A Simple Constellation Precoding Technique over MIMO-OFDM Systems

Fuh-Hsin Hwang†, Tsui-Tsai Lin, Chih-Wen Chan and Cheng-Yuan Chang

Abstract—This paper studies the design of a simple constellation precoding for a multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) system over Rayleigh fading channels where OFDM is used to keep the diversity replicas orthogonal and reduce ISI effects. A multi-user environment with K synchronous co-channel users is considered. The proposed scheme provides a bandwidth efficient transmission for individual users by increasing the system throughput. In comparison with the existing coded MIMO-OFDM schemes, the precoding technique is designed under the consideration of its low implementation complexity while providing a comparable error performance to the existing schemes. Analytic and simulation results have been presented to show the distinguished error performance.

Keywords— coded modulation, diversity technique, OFDM, MIMO, constellation precoding

I. INTRODUCTION

THE fourth-generation (4G) wireless mobile systems are ambitiously expected to provide reliable and very high data rate services such as video conferencing, multimedia internet access and wide area network in wideband transmission channels. An effective approach in enhancement of data rate over wireless channels is to employ multiple transmit and receive antennas (MIMO) [2], [3]. Under certain conditions, the channel capacity can increase linearly with the number of transmit antennas in MIMO systems due to multiple and independent transmission channels. Orthogonal frequency division multiplexing (OFDM) is known to offer many advantages over single-carrier schemes for high data rate transmission in time-disperse channels since it can overcome the problem raised by multipath distortion without the need of complex equalization (e.g. [7], [8], [12], [13]). For these reasons, systems employing the combination of MIMO and OFDM techniques can enjoy the advantages of the spectral efficiency and high data rate at the same time. Space-time modulation (STM) (e.g. [1]) is mainly designed to exploit the spatial diversity created by multiple transmit antennas for flat fading channels. The large delay spreads over frequency selective fading may destroy the orthogonality of the received signals. By using OFDM modulation, frequency selective channels are transformed into multiple flat fading subchannels [10], and thereby system performance can be improved effectively by the space-time processing even in the fading channels with large delay spreads. To this end, here the design of a coded MIMO-OFDM scheme is studied for a high data-rate wireless communication over fading channels. Typical examples of space-time processing for the coded MIMO-OFDM include space-time trellis codes (STTC) and space-time block codes (STBC) [8], [10], [14], [15]. STTC provides maximum diversity and large coding gains at the expense of implementation complexity which grows exponentially in the transmission rate. For this reason, STTC is unfavorable to usage of large size constellation. STBC offers maximum transmit diversity with low-complexity linear decoding, but it comes with reduced transmission rates while more than two transmit antennas being used.

Originated from the symbol-interleaved coded modulation for single-antenna systems [4], [5], the constellation precoding is an alternative transmit diversity scheme without sacrificing the transmission rate. It can improve error performance over fading channels by maximizing the channel cutoff rate or minimum product distance [4], [5]. Further, the literatures concerning the design of constellation precoding for MIMO schemes can be found in [6], [11], etc. In this paper, the considered constellation precoding code [11] is shown to have a high spectral efficiency, and operate at a low SNR for single-carrier systems over fading. Since OFDM is an efficient modulation for multi-carrier transmission, the constellation precoding technique is combined with MIMO-OFDM in a nature manner, and thus the system with a high data rate for wideband channels is proposed. As any form of interleaving is not required for the precoding technique, the proposed system is especially suitable for the applications which are sensitive to processing delay.

