Abstract—Producing companies aspire to high delivery availability despite appearing disruptions. To ensure high delivery availability safety stocks are required. However, safety stock leads to additional capital commitment and compensates disruptions instead of solving the reasons. The intention is to increase the stability in production by configuring the production planning and control systematically. Thus, the safety stock can be reduced. The largest proportion of inventory in producing companies is caused by batch inventory, schedule deviations and variability of demand rates. These reasons for high inventory levels can be reduced by configuring the production planning and control specifically. Hence, the inventory level can be reduced. This is enabled by synchronizing the lot size, straightening the demand as well as optimizing the releasing order, sequencing and capacity control.

Keywords—inventory level, inventory management, production planning and control, safety stock

I. INTRODUCTION

The cost pressure for producing companies increases constantly [1]. To remain competitive the aim of these companies is to reduce their costs [2]. The challenge of logistic is to fulfill the classic logistic targets: low stocks, short throughput times, high adherence to delivery dates and utilization of capacity [3]. These targets are competitive such as reaching a high utilization of capacity by low stocks, because thus disruptions cannot be absorbed. This dilemma is reflected in the dimensioning of safety stock. On the one hand a high safety stock can compensate disruptions [4, 5] and thus enables short throughput times and high adherence to delivery dates [6]. But on the other hand high stocks lead to high capital commitment costs [7], which can be reduced by a low inventory level. Therefore the safety stock must be adjusted to the given constraints of the production [8].

The experiences with industry projects of the Laboratory for Machine Tools and Production Engineering of Aachen University confirm that three most relevant reasons for stocks can be amounted, which are displayed in Fig. 1. Variable replenishment times (schedule deviations) and reject are among the most significant disruptions as well as variable demand rates, which also interfere with the inventory management and can be handled more easily using safety stocks.

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II. DIMENSIONING OF SAFETY STOCK

Safety stocks are to compensate uncertainties along the supply chain and to ensure the ability to deliver of the accordant stock towards the downstream processes. Such uncertainties can be described as deviations in demand, deviations of the expected demand curve as well as deviations of delivery date, quantity and quality. The safety stock is not permitted to serve the planned demand but is rather generated as an additional inventory [9].

To determine the required safety stock the service degree must be defined. The service degree is a logistic target, which describes the ability of any warehouse to supply. It states the proportion of the demand which could be served by the available inventory [10, 11]. If all demands to the stock are served immediately, the service degree amounts 100%. Delays in delivery imply a service degree of less than 100%. If the demands served delayed increase, stockouts as well as defaults in delivery appear and the service degree decreases further. Thus the service degree depends significantly on the input and output processes.

Fig. 2 displays a high dependence of the variability in the production on the service degree. The higher the variability the higher is the required inventory level to fulfill a demanded service degree.

The service degree increases as the average inventory level rises. From a defined height of the mean inventory level, which is called minimal inventory, all demands can be directly served by the stock. The service degree amounts 100% and the mean delay in delivery reaches zero. If the inventory level is increased further, the logistic performance cannot be improved anymore. Methods to improve the stock performance require different efforts. It is necessary to follow a defined sequence in improving the performance to ensure a great input/benefit ratio.
The service degree depends directly on the average inventory level. The higher the inventory level the higher is the service degree. The inventory level can be determined depending on the analyzed disruptions occurring in the production and the defined service degree. It can be calculated for a service degree of 100 % as follows where IN represents the mean input lot size, OUT the mean output lot size, TR the replenishment time, T Rmax the average replenishment time, T Rmax the maximum replenishment time, ΔV the maximum deviation in input quantity, DRm the average demand rate and DRmax the maximum demand rate.

\[
I_{100\%} = \frac{IN - OUT}{2} + \sqrt{\left(\frac{(TR_{max} - TR_m) * DR_m}{2} + (\Delta V_{max})^2 + (\Delta R_{max} - DR_m) * TR^2\right)}
\]

The first summand describes the lot inventory whereas the second summand defines the safety stock, which is necessary to ensure the delivery availability even with disruptions. Three different kinds of disruptions are considered in the following paragraphs. The equation for the calculated safety stock SS can be derived from (1) as

\[
SS = \sqrt{\left(\frac{(TR_{max} - TR_m) * DR_m}{2} + (\Delta V_{max})^2 + (\Delta R_{max} - DR_m) * TR^2\right)}
\]

With (1) an inventory level for a service degree of 100 % is calculated. Today's situation in the industry shows that a service degree of 100 % often is not required. A high amount of inventory level leads to high costs, which is not intended. Moreover a high effort is necessary to reach a service degree of 100 %. It is known that for many events, the major part of the effects come from the minority of the causes. A commonly demanded service degree in the industry amounts 95 %. Therefore the C-norm-parameter is essential to calculate the inventory level needed for a service degree less than 100 %. The inventory level for a service degree of 95 % is described as follows where I95\% represents the average inventory level at a service degree of 95 %, I0 the lot inventory and SD the required service degree.

