Numerical Analysis of Flow through Abrasive Water Suspension Jet: The Effect of Garnet, Aluminum Oxide and Silicon Carbide Abrasive on Skin Friction Coefficient Due to Wall Shear and Jet Exit Kinetic Energy

Deepak D, Anjaiah D, and Yagnesh Sharma N.

Abstract—It is well known that the abrasive particles in the abrasive water suspension has significant effect on the erosion characteristics of the inside surface of the nozzle. Abrasive particles moving with the flow cause severe skin friction effect, there by altering the nozzle diameter due to wear which in turn reflects on the life of the nozzle for effective machining. Various commercial abrasives are available for abrasive water jet machining. The erosion characteristic of each abrasive is different. In consideration of this aspect, in the present work, the effect of abrasive materials namely garnet, aluminum oxide and silicon carbide on skin friction coefficient due to wall shear stress and jet kinetic energy has been analyzed. It is found that the abrasive material of lower density produces a relatively higher skin friction effect and higher jet exit kinetic energy.

Keywords—Abrasive water suspension jet, Skin friction coefficient, Jet kinetic energy, Particulate loading, Stokes number.

I. INTRODUCTION

ABRASIVE Water Suspension Jet (AWSJ) technology is evolving rapidly from last decade. Parallely the material for AWSJ nozzle and the type of abrasive suspension are being explored in greater detail using tools of experimental and numerical methods. The rapid advances in AWSJ machining is due to high capability of the process to machine complex shapes that need to be produced from brittle and heat sensitive materials and also from the need to machine different variety of composites. The advent of computer numerical control for the AWSJ motion on the work part complex profiles with better surface quality and precision can now be achieved. One of the plaguing problems faced by AWSJ machining is nozzle wear.

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扭矩t_n，系统响应时间(s)  
V Velocity of phase (m/s)

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>d</td>
<td>Focus tube diameter (mm)</td>
</tr>
<tr>
<td>d_p</td>
<td>Diameter of abrasive particles (µm)</td>
</tr>
<tr>
<td>D</td>
<td>Inlet diameter of nozzle (mm)</td>
</tr>
<tr>
<td>F_{Lift}</td>
<td>Lift force (N)</td>
</tr>
<tr>
<td>F_b</td>
<td>External body force (N)</td>
</tr>
<tr>
<td>F_v</td>
<td>Virtual mass force (N)</td>
</tr>
<tr>
<td>K</td>
<td>Momentum exchange co-efficient</td>
</tr>
<tr>
<td>l</td>
<td>Length of flow domain (mm)</td>
</tr>
<tr>
<td>L</td>
<td>Particle spacing (mm)</td>
</tr>
<tr>
<td>m</td>
<td>Mass flow rate of mixture (m³/s)</td>
</tr>
<tr>
<td>S_t</td>
<td>Stokes number</td>
</tr>
<tr>
<td>t_s</td>
<td>system response time (s)</td>
</tr>
<tr>
<td>V</td>
<td>Velocity of phase (m/s)</td>
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**II. THEORETICAL FORMULATION**

**A. Numerical Model and Assumptions**

The numerical region for flow analysis is made up of flow geometry as depicted in the figure 1 for the multistep AWSJ nozzle. Computational domain consists of three converging steps of nozzle of length 4 mm each between which 4 mm straight length duct is introduced. There is a focus tube of diameter 1.3 mm and length 17mm. The conical steps of the nozzle are of half cone angle of 50 for first and second section followed by 10° for third section. The Abrasive water suspension mixture is let into the nozzle at the inlet and is carried down through the converging cone to the focus tube and exits as coherent jet at the nozzle exit, in which the focus tube is used for stabilizing the flow.

**Fig. 1 Flow domain used for AWSJ nozzle**

The numerical model adopted closely follows the work of G.Hu et.al [7] in which liquid solid two-phase flow is considered and the following assumptions are valid for the present work.

