World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
Vol:3, No:9, 2009

Comparison of frequency estimation methods for reflected signals in mobile platforms

Kathrin Reinhold

***Abstract*—Precise frequency estimation methods for pulse-shaped echoes are a prerequisite to determine the relative velocity between sensor and reflector. Signal frequencies are analysed using three different methods: Fourier Transform, Chirp Z-Transform and the MUSIC algorithm. Simulations of echoes are performed varying both the noise level and the number of reflecting points. The superposition of echoes with a random initial phase is found to influence the precision of frequency estimation severely for FFT and MUSIC. The standard deviation of the frequency using FFT is larger than for MUSIC. However, MUSIC is more noise-sensitive. The distorting effect of superpositions is less pronounced in experimental data.**

***Index Terms*—frequency estimation, pulse-echo-method, superposition, echoes**

I. Introduction

REMOTE sensing on mobile platforms can be used to measure the relative velocity of sensor and reflector via the Doppler effect. For velocities much smaller than the propagation velocity ($v \ll v_\mathrm{p}$), the frequency shift $\Delta f = f_r - f_e$ between the spectral maximum of the emitted pulse f_e and the reflected signal f_r in both directions can be calculated via

$$\Delta f = 2\,\frac{v}{v_\mathrm{p}}\,f_e\,[1]. \tag{1}$$

Employing a pulse echo method, the frequency estimation of reflected echoes faces several challenges due to the short duration of the signal, the superposition of echoes and additive noise in the propagation channel. In some special situations a modelling of the echo parameters strongly improves the accuracy of frequency estimation methods. This has been shown by Demirli et al. [2], where methods of maximum likelyhood estimation and expectation maximization are used as well as the assumption of a Gaussian envelope function. Relating to the topic of echo superposition, this has previously been studied by Martinsson et al. in the domain of material characterization [3]. The result is a complete separation of the overlapping echoes in this specific situation after a strong modelling of the data. But there are also disadvantages of strong parametric models: Modelling cases with unknown parameters like the size of the Doppler effect is extremely challenging. Further problems are industrial restrictions as the needs for saving costs and computational time.

In this paper we present the comparison of two methods for time-invariant frequency estimation. The first method is the Fast Fourier Transform (FFT) used with a zero padding (50 times the signal duration) and a rectangular window since a small main lobe of the spectrum is desired. As a second method we use the MUSIC algorithm as described in [4] with the order estimation of one real frequency component. That corresponds to a soft modelling in case of the MUSIC algorithm whereas the FFT does not require any a-priori knowledge. For visualizing the change of frequency components over time, the Chirp Z-Transform (CZT) is used. With this method, the frequency resolution in a selected region of interest is significantly improved compared to the FFT or a short-time FFT [5]. The underlying algorithm for the CZT and possible applications can be found in [6] and [7], respectively. In this paper the spectral maximum of a certain time window is visualized which is a projection of the original time-frequency distribution of overlapping time windows calculated by the CZT.

Modelling the echo shape with a Gaussian function as in [2] or [8] does not match echoes with a longer signal duration. So we propose another approach derived from experiments.

II. Simulation parameters

The simulation assumptions and especially the envelope function are derived from measured signals as visualized in the upper part of figure 1. This echo was reflected on a single sphere shaped reflector. The envelope used for further simulations is based on a Gaussian function with a bandwidth of 0.1 relative to the carrier frequency. A additional flat part at the maximum of the Gaussian function is inserted with a duration of 6 full periods or wavelengths. A simulated echo with the derived envelope function is shown in the lower part of figure 1. The envelope function h corresponds to a signal length of about $500\,\mu$s. So the echo function $e_i(f,t)$ which is dependent on frequency f and time t is calculated as

$$e_i(f,t) = h \cdot \cos(2\pi f t + \phi) + n(t). \tag{2}$$

For simplicity, the simulated echo contains only one frequency component of $f = 48\,\mathrm{kHz}$. Superimposed noise $n(t)$ is simulated by adding random numbers with a Gaussian distribution, so $n(t) \propto N(0,\sigma)$. For further analysis a sampling rate of $f_S = 1\,\mathrm{MHz}$ is used. This value is far away from the limit of the Nyquist theorem and gives a good resolution in the time domain. The simulated echoes contain a second random component in addition to the added noise: a random initial phase generator calculating the variable ϕ. So a uniformly distributed phase between 0 and 2π is the starting point for the cosine carrier frequency.