II. SYSTEM DESCRIPTION

A. Transmitter and Diversity Encoder

Consider an MIMO-OFDM system with \( n_t \) transmit antennas, \( n_r \) receive antennas, and \( n_c \) OFDM modulators with \( K \) subcarriers each, signaling in a Rayleigh fading channel. The framework of the transmitter is depicted in Fig. 1. At the input of the transmitter, the information bit stream is fed into the \( M \)-PSK modulator, and then the output symbols are encoded by a multiphase code of length \( n_t \). Blocks of \( n_t \) coded symbols are produced sequentially by the encoder, and the coded symbol stream is partitioned into \( n_t \) parallel branches. Let each symbol out of the same block be mapped to one subcarrier of a different OFDM modulator, respectively. Since \( n_c \) OFDM modulators are equipped in the transmitter, the \( n_t \) coded symbols of the
same block are distributed to \( n_t \) OFDM modulators in order. Here a multi-user environment with \( K \) synchronous co-channel users is considered. During each OFDM frame, there are totally \( n_t K \) coded symbols transmitted from the \( n_t \) antennas simultaneously.

At each time slot, suppose the serial-to-parallel converter outputs a block \( \mathbf{z} = [z_1, z_2, \ldots, z_K]^T \) where \( z_k = \exp(j2\pi d_k/M) \) denotes one specified \( M \)-ary symbol be with \( d_k \) taking value in \( \mathbb{Z}_N \), the ring of integers modulo \( M \). The output block \( \mathbf{z} \) is encoded by the encoder which maps each information-carrying \( \mathbf{d} \) into a \( \mathbb{Z}_N \)-PSK symbol. The coded OFDM symbol, which is transmitted from the \( j \)-th transmit antenna during each OFDM frame interval. In this way, the coded symbols within a codeword \( \mathbf{x} \) are transmitted from each a different antenna. The coded OFDM symbol, which is written as

\[
\mathbf{x}_j = \mathbf{g}_j \mathbf{d} = \begin{bmatrix} g_{j1} \mathbf{d}_1 & g_{j2} \mathbf{d}_2 & \cdots & g_{jK} \mathbf{d}_K \end{bmatrix}^T
\]

with \( \mathbf{g}_j = [g_{j1}, g_{j2}, \ldots, g_{jK}] \) is called the generating vector with the entities taking value in \( \mathbb{Z}_N \), and the output matrix \( \mathbf{Q} \) can be expressed as

\[
\mathbf{Q} = \mathbf{c}^T \mathbf{g} \mod N. \quad (1)
\]

where \( \mathbf{g} = [g_1, g_2, \ldots, g_n] \) is called the generating vector with the entities taking value in \( \mathbb{Z}_N \), and the output matrix \( \mathbf{Q} \) can be expressed as

\[
\mathbf{Q} = [c_1, c_2, \ldots, c_K]^T \equiv [\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_n]
\]

with \( c_i = d_i \mathbf{g} \) and \( \mathbf{q}_j = \mathbf{g} \mathbf{d}_j \) where \( i = 1, 2, \ldots, K \) and \( j = 1, 2, \ldots, n_t \). The codeword \( \mathbf{c}_i \) is then mapped onto an \( N \)-ary phasor block \( \mathbf{x}_i \) for user \( i \) where \( i = 1, 2, \ldots, K \).

Consider the receiver is in an AWGN channel with Rayleigh fading, and suppose that the fading fluctuations remain static but vary from one OFDM frame to another independently. Thus, \( r_{m,j}[n] \) can be represented by

\[
r_{m,j}[n] = h_{m,j} \gamma_{m,j}[n] \mathbf{x}_j[n] + \omega_{m,j}[n]
\]

where \( h_{m,j}[n] \) is Rayleigh distributed, and the AWGN \( \omega_{m,j}[n] \) is a sequence of i.i.d. circularly symmetric complex Gaussian rv’s with zero mean and unit variance. The AWGN, the fading process and the transmitted signal are assumed mutually independent. At the receiver, \( n_t \) maximal ratio combiners (MRC) are equipped, and each is used to combine the \( n_t \) diversity receptions corresponding to one of the \( n_t \) symbols within a coded block \( \mathbf{x} \). For MRC, perfect channel side information for all diversity paths is supposed to be obtained at the receiver. Let \( y_j[n] \) be the output of the \( j \)-th MRC for combining the received diversity signals in response to \( x_j \) emitted from transmit antenna \( j \).