- Flow is taken to be two-phase flow in which the primary liquid phase mixes homogeneously with the particles of equal diameter, constituting the solid phase.
- The primary liquid phase is continuous and incompressible.
- Two-phase flow assumed is steady and characterized by turbulent flow.

**B. The computation of Particulate loading and Stokes number**

Particulate loading and the Stokes number are important parameters that help to identify the appropriate multiphase model. Particulate loading has a major impact on phase interactions and is defined as the mass density ratio of the dispersed phase to that of the carrier phase. The Particulate loading for garnet abrasive is,

\[
\beta = \frac{\alpha \rho_s}{\alpha \rho_l} = \frac{0.1 \times 2300}{998} = 0.230
\]

(1)

The degree of interaction between the phases is intermediate loading, the coupling is two-way i.e., the fluid carrier influences the particulate phase via drag and turbulence, but the particles in turn influence the carrier fluid via reduction in mean momentum and turbulence. All multiphase models can handle this type of problem but it is found that the Eulerian multiphase model seems to be the most accurate one [15]. The average distance between the individual particles of the particulate phase can be estimated by equation developed by Crowe et al.[14].

\[
\gamma = \frac{\rho_s}{\rho_l} = 2.3
\]

(2)

\[
k = \frac{\beta}{\gamma} = 0.100
\]

(3)

\[
\frac{L}{d} = \left(\frac{\pi}{6} + \frac{1}{k}\right)^\frac{1}{3} = \left(\frac{\pi}{6} + \frac{1}{0.10}\right)^\frac{1}{3} = 1.7925
\]

(4)

The average distance between the individual particles of the particulate phase is calculated from equation (4) for an abrasive particle size of \(d_p=63\,\mu m\).

\[
L = 1.7925 \times d_p = 1.7925 \times 0.063 = 0.1129\,mm
\]

Estimating the value of the Stokes number helps to select the most appropriate multiphase model. The Stokes number is defined as the ratio of the particle response time to the system response time is calculated below.

\[
\tau = \frac{d^2 \rho_s \rho_l}{18 \mu \rho_l} = \frac{2300 \times (63 \times 10^{-6})^2}{18 \times 0.001004} = 5.05129 \times 10^{-4}
\]

(5)

\[
t = \frac{l}{v} = \frac{0.0364}{25.6} = 1.4218 \times 10^{-3}
\]

(6)

\[
S = \frac{\tau T}{t_s} = \frac{5.05129 \times 10^{-4}}{1.4218 \times 10^{-3}} = 0.3552
\]

(7)

For Stokes number less than unity, particles will closely follow the fluid flow and any one of the three multiphase models namely Volume of fluid model, Mixture model or Eulerian multiphase model is applicable. Also from the calculation of the effect of particulate loading it is clear that coupling between two phases is intermediate [15]. Hence present numerical simulation is carried using Eulerian multiphase model which though is most expensive in computation, but gives most accurate results. Eulerian Multiphase model is embedded in Fluent software. Fluent solves a multi-fluid granular model to describe the flow behavior of fluid solid mixture. The stresses induced in the solid phase are deduced through an analogy between the random particle motion arising from particle to particle collisions and the thermal gradient of molecules in the fluid.
stream taking into effect the inelasticity of the granular phase. Intensity of the particle velocity fluctuations determines the stresses, viscosity and pressure of the solid phase [15].

The governing equations for mass and momentum conservation are solved for the steady incompressible flow. The coupling between velocity and pressure has been attempted through the phase coupled SIMPLE algorithm developed by Patankar S.V [16] using the power law scheme for the solution. The turbulence is modeled using Realizable $k$-$\varepsilon$ turbulence model. The governing partial differential equations, for mass and momentum conservations are detailed below.