III. Results

At first, the FFT and the CZT are applied to recordings on three static triple reflectors ($v = 0\,\mathrm{km/h}$) with a sampling

K. Reinhold is employed at Robert Bosch GmbH, Leonberg, Germany, e-mail: (Kathrin.Reinhold@de.bosch.com).

Manuscript received June 3, 2009; revised June 19, 2009.

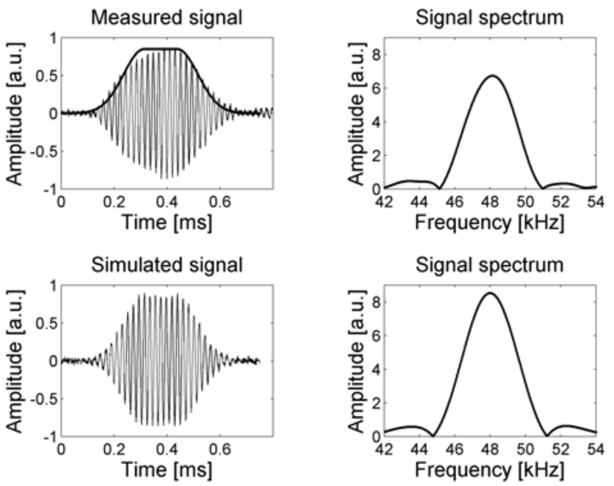

Fig. 1. Top: Visualisation of a measured echo from a single sphere shaped reflector with the corresponding spectrum calculated by FFT. Bottom: Derived simulated signal with additional noise and its FFT spectrum.

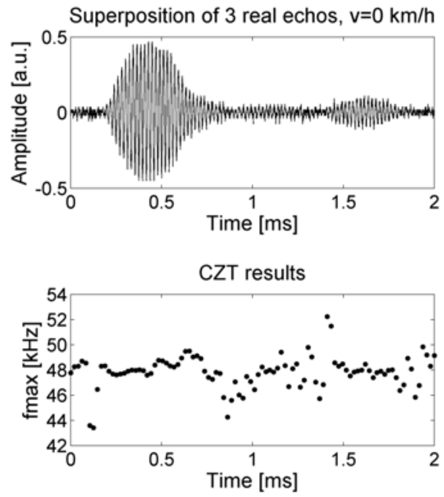

Fig. 2. Top: Experimental data of a superposition of real echoes in a static case. Bottom: Results of the CZT showing the course of frequency over time.

rate of 1 MHz (for original data see figure 2, upper part). Two echoes are superimposed at about 0.5 ms, and a third small echo is observed at about 1.5 ms. The CZT (lower part of figure 2) is calculated with the following parameters: The zoom region was chosen as the frequencies between 42 and 54 kHz. Using 600 steps, this results in a frequency binning of 20 Hz. The time segments have a size of $100\,\mu\mathrm{s}$ with an overlap of $80\,\mu\mathrm{s}$. A Hamming window is used before the transform. One observes a nearly constant frequency of roughly 48 kHz with less spread during the middle part of the echoes around 0.4 ms and 1.5 ms while the spread rises for parts where only noise contributes to the signal. At about $t = 0.5\,\mathrm{ms}$ two small oscillations in the spectral maximum are visible. This happens at a time where the echoes probably overlap.

The resulting spectrum calculated by FFT is shown in the upper part of figure 3. It contains three or more main peaks. In contrast to the simulations shown below, the highest maximum is positioned at 47.93 kHz, thus being close to the expected frequency of 48 kHz.[1] The lower part of figure 3 shows the pseudospectrum calculated by the MUSIC-algorithm. According to the model assumption of a single real frequency component in the signal, one broad spectral peak appears without the oszillations shown in the FFT spectrum. The MUSIC algorithm yields a result of 47.73 kHz.

Furthermore, a measured reflection from three triple reflectors is evaluated with a relative velocity of about 28 km/h between sensor and reflectors. In figure 4 the superposition of three real echoes and the results of the CZT are shown. Here, the three echoes can be separated visually. Considering the results of the CZT, an almost constant value for the spectral maximum is found during the duration of the three echoes, while the spread increases in regions of noise at the end of the signal. There is one exception at about 0.95 ms at the overlap between echo 2 and 3 where a large variation in the spectral maximum is observed.

[1]There are small variations in the emission frequency of the sensors.

