**Abstract**—Due to the liberalization of countless electricity markets, load forecasting has become crucial to all public utilities for which electricity is a strategic variable. With the goal of contributing to the forecasting process inside public utilities, this paper addresses the issue of applying the Holt-Winters exponential smoothing technique and the time series analysis for forecasting the hourly electricity load curve of the Italian railways. The results of the analysis confirm the accuracy of the two models and therefore the relevance of forecasting inside public utilities.

**Keywords**—ARIMA models, Exponential smoothing, Electricity, Load forecasting, Rail transportation.

## I. INTRODUCTION

The liberalization of countless electricity markets has been forcing many firms (producers, electric utilities, big consumers and traders) to change the way of buying and selling the energy needed to function in the current market scenario. This scenario makes forecasting activity crucial for an increasing number of companies. Forecasting model can translate in significant savings or seriously affect the operational cost structure of the firm [1]-[2]. Several models have been proposed in terms of functional representation and estimation procedure [1], [3]-[5]. The results obtained have been very different in terms of accuracy and robustness. This is due to the wide range of cases treated in literature. Statistic approaches try to obtain forecasts through a mathematical combination among past observed values and some other exogenous variables. On the other hand, non parametric models can take into account complexity and non linearity of the data structure but they remain a sort of "black box" [6]-[11].

There are two reasons for writing this article. Firstly, the recent reformation of the Italian electricity market oblige companies to carefully evaluate each contract and market request. This can happen only if the load curve features with respect to time are known. The second reason is related to the opportunity to apply, for the first time in Italy, some statistical models to analyse the load curve of the Italian railways, the first consumer of electricity in the country. Using a particular class of models, the goal of this work is both theoretical and practical, time providing an actual base for all those businesses subject to epochal changes in their own market sectors.

The paper is organised as follow: as the growing importance of time horizon from a forecasting point of view, Section II presents a short literature distinguishing between short, medium and long term forecasting. Section III presents a comprehensive review of the literature where the application of different methodologies to the electricity sector is reported. Section IV is dedicated to the application of two quantitative methods to the Italian railways. The data set is presented in Subsection A while Subsections B and C describe respectively the results deriving form the application of the exponential smoothing model and time series analysis techniques. Conclusions and main results are reported in Section V.

## II. SHORT REVIEW OF THE LITERATURE FROM THE TIME HORIZON POINT OF VIEW

Choosing which model and methodology to apply depends on the forecast. In the field of long-medium term forecast, both end-use and econometric approaches are commonly used and even, where possible, a combination of the two. The first class of models requires major data in order to guarantee a good performances [12] and is sensitive with respect to the quality and quantity of the data set. On the other hand, the econometric approaches are predominantly interested only in economic variables. Combinations of these two models are also possible. Sometimes the two methodologies just described require major human activity. Because of these, and other, hitches, different solutions have been suggested [13]-[14].

On the other hand a great array of models has been developed for short term forecasting, based on statistical science and artificial intelligence. A first class considers the similarities among historical data of the same day (similar day method). The forecasted values are calculated by elaborating the information gathered for similar days. The preferred statistical technique is by far the regression analysis. Reference [15] proposes some models for forecasting the peak load of the electricity load curve for the following day (see also [16]-[19] on the subject). Another class of models is based on the use of stochastic models for the analysis of time series. Because of the importance of climatic and temporal variables ARIMAX (autoregressive integrated moving average with exogenous variables) models are widely used in the field of electricity load forecasting [20]-[22]. The artificial neural networks used in the electricity sector since the 90s play a significant role as they also allow for the introduction of non linear relations [23]. Examples of electricity load forecasting by neural networks can be found in [24], [25] and [26]. A different approach used in short term forecasting is based on expert systems which feed rules and procedures through an informatics procedure in order to turn out forecasts.
without human support. An interesting application of this technique is offered in [27] and [28]. Electricity load forecasting is also implemented with the use of the fuzzy logic approach resulting in an excellent outcome without the need to mathematically represent inputs and outputs [29]-[31]. Lastly we mention the Support Vector Machine as powerful technique for resolving classification and regression problems [32]. This method has been successfully applied for solving short term problems related to forecasting [33]-[36].

III. LITERATURE REVIEW ON ELECTRICITY LOAD FORECASTING

Electric companies have been using forecasting models for a long time and during the last ten years such models have been perfected by adding structures an applications. In addition, major attention has been given to the horizon and short term forecasting has become more and more relevant as a direct consequence of market liberalization throughout the world.