### Continuity Equation

\[
\frac{1}{\rho} \left[ \frac{\partial}{\partial t} \left( \alpha \rho \right) + \nabla \cdot \left( \alpha \rho \mathbf{v} \right) \right] = \sum_{p=1}^{N} \left( \frac{m_{pq}}{pq} - \frac{m_{qp}}{qp} \right) \quad (8)
\]

### Fluid-Solid Momentum Equation

The conservation of momentum equation for the solid phase is as follows.

\[
\frac{\partial}{\partial t} (\alpha \rho_s \mathbf{v}_s) + \nabla \cdot (\alpha \rho_s \mathbf{v}_s^2) = - \alpha \mathbf{g} + \mathbf{v}_s \cdot \nabla \mathbf{p} + \sum_{l=1}^{N} \left[ k_{ls} (\mathbf{v}_l \cdot \mathbf{v}_s) + (m_{ls} \mathbf{v}_s - m_{sl} \mathbf{v}_l) \right] + (F_s + F_{lift,s} + F_{vm,s})
\]

The conservation of momentum equation for the fluid phase is as follows.

\[
\frac{\partial}{\partial t} (\alpha \rho_f \mathbf{v}_f) + \nabla \cdot (\alpha \rho_f \mathbf{v}_f^2) = - \alpha \mathbf{g} + \mathbf{v}_f \cdot \nabla \mathbf{p} + \sum_{p=1}^{N} \left[ k_{pq} (\mathbf{v}_p \cdot \mathbf{v}_q) + (m_{pq} \mathbf{v}_q - m_{qp} \mathbf{v}_p) \right] + (F_f + F_{lift,f} + F_{vm,f})
\]

### III. METHOD OF SOLUTION

#### A. Numerical Scheme

Conservation equations are solved for each control volume to obtain the velocity and pressure fields. Convergence is affected when all the dependent variable residuals fall below 0.00001 at all grid points. Computational domain is modeled using commercially available pre-processor routine called GAMBIT and meshing is carried out using linear quad paved mesh. Wall region in the flow domain is closely meshed using the boundary layer mesh concepts for extracting high velocity gradients near the boundary walls. According to the structure of nozzle and jet characteristics, computational domain is built as axi-symmetric model. Figure 2 shows the computational domain. The grid independence test is performed to check the quality of mesh for solution convergence as shown in figure 3. It is clear from the graph that there is almost negligible variation (not more than 1 %), in the axial velocity distribution for the between mesh geometries consisting of 58320, 87480 and 116640 control volumes. Hence considering lesser computational time required, a mesh geometry consisting of 58320 control volumes has been adopted in this work.

![A portion of the meshed domain near the critical section of AWSJ nozzle](image1)

![Results of grid independence test for multi-step AWSJ nozzle](image2)

#### B. Boundary Conditions and Operating Parameters

Suitable boundary conditions are imposed on the computational domain, as per the physics of the problem. Inlet boundary condition is specified by the operating pressure entering the nozzle. It is assumed that velocity at inlet is uniform across the cross section. At the exit, static pressure of effluxing flow is taken to be zero (gauge), so that the computation would proceed by the relative pressure differences across the grid volumes for the entire domain of the flow. Wall boundary conditions are imposed to bound fluid and solid regions. In viscous flow models, as in the present case, velocity components at the wall are set to zero in accordance with the no-slip and impermeability conditions that exist on the wall boundary. The axis of the nozzle is used to solve the computational domain as axisymmetric problem and suitable boundary conditions are imposed for the same i.e., the gradient of fluid properties are set to zero across the axis line. In the present numerical simulation, mixture of water...
and suspension liquid is treated as primary phase (Suspension liquid) and abrasive is treated as secondary phase.

IV. RESULTS AND DISCUSSION

A. Validation of the Numerical Model

The numerical validation for the flow analysis through the AWSJ nozzle is carried out as cited in the references [7-8] for the purpose of numerical calibration of the computational scheme for a single step nozzle has been used for comparison with the work as cited in the literature shown in figure 4 and figure 5.