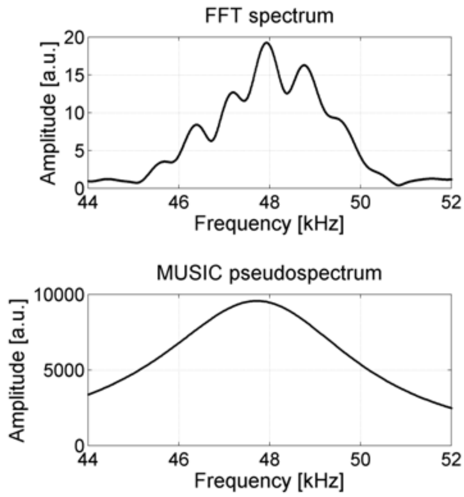

Fig. 3. Corresponding spectra to figure 2, calculated using FFT and MUSIC. The results are $f = 47.93\,\mathrm{kHz}$ for the FFT and $f = 47.73\,\mathrm{kHz}$ for the MUSIC algorithm.

The corresponding FFT spectrum for moving reflectors is given in the upper part of figure 5. The maximum value is found to be $f = 50.13\,\mathrm{kHz}$ whereas a frequency change of $f = 50.11\,\mathrm{kHz}$ was predicted by the Doppler effect. The MUSIC frequency estimation, shown in the lower part of the figure, gave a result of 49.77 kHz (predicted: 49.94 kHz)[2]. As in the other real spectrum for the static case shown in figure 3 the spectral maximum is again in the range predicted by the Doppler effect for both methods.

After the analysis of experimental data, the simulation of echoes described in section II can be extended from one echo to multiple echoes. Assuming a linear superposition, the reflected signal is calculated by adding echoes e_i with different amplitudes a_i and Gaussian noise $n(t) \propto N(0, \sigma)$ with a

[2]For a given initial frequency of 48 kHz, a Doppler shift of 1 kHz translates into a relative velocity of 13 km/h (cf. formula 1). The resulting frequencies of the static case are the basis of the predictions for both evalution methods.

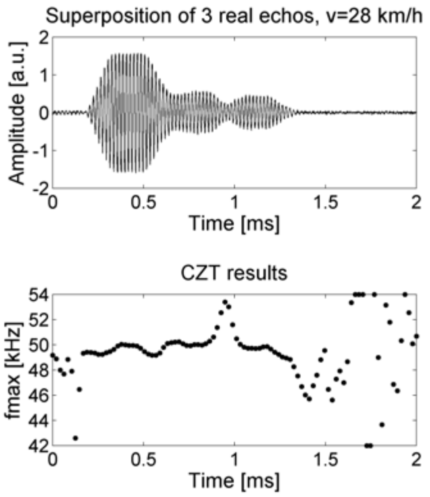

Fig. 4. Top: Experimental data of a superposition of three echoes in a dynamic case. Bottom: Results of the CZT showing the course of frequency over time.

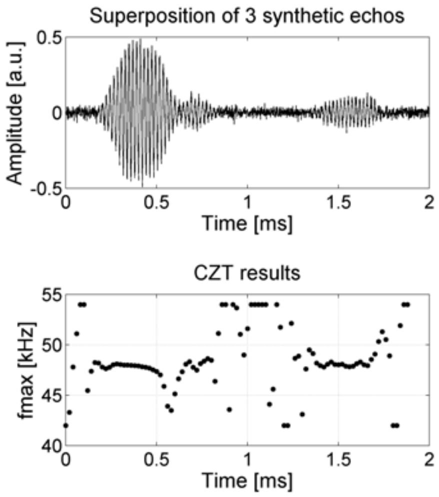

Fig. 6. Top: One example for simulated data of a superposition of three echoes in a static case. Bottom: Results of the CZT showing the course of frequency over time.

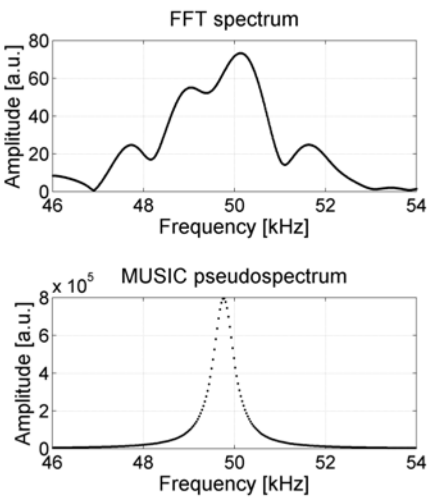

Fig. 5. Corresponding spectra to figure 4, calculated using FFT and MUSIC.The results are $f = 50.13\,\mathrm{kHz}$ for the FFT (predicted: $f = 50.11\,\mathrm{kHz}$) and $f = 49.77\,\mathrm{kHz}$ for the MUSIC algorithm (predicted: $f = 49.94\,\mathrm{kHz}$).