Fig. 1. Transmitter Model.

Fig. 2. Receiver Model.

B. Receiver and Maximum-Likelihood Receiver

Fig. 2 depicts the receiver model. During each OFDM frame, the reconstucted signal at each receive antenna is a noisy superposition of the coded OFDM signals corrupted by fading. The coherently demodulated output of each receive antenna is then fed into an OFDM demodulator to extract the component of the received signals corresponding to the phasor block \( \mathbf{x}_j \) for user \( i \) where \( i = 1, 2, \ldots, K \). Given a specific coded block \( \mathbf{x} = [x_1, x_2, \ldots, x_n] \), \( r_{m,j}[n] \) denotes the demodulated signal in response to \( x_j \), which is transmitted from the \( j \)-th transmit antenna and received by the \( m \)-th receive antenna at the time index \( n \). Consider the receiver is in an AWGN channel with Rayleigh fading, and suppose that the fading fluctuations remain static but vary from one OFDM frame to another independently.
TABLE I.
PARAMETERS FOR THE GOOD CODES

<table>
<thead>
<tr>
<th>n_s</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>z_{min}</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>g</td>
<td>[4 7]</td>
<td>[1 2 4]</td>
<td>[1 5 7 13]</td>
</tr>
</tbody>
</table>

Remark

Q_{z1}^2, Q_{z2}^2, Q_{z3}^2

optimum in the sense of achieving minimum block error probability. Assume all of the diversity paths suffer from independent fading and uncorrelated additive white Gaussian noise. Since \( y_{mj}[n] \) are known due to perfect CSI at the receiver site, the ML decision is equivalent to the maximum likelihood sequence estimation (MLSE) in an AWGN channel. The decision rule is to choose the block \( \epsilon \) iff \( \eta(\hat{\epsilon}) = \max\{\eta(\epsilon)\} \) with the decision metric

\[
\eta(\epsilon) \triangleq \sum_{j=1}^{n_c} [y[n]p_j[n] + y^*_n[n]p_j^*[n]].
\]  

III. ERROR ANALYSIS

A. Pairwise Symbol Error Probability

The performance analysis for the considered MIMO-OFDM scheme is here derived by investigating the pairwise block error probability (PEP) for each user. Let \( \epsilon \) and \( \epsilon' \) be two distinct codewords transformed from the information carrying \( M \)-ary symbols \( e^{j2\pi df/M} \) and \( e^{j2\pi dfe/M} \), respectively. \( \Pr(\epsilon \rightarrow \epsilon') = \Pr(\eta(\epsilon') - \eta(\epsilon) < 0 | \epsilon) \) is the conditional probability that the metric \( \eta(\epsilon') \) is smaller than \( \eta(\epsilon) \) given that \( \epsilon \) was indeed transmitted. Given \( \epsilon \) and \( \gamma \triangleq [y_1, y_2, \ldots, y_{n_c}], \eta(\epsilon) - \eta(\epsilon') \) is a conditional Gaussian random variable whose mean and variance are both equal to \( 2 \sum_{j=1}^{n_c} \gamma_j \kappa_j(\epsilon, \epsilon') \) with \( \kappa_j(\epsilon, \epsilon') \triangleq \sin^2 \left( \frac{\pi(\epsilon_j - \epsilon'_j)}{n} \right) \). Thus, the conditional probability is written as

\[
P(\eta(\epsilon) - \eta(\epsilon') < 0 | \epsilon, \gamma) = Q \left( \sqrt{2 \sum_{j=1}^{n_c} \gamma_j |\kappa_j(\epsilon, \epsilon')|} \right)
\]  

where \( Q(\cdot) \) denotes the Gaussian tail integral. Averaging (7) over \( \gamma \), the PEP results as

\[
Pr(\epsilon \rightarrow \epsilon') = \frac{1}{\pi} \int_0^{\pi/2} \prod_{j=1}^{n_c} \left[1 + \Omega \kappa_j(\epsilon, \epsilon') \csc^2 \theta \right]^{-n_{s_j}} \sin \theta d\theta
\]