Statistical approaches are based on a mathematical representation of the load curve where the curve depends on time, climatic conditions and some others variables. The similar day method consists of searching historical data in the same time frame of the day which is being forecast. Such similarities can be found by comparing the same days of the week or year, or by considering the dynamics of meteorological variables. The load curve chosen as a function of its correspondences represents the forecast at the time t. This methodology is often used as a benchmark for more complex models.

Regression models are the most frequent techniques to short term forecasting, using, as main variables climatic conditions, type of customer and day of the week. Linear regression models and weighted least squares are very frequently applied by a large number of electricity companies. Interesting contributions can be found in [17] and [18]. A quantification of the different components of the load curve for electrical substations by means of a least squares technique is presented in [37] while a regression model which explicitly considers the climatic conditions can be found in [38]. Reference [16] presents a two day ahead forecasting model which considers sensitivity variable depending on the weather. The regression analysis for investigating the features of different types of customers in several months and days of the year is proposed in [39]. An application of a regression model for the Saudi Arabia east zone using weather data, solar radiation, population and gross domestic product as explicative variables is presented in [40] while in [19] the probability density function and the factors influencing it is revised. Reference [41] presents an annual regression model considering holidays and a series of seasonal features contributing to overall yearly electricity demand. In [42] a semi parametric Bayesian regression model for short term forecasting in the gross market is developed.

Exponential smoothing is the classical model used in the field of load curve forecasting. As far as the electrical sector is concerned, this methodology has produced satisfactory results and therefore lots of companies and utilities have been using this technique. As pointed out in [43] exponential smoothing is often used in the Holt-Winter version with the additional element of periodicity (for comparative studies see [44] and [45]). The application of the Holt-Winter method in areas characterized by strong growth rates is discussed in [46] while [47] offers a hybrid approach where exponential smoothing, spectral analysis and autoregressive additive structure are intermingled. Reference [43] proposes an approach in which firstly the seasonal component is removed by a Holt-Winter model and then an ARIMA structure for the residual is implemented. A new technique based on optimal smoothing for removing trend is proposed, with few encouraging results, in [48].

It is common knowledge that statistical models based on time series analysis are founded on the assumption that data own an internal structure. This set of forecasting models aims to explore and recognize said structure. However, because of the peculiar features of the load curve, various authors point out that the use of time series based models can be misleading, for example, when used to analyse fast growing geographical areas. Even though these limits, time series analysis has been widely used among a variety of operators in the electrical sector. Autoregressive models express the load curve level by means of a linear combination of its past values (see [49] and [50]). In [51] an autoregressive model with partial autocorrelation analysis is suggested while [52] analyses an autoregressive model where the minimum number of parameters for the casual component, the refusal of subjective judgments and the accuracy of the forecast are computed by means of a particular algorithm. An interesting analysis is presented in [53] where it is proposed a forecasting model for every hour of the day. The combination of autoregressive and moving average leads to a very relevant class of models in the field of load curve forecasting, namely the ARMA model. A methodology that considers temperature and breaks down the deterministic and the stochastic component, the latter modelled as an ARMA process, is presented in [54]. Reference [20] shows an ARMA structure in which the parameters are estimated and updated by means of a recursive procedure based on weighted recursive least squares while in [55] is presented an ARMA model where forecasting errors are employed to update the model. An adaptive model which takes into account cyclical aspects and where parameters, structure and the model’s order can change according to different conditions can be found in [56]. Reference [57] proposes an ARMA on deseasonalized data with hyperbolic noise to forecast the load curve of a Californian electric utility. The results presented in that work suggest the superiority of the ARMA model compared to the one used by the Californian electric operator. Under the hypothesis of non normal residuals it is suggested an iterative procedure for calibrating an ARMA process obtaining daily and weekly forecasts with better results then the ones obtained under the assumption of normal residuals in the ARMA representation [58]. The class of ARIMA models is very widely used in the
electricity sector mainly for short term forecasting. A seasonal ARIMA model that can be used for forecasting the load curve in presence of seasonal variations is presented in [46]. Reference [45] calibrates a seasonal ARIMA model on hourly data and compares the forecasting results with the following techniques: exponential smoothing; multiple linear regression; transfer function model; state-space with Kalman filter and an expert system. The ARIMA model offers fairly satisfying results, at least during typical summer days, but underperforms when compared to the best model (transfer function). The results are somewhat different on winter days, due to the different temperature profile. In [59] it is used the trend component for estimating the growth of the system load curve, climate parameters for estimating the load curve component susceptible to climate variations, and an ARIMA model for generating the cyclical component of the weekly pick load, which is not susceptible to temperature. Another comparative study [21] tests an ARIMA model, a transfer function model and a regressive one for four different class of consumers in Taiwan (residential, commercial, office and industrial). The transfer function model, which also requires temperature data, offers the best results. Some other authors examine a seasonal ARIMA model with double cycle [8]. The results achieved with the ARIMA model are more satisfactory than with a neural network model, but cannot compete with an exponential smoothing model with double seasonality. For a comparison between a SARIMA model and one composed of 24 autoregressive models for each hour of the day one can see [53]. The ARIMA model proposed in [60] is used for load peak forecasting of the hourly load curve in Iran. This model is a set of 8 autoregressive models with two exogenous variables: temperature and load curve, as estimated by the system operator. For a real time ARIMA model for the Spanish market including climate conditions as an explicative variable see [61]. A different class of models based on time series analysis evaluates the information coming from other time series. These cases are common in the electricity field where the load curve can depend on exogenous variables, such as the climate conditions under which the electricity service is supplied. For estimation procedures in the ARMAX case see [62] and [20]. For an approach based on evolutionary programming for daily forecasting up to a week in advance see [22]. The model, used for hourly forecast on a weekly timetable, has been tested in Taiwan with - compared to other evolutionary programming models - better performances both in terms of convergence and computational time.