![Fig. 4 The velocity distribution along the length of the single step nozzle as given in reference [7]](image)

![Fig. 5 The velocity distribution along the length of the single step nozzle [8]](image)

B. The Effect of Garnet, Aluminum Oxide and Silicon Carbide Abrasive on Skin Friction

The results of simulation carried out on AWSJ multistep nozzle corresponding to the use of different abrasive particles namely garnet, aluminum oxide and silicon carbide at various volume fractions is described in figure 6, 7 and 8. Volume fraction of 5%, 10% and 15% are considered for the analysis. The inlet operating pressure is maintained at 40 MPa and abrasive size is also kept 63 microns for different abrasive materials in all the simulation that are considered in figure 6, 7 and 8. The variation of skin friction coefficient along the length of AWSJ multistep nozzle is described with respect to different volume fraction corresponding to garnet abrasive material in figure 6. Similarly the plot of effect of aluminum oxide and silicon carbide abrasives on skin friction coefficient for different volume fraction is described in figure 7 and 8 respectively. It is seen from these figures that, the skin friction coefficient increases with reduction in cross section of the nozzle along the length of the nozzle. But, it is to be noted that, at converging sections of the nozzle with low angle of cone about 5°, the skin friction raises, but does not attain peak values. The skin friction rises rapidly with the increase in converging angle of the conical section (cone angle 10°) of the nozzle. This is due to higher velocity gradient at this section. The overall pattern of skin friction produced by various abrasive particles at different volume fractions on nozzle surface follows similar trends with difference in magnitude.

![Fig. 6 Plot of effect of garnet abrasive on skin friction coefficient](image)

![Fig. 7 Plot of effect of aluminum oxide abrasive on skin friction coefficient](image)
The maximum skin friction produced for different abrasive materials at various volume fractions is plotted in Figure 9. It is observed from the figure that for each abrasive, the increase in volume fraction results in a decrease in skin friction coefficient produced. This is due to the fact that increased in volume fraction results in an increase in the particle concentration in the water suspension liquid. To transport these abrasive particles in fluid medium a part of the pressure energy will be lost, which leads to reduction in the jet velocity. Hence skin friction coefficient reduces.

Further, the investigation is made on maximum skin friction produced by different abrasive particles namely garnet, aluminum oxide and silicon carbide on skin friction at different volume fraction. It is seen from the figure 9 that, for the same inlet operating pressure at every volume fraction, the skin friction produced by garnet abrasive particles is highest and it is followed by aluminum oxide and silicon carbide abrasive particles. This trend is due to the effect of density of abrasive particles (silicon carbide $\rho = 3170$ kg/m$^3$, aluminum oxide $\rho = 2719$ kg/m$^3$ and garnet $\rho = 2300$ kg/m$^3$). Increase in the density of abrasive particles results in corresponding reduction velocity of the flow due to mass inertial effect and hence there is reduction in skin friction produced. Numerical simulation indicates that the skin friction produced by garnet abrasives is highest which is followed by aluminum oxide and silicon carbide abrasives.

C. The Effect of Garnet, Aluminum Oxide and Silicon Carbide Abrasive on Jet Exit Kinetic Energy

The maximum jet exit kinetic energy produced for different abrasive materials at various volume fractions is plotted in Figure 10. It is seen from the figure 10 that there is a marginal decrease in jet exit kinetic energy with increase in volume fraction of abrasive particles. As explained previously, higher the concentration and density of abrasive particles in the fluid, higher is the fluid inertial resistance while transporting these abrasive particles leading to a reduction in exit velocity. Hence there is a corresponding decrease in jet kinetic energy with increase in volume fraction and density of abrasive particles. It is seen that highest jet exit kinetic energy is produced at lower volume fraction. At each volume fraction, the jet exit kinetic energy produced by garnet abrasive particles is highest which is followed by aluminum oxide and silicon carbide abrasives respectively.

V. Conclusion

Following conclusions are drawn from the present numerical simulation. Increase in abrasive concentration and density results in increase in skin friction coefficient and reduce in jet exit kinetic energy. Numerical simulation indicates that garnet abrasives produce better jet exit kinetic energy than aluminum oxide and silicon carbide. It is expected that higher wall shear stress will induce higher wear of the wall. As shown in figure 9 it may be concluded that garnet as the abrasive particle will induce higher rate of wear of the nozzle.
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