A union bound to the symbol error probability (SEP) of the ML rule can be expressed as

\[
P_s \leq 2^{-n_t} \sum_{\epsilon} \sum_{\epsilon' \neq \epsilon} Pr(\epsilon \rightarrow \epsilon')
\]  

B. Chernoff Bound

Using the Chernoff bound technique, \( \Pr(\epsilon \rightarrow \epsilon) \) can be upper bounded as follows:

\[
P(\eta(\epsilon, \epsilon') \geq 0 | \epsilon) \leq E \left[ e^{\lambda \eta(\epsilon, \epsilon')} \right] | \epsilon
\]

where \( \eta(\epsilon, \epsilon') \triangleq \eta(\epsilon) - \eta(\epsilon') \), \( E[\cdot] \) denotes expectation, and \( \lambda \) is a Chernoff bound parameter to be optimized. Given \( \epsilon \) and \( \gamma \), the conditional expectation of \( e^{\lambda \eta(\epsilon, \epsilon')} \) can be derived as

\[
E \left[ e^{\lambda \eta(\epsilon, \epsilon')} | \epsilon, \gamma \right] = \prod_{j=1}^{n_c} e^{(\lambda^2 - 2\lambda) \gamma_j \kappa_j(\epsilon, \epsilon')}
\]  

Averaging (10) over \( \gamma \), it then follows that

\[
E \left[ e^{\lambda \eta(\epsilon, \epsilon')} | \epsilon \right] = \prod_{j=1}^{n_c} \left( \frac{\Gamma(n_c)}{\Omega^{n_c}} \int_0^{\infty} e^{(\lambda^2 - 2\lambda) \gamma_j \kappa_j(\epsilon, \epsilon') - \gamma_j/\Omega} \gamma_j^{-n_c - 1} \gamma_j d\gamma_j \right)
\]

where \( \Gamma(\cdot) \) is the gamma function. Employ the formula

\[
\int_0^\infty x^p e^{-\beta x} dx = \frac{\Gamma(p+1)}{\beta^{p+1}}, \beta > 0, \text{ and (11) results as}
\]

\[
E \left[ e^{\lambda \eta(\epsilon, \epsilon')} | \epsilon \right] = \prod_{j=1}^{n_c} \left( 1 - (\lambda^2 - 2\lambda) \kappa_j(\epsilon, \epsilon') \Omega \right)^{-n_{s_j}}
\]

The Chernoff bound to \( \Pr(\epsilon \rightarrow \epsilon) \) can be represented by

\[
\Pr(\epsilon \rightarrow \epsilon) \leq \prod_{j=1}^{n_c} \left( 1 + \kappa_j(\epsilon, \epsilon') \Omega \right)^{-n_{s_j}}
\]  

as the Chernoff bound parameter \( \lambda \) is optimized to one. Substitute (13) into (8), and another upper union bound to SEP is achieved.

IV. CODE SEARCH

The cutoff rate provides a valid measure while comparing the coded modulation schemes. Using the Chernoff bound (13), the cutoff rate \( C_R \) for the proposed scheme in bits/transmitted block can be derived as [9]

\[
C_R = 2n_t \log_2 \left( \sum_{\epsilon \neq \epsilon'} \prod_{j=1}^{n_c} \left( 1 + \kappa_j(\epsilon, \epsilon') \Omega \right)^{-n_{s_j}} \right).
\]  