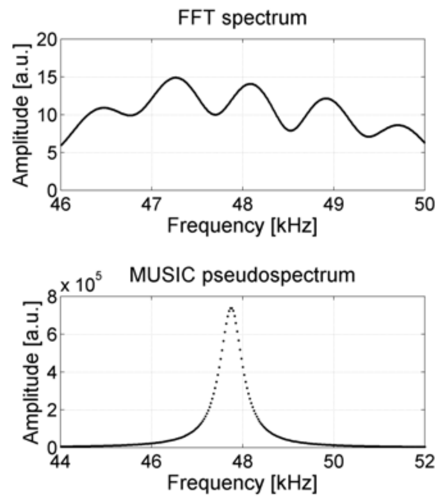

Fig. 7. Corresponding spectra to figure 6, calculated using FFT and MUSIC. The results are $f = 47.27\,\mathrm{kHz}$ for the FFT and $f = 49.76\,\mathrm{kHz}$ for the MUSIC algorithm (prediction for both methods: $f = 48.00\,\mathrm{kHz}$).

variable standard deviation σ:

$$E(t) = \sum_{i=1}^{3} a_i \cdot e_i(f,t) + n(t), \tag{3}$$

where single echoes e_i are generated by formula 2.

In figure 6 one possible resulting signal of a synthetic linear superposition of three echoes with a small noise component of $\sigma = 0.02$ is shown. The envelope curves and the temporal overlap are chosen in a way to maximize the resemblance with the measured data of figure 2.

In the lower part of figure 6, the time-dependent variation of the spectral maximum calculated by the CZT is visualised. One observes the following effects: There are two regions with an almost constant frequency between 0.2 and 0.5 ms and between 1.4 ms and 1.7 ms. In these constant parts of the signal there is no visible overlapping between different echoes. The boundaries of the time segment are of little relevance as the signal amplitude there is very low. Additionally, one finds a big spread of the spectral maximum between the constant parts in the middle of the signal. This is most likely due to the added noise in the signal and the lack of an echo with a defined frequency in that region. Moreover a nicely resolved minimum of the signal is found at about 0.6 ms, where echoes 1 and 2 overlap. One possible interpretation is that the superposition of the echoes induces additional frequency components into the signal due to rapid phase changes.

The resulting FFT spectrum for the generated signal of figure 6 is shown in the upper part of figure 7. Instead of one single peak at 48 kHz as observed for one reflector, three spectral peaks are visible which are very similar in their amplitudes. In this extreme example, the absolute spectral maximum is found at a frequency of 47.27 kHz instead of the

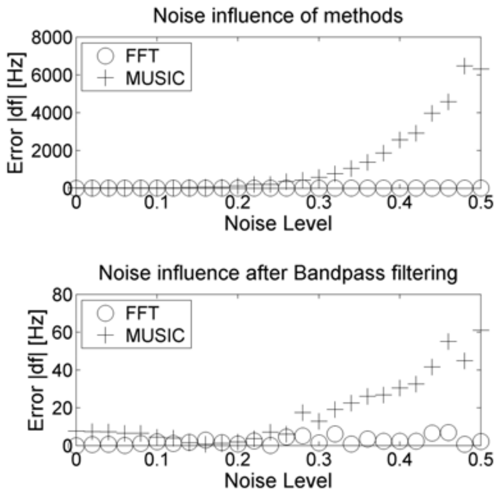

Fig. 8. Comparison of the noise dependent error in frequency estimation of the FFT (circles) and MUSIC-algorithm (crosses) before and after the application of a bandpass filter. Note different scales.

correct value of 48 kHz used in the simulations. The frequency estimation of the MUSIC algorithm yields a result of 47.76 kHz which is much closer to the correct value. It contains only a single peak visible with the resulting frequency, according to the model assumption.