\[
I_{95\%} = \frac{I_0 * (\frac{SD}{100})^2 + (I_{100\%} - I_0) * c}{1 - (1 - \frac{SD}{100})^c}
\]

The C-norm-parameter has a high impact on the curve of the service degree. Low values of C are preferable. A lean distribution of all deviations from the plan leads to low values of C. In literature a value of 0.33 to 0.37 for the value of C is given, if deviations from plan are moderate dispersive[13].

III. CAUSES OF INVENTORY

However safety stocks only compensate the indications for a high inventory level and do not solve the causing problems. The described causes can be influenced by the production control. A specific configuration of the production control should influence the reasons. Thus the safety stocks as well as the capital commitment costs can be reduced.

The derivation of suitable measures requires analysis of the determining influences for the target inventory level's height. Based on the share of the single plan deviations in the height of the minimal inventory the share in the target inventory can be determined. The analysis of the target inventory level for the regarded range of articles results in the stock proportion's distribution shown in Fig. 3. The average shares of the inventory level can also be derived from (1). Lot inventory, schedule deviations, variability of demand rates and reject the most relevant reasons for inventory [14]. They are displayed in Fig. 3.

![Fig. 3 Typical shares of the inventory level](image-url)

The inventory level caused by a production in lots I0 is defined as follows. The lot inventory is the result of input and output. The lot inventory increases with the amount of the input level. Frequently it corresponds to the manufacturing lot size.

\[
I_0 = \frac{IN - OUT}{2}
\]
A production in lots harmonizes the material flow as well as the value-added process along the supply chain and makes the entire production system more economical. The lot size is determined differently to fulfill different constraints. The lot size can be defined to reach high capacity utilization or to decrease the capital commitment costs. The height of hold capacity and transportation systems also influences the choice of the lot size. Moreover urgent orders of important customers claim a small lot size.

Schedule deviations in additions to stock are the reason of schedule deviations on the production processes. The schedule deviation $SD$ is calculated as follows.

$$SD = (TR_{max} - TR_{m}) \cdot DR_{m}$$ \hspace{1cm} (5)

If an expected order demand is delayed and thus the inventory level could not serve the demands, they have to be served out of the safety stock. The needed safety stock increases with the number of schedule deviations of the upstream process and the average consumption rate. The inventory level must be increased, if schedule deviations of the addition to stock are often existent. The additional inventory accommodates the demand during the period of the maximal positive schedule deviation. The schedule deviation of the addition to stock is determined by the schedule deviation of usage of the production.

The delivery availability is limited, if the planned delivery quantity is undercut. To avoid poor delivery availability, a safety stock is required in the height of the maximal negative quantity deviation of the upstream process. These quantity deviations can occur as a consequence of quality problems. This reject is defined by $DV_{max}$. Inventory defined as reject cannot serve the purpose sufficiently it used to. Usually a manufacturing of these materials in the production is often impossible. Thus they must be disposed. The arising stockout must be compensated to fulfill the demand. The upstream process has to deliver the accordant amount again. This leads to longer throughput times and schedule deviations. Hence reject is always followed by additional costs.

The variability of demand rates $VDR$ is calculated as follows.

$$VDR = (DR_{max} - DR_{m}) \cdot TR$$ \hspace{1cm} (6)

If products are demanded irregular, variabilities of demand rates occur. Variabilities of demand rates cause variations in the capacity requirement of the production.

In general the demand is not accepted to be constant, but it varies from a mean value. If the demand during the replenishment time is higher than it was planned, some parts of the demand must be served by the safety stock. Therefore the required safety stock depends on the replenishment time and the maximum demand rate. For high maximum demand rates and long replenishment times a high safety stock is necessary.

All these parts of inventory can be influenced by the production control. Hence the production control can impact on the inventory. The aim is to stabilize the production by configuring the production planning and control accordingly. Thus the inventory can be reduced. The production control influences the required safety stock in two aspects: On the one hand average value and statistical spread of the schedule deviation of the upstream process determine the maximum delivery date deviation. On the other hand the throughput time makes a major contribution to the delivery and replenishment time. If improvements are achieved towards these targets, the required safety stock will be reduced significantly.