The expression (14) of the cutoff rate includes a summation of the pairwise product terms. For the pairwise \( \epsilon \rightarrow \epsilon \), the product term in the summation of (14) can be approximated as

\[
\prod_{j=1}^{n_c} \left( \kappa_j(\epsilon, \epsilon') \Omega \right)^{-n_{s_j}}
\]  

for \( \Omega > 1 \). Define the effective length of the pairwise \( \epsilon \rightarrow \epsilon \) as

\[
\Omega^{-n_{s,\text{min}}} \left( \rho_{\text{min}} \right)^{-n_{s_j}}
\]  

where \( n_{s,\text{min}} \) is the nonzero entry number of the codeword difference. Thus, the approximated product term which dominates the summation of (14) is expressed by

\[
\Omega^{-n_{s,\text{min}}} \left( \rho_{\text{min}} \right)^{-n_{s_j}}
\]  

International Scholarly and Scientific Research & Innovation 4(5) 2010 904 scholar.waset.org/1999.5/1420
where \( \Xi_{\min} \) denotes the minimum value of the effective length among all pairwise \( c \rightarrow e \), and

\[
\rho_{\min} = \min_{c \rightarrow e} \left\{ \Xi_{\min} \right\}
\]

with \( \kappa_j(c, e) \) being the \( \kappa_j(c, e) \) in response to the pairwise with \( \Xi_{\min} \). Conveniently, here the code merit figure is defined as \( \rho_{\min} \) which represents the minimum value of the product terms for all pairwise \( c \rightarrow e \) with the effective length being \( \Xi_{\min} \). The approximated dominant term (16) is inversely proportioned to \( n_\Omega \Xi_{\min} \) to the power of \( \Omega \). This implies the diversity order can reach a maximum value of \( n_\Omega n_r \) as the minimum effective length \( \Xi_{\min} \) equals to the number of transmit antennas. Since a code with a larger code merit figure \( \rho_{\min} \) will have a larger approximated cutoff rate, the code search criterion is to search the good codes achieving the maximum \( \rho_{\min} \). For a specified \( Q_{n_r} \), the code family with the code length of \( n_\Omega \) and the entries taking in \( \mathbb{Z}_N \). Given a specified \( Q_{n_r} \), for the sake of finding the good codes, one has to exhaustively search for all admissible forms of the generating vector \( g \). The complete search steps for a code family \( Q_{n_r} \) are presented as follows:

1. **Step 1**: Determine the values of \( \Xi_{\min} \) for all codes in \( Q_{n_r} \).
2. **Step 2**: Select the candidate codes with the largest minimum effective length \( \Xi_{\min} \) in \( Q_{n_r} \).
3. **Step 3**: From the candidates, pick out the good code(s) yielding the largest \( \rho_{\min} \).

### V. NUMERICAL AND SIMULATION RESULTS

Without loose of generality, the cases with \( M = 2^{n_r} \) are considered in this section. Following the steps of code search in Sec. IV, some good codes have been found through a complete computer search. The codes remarked by \( Q_{16}^{27}, Q_{9}^{3} \) and \( Q_{16}^{4} \) are the best ones found in the code families with \( n_r = 2, 3 \) and 4, respectively. Table 1 lists the generating vectors \( g \) and minimum effective length \( \Xi_{\min} \) for the codes. The computer simulation of the constellation precoded MIMO-OFDM system is realized by Monte-Carlo approach. The subcarrier number \( K \) for each OFDM modulator is 256 in the considered system. Fig. 3 shows the cutoff rate values of the codes \( Q_{16}^{27}, Q_{9}^{3} \) and \( Q_{16}^{4} \) with \( n_r = 1, 2 \) and 3, respectively. Fig. 4 presents the SEP union bounds of the three MIMO-OFDM systems employing the codes \( Q_{16}^{27}, Q_{9}^{3} \) and \( Q_{16}^{4} \) with \( n_r = 1, 2, 3 \), respectively, over Rayleigh fading. The error probability of uncoded BPSK for OFDM system without diversity scheme is also included in the figure for performance comparisons. It is shown that distinguished diversity gain can be achieved by the
proposed MIMO-OFDM scheme with the three codes. For instance, gains of 16 dB, 17 dB and 18.2 dB are achieved at the SEP union bound of 10^{-3} over the uncoded OFDM system by using the the codes Q_{2}^{b}, Q_{2}^{a} and Q_{2}^{ma}, respectively, for the proposed MIMO-OFDM schemes with two receive antennas. Fig. 5 investigates the impact of multipath fading on the error performance of the proposed scheme. Here the Rayleigh fading is supposed to have a channel length of 4. The simulation results of the error probability for the proposed MIMO-OFDM system using the codes Q_{2}^{b} are depicted in Fig. 5 for the delay spreads of 100 ns, 200 ns, and 300 ns, respectively. The more the delay spread is, the more the performance degradation will be. This is because the codes are not optimized for the multipath fading. Nevertheless, the scheme is shown to be robust over multipath fading with channel length of 4 while the delay spread is small than 100 ns even the code is not optimized for the multipath case.

