMENT**

This work is supported by the Scientific Research Foundation of Yunnan Education Department of China (2010Y245) and the Scientific Research Foundation of Yunnan University (2010YB021).
REFERENCES