IV. INFLUENCE OF PRODUCTION PLANNING AND CONTROL ON THE INVENTORY LEVEL

In the past years the production planning and control has been dominated by constantly growing markets and an insufficient supply of production capacity in many industry sectors. Against this background inventory seemed not that important and industry focused on adherence to delivery dates and resource efficiency. This target system inhibited the improvement of logistic skills which can be seen in many companies who increased inventory even though sales decreased. Today the focus on inventory is sharpened and the inventory level can be reduced by improving the production planning and control towards low inventory levels. The configuration of production planning and control influences the four tasks order generation, order release, sequencing and operative capacity planning [15]. It determines the inventory levels based on process stability. But even more important is the transparency about causes for inventory. Especially lot size changes are a major and underestimated reason for inventory levels.

A. Harmonization of lot sizes

A production in lots has the highest impact on the inventory and makes up the largest amount of inventory. As the lot size decreases the inventory level decreases and the upstream process must deliver more frequently [16]. The general stock model, which is shown in Fig. 4, describes a linear demand curve (saw tooth) [17].

The lot inventory describes the mean inventory level, where no delayed delivery occurs and the service degree amounts
100%. It is assumed that ideal process conditions are available. The delay in delivery is maximal with a mean inventory level of zero. The service degree amounts 0% in this case. With an increasing inventory a value \( I_{L0} \) is reached, where no delayed delivery is achieved under ideal conditions.

To determine the lot inventory \( I_{L0} \) the input and output values are necessary. The input corresponds to the lot size of the previous process \( IN \). The lot inventory \( I_{L0} \) is described by

\[
I_{L0} = \frac{IN}{2}. \tag{7}
\]

However the demand curve cannot be defined as linear, if the production is by batch, as shown in Fig. 5. There the development of inventory over time is displayed for a real process with discrete input and output quantities. The outward stock movement is continuous with a constant lot size. Therefore the lot inventory cannot be determined as in (7). Depending on the output lotsize \( OUT \) the inventory caused by a production in lots can increase largely.

A synchronization of lot sizes in up and downstream processes has a high impact on the inventory level. The harmonization of lot sizes has a synchronization effect, which implies a reduction of the inventory level. Hence the first step in reducing the inventory level is the harmonization of the lot size. In the following section two production processes in sequence are analyzed. The input lot size, which is determined by the upstream process, is held constant at a value of 50 units. Fig. 6 displays the effect of different lot sizes of the output lot size, which is determined by the downstream process in sequence processes on the lot inventory. The lot inventory \( I_{L0} \) is calculated as follows where LCM represents the least common multiple.

\[
I_{L0} = \frac{\text{IN} + \text{OUT}}{2} - \frac{\text{IN} \times \text{OUT}}{\text{LCM}} \tag{8}
\]

It is obvious that the output lot size ideally equals the input lot size. Good results for the mean inventory level will also be achieved, if the input and output lot sizes have a common divisor. The synchronization of lot sizes of up and downstream processes must be evaluated along the whole value stream to achieve low inventory levels.

### B. Implementation of optimal lot sizes

If the lot sizes of upstream and downstream processes are synchronized a first step is made towards a lower inventory level caused by a production in lots. If the lot size of the upstream process is not constant but can be adapted, it is advisable to determine the optimal lot size in case of low inventory levels. Common calculation methods determine the lot size considering the consequences in production insufficiently. The formula by Andler, which is often used in practice, is calculated as follows: \( L_{S_{\text{opt}}} \) represents the optimal lot size, \( C_{\text{preproduction}} \) represents the preproduction costs and \( C_{\text{inventory}} \) determines the inventory costs [19].

\[
L_{S_{\text{opt}}} = \frac{2 \times DR \times C_{\text{inventory}}}{C_{\text{preproduction}}} \tag{9}
\]

In general batch building is considered to be a compromise between preproduction costs and inventory costs (capital commitment costs). Preproduction costs are often much higher than inventory costs because of high personnel costs. The consequences are high lot sizes reducing the preproduction costs but at the same time increasing the inventory level. Thus the throughput times rise and the adherence to delivery dates decreases. These circles lead to higher safety stocks to compensate the missing adherence to delivery dates. As the synchronization of the lot sizes straighten the production, a limitation of the height of lot sizes reduces the work in process and the throughput time.

The connection between lot size and setup time can be described as follows. With a given number of orders the total process time is defined, whereas the total setup time can differ depending on the lot size. A project example showed that the amount of total setup time is about 15% of the total process time. In this case 20% of the orders caused 80% of the process time. Decreasing the lot size increases the number of orders causing 80% of the process time, while the setup time increases. Therefore small lot sizes are refused, because the total setup time rises. However small lot sizes stabilize the
production and lead to a decreasing inventory level. The positive consequences of a reduction of lot sizes are reduced throughput times, improved delivery reliability and decreased inventory levels. They outweigh the negative consequences of higher setup times. The fulfilling of the classic logistics targets has priority in today’s production planning and control. From this perspective, a lot size of one is preferred. Because of strongly increasing costs and setup times, a lot size of one is not practicable in the industry. It is necessary to find a compromise between a lot size calculated as in (8) and a lot size of one to decrease the inventory without causing uneconomical costs.