Due to the random components of the signal, the spectra of simulated signals generally vary in shape. As the FFT evaluation is not very sensitive to noise, the relevant factor is the random phase generator for the initial phase of the echoes. For this reason, the same signal with the same envelope functions, noise level and temporal overlap as in figure 6 is simulated 1000 times and a statistical evaluation is performed. All other settings remain constant. The result is a mean value of $f_{max} = 47,982 \pm 0.441\,\mathrm{kHz}$ for the FFT frequency estimation. After repeating the same number of simulations and evaluating the signals with the MUSIC algorithm, a result of $f_{max} = 48,004 \pm 0.185\,\mathrm{kHz}$ is obtained. Hence, the standard deviation of the FFT is found to be more than twice the standard deviation of the MUSIC algorithm. The enormous size of the standard deviations is most likely due to the frequency oszillations at the overlap of echoes. This points as a possible improvement to cutting out parts of the signal for the evaluation instead of taking the whole signal. One selection criterion could be a minimal variance in the instantaneous frequencies.

Furthermore, the noise influence onto the frequency estimation methods is evaluated. The results are shown in figure 8. The MUSIC algorithm was found to be much more noise sensitive than the FFT method, resulting in a much higher frequency estimation error for the same noise level. After the application of a bandpass around the frequency region of interest, the performance of the MUSIC algorithm is strongly improved whereas the influence of the bandpass filtering onto the FFT is small. However, even after the filtering, the frequency estimation error of the MUSIC algorithm for high noise levels is still larger than the error of the FFT method.

IV. Conclusion and Discussion

The motivation for this work is to understand the impact of superimposed echoes (from multiple reflectors) on frequency estimation methods. In simulations, one important influence factor is found: the increased error in the frequency estimation of superimposed echoes is most likely due to the phase change at the overlap of the echoes. Qualitatively, we find that the resulting spectra of the FFT vary in shape according to the corresponding initial phase of the three simulated echoes and obtain at least three spectral maxima of comparable sizes. Quantitatively, the standard deviation of the spectral maximum calculated by the FFT is more than twice as large as the standard deviation of the MUSIC algorithm for a low noise level. The MUSIC algorithm, however, shows an increased error for high noise levels. So the absolute size of the standard deviation for both methods is in the order of magnitude of the Doppler effect that one desires to observe. Therefore, the frequency estimation errors for multiple reflectors cannot be neglected. According to the results of the simulations, it should be avoided to insert signal segments with highly overlapping parts into one of the analyzed frequency estimation method in order to reduce the error.

Real superimposed echoes of three reflectors for both static and dynamic situations are also tested. At first sight, the results of frequency estimation methods in measured data seem to be slightly better than the results found in simulations. A larger statistic sample of experimental data has to be analyzed in order to find out whether these experimental results are representative.

References

[1] H. Lindner, *Physik für Ingenieure*, Fachbuchverlag Leipzig im Carl-Hanser-Verlag, Muenchen, Wien, 15th edition, 1999.
[2] R. Demirli and J. Saniie, "Model-based estimation of ultrasonic echoes. part i: Analysis and algorithms," *Ultrasonics, Ferroelectrics and Frequency Control, IEEE Transactions on*, vol. 48, no. 3, pp. 787–802, May 2001.
[3] J. Martinsson, F. Hägglund, and J.E. Carlson, "Complete post-separation of overlapping ultrasonic signals by combining hard and soft modeling," *Ultrasonics*, vol. 48, no. 5, pp. 427 – 443, 2008.
[4] S.M. Kay, *Modern Spectral Estimation: Theory and Application*, 1988.
[5] T.T. Wang, "The segmented chirp z-transform and its application in spectrum analysis," *Instrumentation and Measurement, IEEE Transactions on*, vol. 39, no. 2, pp. 318–323, Apr 1990.
[6] L. Rabiner, R. Schafer, and C. Rader, "The chirp z-transform algorithm," *Audio and Electroacoustics, IEEE Transactions on*, vol. 17, no. 2, pp. 86–92, Jun 1969.
[7] L. Rabiner, R. Schafer, and C. Rader, "The chirp z-transform algorithm and its applications," *Bell Sys. Tech. J*, 1969.
[8] G. Cardoso and J. Saniie, "Ultrasonic data compression via parameter estimation," *Ultrasonics, Ferroelectrics and Frequency Control, IEEE Transactions on*, vol. 52, no. 2, pp. 313–325, Feb. 2005.