VI. CONCLUSION

In this conference paper, the design of a simple constellation precoding for MIMO-OFDM systems in a wireless Rayleigh fading environment is studied. The proposed technique exploits the maximum diversity \( n_{r}n_{t} \) given by \( n_{r} \) transmit and \( n_{t} \) receive antennas. For the found good codes, analytical and simulation results has been conducted to assist performance evaluation. It shows that the proposed MIMO-OFDM system is capable of providing a high data-rate and reliable transmission at the cost of a little increased baseband implementation complexity and processing delay.

ACKNOWLEDGMENT

This work was sponsored by the National Science Council, R.O.C., under the Contract NSC 98-2221-E-017-006-

REFERENCES


Fuh-Hsin Hwang was born in Chayi, Taiwan, in 1968. He received B.S. and M.S. degrees, both in telecommunication engineering, from National Chiao Tung University at Hsinchu, Taiwan in 1991 and 1993, respectively and the Ph.D. degree in electrical engineering, from National Central University at Jhong-Li, Taiwan in 2003. From 1995 to 2004, he was with the Department of Electrical Engineering, Hwa-Hsia Institute of Technology, Taipei, Taiwan. In August 2004, he joined the faculty of the Department of Optoelectronics and Communication Engineering, National Kaohsiung Normal University, Kaohsiung, Taiwan as an Associate Professor. His research interest is focused on the wireless communications, including coded modulation, diversity schemes for MIMO channels and multicarrier communications.

Tsiu-Tsai Lin was born in Taoyuan, Taiwan, in 1967. He received the B.S., M.S., and Ph.D. degrees from National Chiao-Tung University, Hsinchu, Taiwan, in 1991, 1993, and 1997, respectively, all in communication engineering. From 1997 to 2000, he was in the Computer & Communications Research Laboratories, Industrial Technology Research Institute. In 2001, he joined the faculty of National United University, Miaoli, Taiwan, where he currently holds a position as Associate Professor in the Department of Electronic Engineering. His current research interests include communication signal processing, geo-location, and smart antenna for wireless communications.

Chih-Wen Chan was born in Taipei, Taiwan, in 1983. She received the B.S. degree from United University, Miaoli, Taiwan in 2006, and M.S. degree from National Kaohsiung Normal University, Kaohsiung, Taiwan, in 2009. In 2009, she was with Chiao-Tung University, Taiwan, where she currently works as a research assistant in the Institute of Communication Engineering. Her current research interests include wireless communication and channel coding techniques.

Cheng-Yuan Chang was born in Taiwan, R.O.C., in 1968. He received the B.S. and M.S. degrees in control engineering from National Chiao-Tung University, Hsin-Chu, Taiwan, R.O.C., in 1990 and 1994, and the Ph.D. degree in electrical engineering from National Central University in 2000. From 1994 to 2007, he was with the Department of Electronic Engineering, Chung Yuan University. In August 2007, he joined the Department of Electrical Engineering, Chung Yuan Christian University, Jhongli, Taiwan, R.O.C. He is currently an Associate Professor. His research interests are in the area of digital signal processing application and filter design.