C. Straightening of demands

Besides the harmonization of the lot size and the implementation of the optimal lot size the straightening of the demands also have a positive impact on the inventory level. The variability of demand rates are in particular caused by variable customer demands [20, 21]. However the variability of demand can be compensated by simple mechanisms in the production control and sales. The method of straightening the demand is supposed to reduce the varying amounts in the production.

The variability of demand rates \( VDR \) is calculated as in (6). Accordingly the adjusting lever to reduce the variability of demand rates are the maximum demand rate \( D_{\text{Rmax}} \) and the replenishment time \( T_R \). The maximum demand rate is reduced by straightening the peaks in demand. The replenishment time within the production can be reduced by implementing a high frequency production. Concerning this the production planning and control must be configured accordingly.

In Fig. 7 an example of an industrial project of the Laboratory for Machine Tools and Production Engineering of Aachen University is displayed. The described peak in order could be split into three orders. The reduction of the peak about 30% leads to a reduction of the safety stock of 40%. The safety stock was reduced from 6324 units to 3681 units.

In the next step the replenishment time was decreased. Therefore smaller lot sizes have been used and a high frequented production was implemented. A reduced replenishment time leads to low variabilities of demand rates during the replacement. The production and ordering interval was reduced from three to two weeks. The reduction of the replenishment time about 30% induced a decrease of the inventory level about 16%.

D. Compensation of schedule deviations

Schedule deviations describe the third reason for high inventory levels. Often schedule deviations are the result of an unsuitable configuration of the production control. On the one hand an uncontrolled order release has a major influence on the spread of throughput times. If orders are released uncontrolled, the work in process (WIP) is not constant. Strongly varying throughput times and thus varying replenishment times are the consequence of a strongly varying WIP. The more the replenishments take the higher the required safety stock to compensate these fluctuations. A constant WIP controlled by an order release of the production planning and control leads to decreasing schedule deviations and allows a low safety stock [22]. On the other hand schedule deviations are the consequences of local setup optimizations. Local setup optimization results in a changing sequence in front of the machine, because the sequence is changed to reduce the setup time. Thus the total setup time can possibly be decreased but the sequence is varied. Therefore the adherence to delivery dates is lowered.

Another possibility to compensate schedule deviations caused by disruptions is the capacity utilization control. A temporarily increased use of resources can prevent replenishment times.

Here, the possibilities of the capacity utilization control depend on the capacity of the company’s flexibility. A compensation of disruption by the use of additional capacity represents an active form of decoupling and is particularly appropriate when disturbances only occur rarely.

For decoupling additional stocks can also be implemented. Such stocks are known as decoupling stocks. If the production control is not able to reach the required delivery reliability by the downstream process in terms of adherence to delivery dates, quantity and quality, such safety stocks are necessary. Basically they are applied to compensate time deviations. The time deviations being compensated are earliness and lateness of upstream processes. In case of set-up optimization orders located in the decoupling stock can also be used at earlier dates.

Before implementing decoupling stocks, their position in the value stream and their required inventory level must be defined. Their position is influenced by upstream processes.
with poor adherence to delivery dates. Such processes should be followed by a decoupling point. The required inventory level depends on the adherence to quantity of the upstream process.

Decoupling stocks must be designed to compensate all occurring disruptions and to ensure the required ability to supply. Otherwise they would not counteract disruptions but only lead to rising inventory levels in the production process. Well implemented decoupling stocks have a strong impact on achieving the logistic targets. The targeted building of inventory guarantees shorter delivery times for certain products. The productions capacities are decoupled from the demands.

A much lower impact on the inventory level is caused by reject. Reject is a consequence of a poor quality. Therefore measures to improve the quality influence the inventory level of the production positively.

V. CONCLUSION

In this paper, the reasons for high inventory levels are presented. Instead of compensating the variability in production companies by the required safety stock, the paper explains how to configure the production planning and control to achieve a high stability of the whole production, leading to reduced inventory levels. The described approach has been applied in industry cases by the Laboratory for Machine Tools and Production Engineering of Aachen University, showing its great potentials. Nevertheless the constraints in every production system differ and must be defined before using this approach to achieve best performance. Recognizing constraints given by the logistic targets such as short throughput times and a high adherence to delivery dates increases the complexity but is necessary to use the full potential of methods to reduce the inventory level. Therefore the described orientation towards value stream is useful to manage complexity.

REFERENCES