From a different point of view artificial intelligence based models or nonparametric models can also take into account both flexibility and complexity, and their recent use, starting form the '90, has grown along with the potential offered by modern computers. From a theoretical perspective and because of their good performances, methods based on neural network have received the most attention. Others nonparametric techniques (fuzzy logic, expert systems and support vector machines) have been mainly used along with statistical and neural network models. The relative simplicity of these models is also their limit, as only rarely the operator is able to introduce specific relations among variables. As discussed in recent literature, the robustness of this method hasn’t been entirely proved. On the other hand, empirical evidence suggests that this class of models can produce interesting and promising results [63]. Conflicting conclusions can be found in [64] and [8], whose analysis show the superiority of a seasonal exponential smoothing with double cycle compared to the neural network model, at least for hourly forecasting. A second class of models, which can operate with the new available information, is the expert systems one. In this case the expert’s capacity to convey his decision process to the programmer is crucial. An application for short term forecasting is proposed in [65]. Reference [66] develops a forecasting technique for different geographical areas by formalizing the load curve available information and exogenous variables in terms of parametric rules and adding some specific information for each geographical area. In this case the model is not set up using specific knowledge on the different areas. Expert systems are not used in an isolated way but together with other models [67]. Reference [68] combines different non linear structures based on fuzzy logic, neural network and expert systems to set an automatic short term forecasting model. An additional class of non linear models is represented by all models based on Boolean logic. Even in this class of models, hybrid models are widely used [69]-[71]. Among non linear models those based on support vector machines which are founded on the fundamental work of Vladimir Vapnik [72] are worth mentioning. Examples of such applications in the electricity sector can be found in [33], [34], [69] and [73].

The next part of this paper is dedicated to the application of two specific statistical models per category of electrical company. Railway undertakings, in fact, have to tackle a double challenge: on the one hand they have to operate in a competitive (both for the transport of passengers and freight) market, on the other hand, it must manage a series of strategic variables from which the company’s profitability depends. The ultimate goal is to demonstrate the potential of forecasting activity, which can offer a more comprehensive knowledge of the market, allowing for a successful outcome for the company as a whole.

IV. HOURLY LOAD FORECASTING IN THE ITALIAN RAILWAY BUSINESS

Railway companies are among the bigger consumers of electricity. For this and other reasons the application of quantitative forecasting models in this sector can represent a very interesting test for the goodness of a particular model. As known, in the Italian railway market there is separation between greed operator and companies which supply with transport services and can freely choice, at least in theory, the energy supplier. As the railway sector as a whole is the first Italian electricity consumers then it is quite obvious that the introduction of a market for organised exchange put a great deal of attention on the forecasting problem. The next section
is dedicated to the problem of forecasting the load curve of the Italian railway system: such a curve is the sum of the demand for different kind of transport services (long distance, regional and freight transport) over time. The goal is comparing two statistical models which can be appropriated both from an operational point of view and in term of scientific research. The selection of these models reflects the analysis of the previous sections, where the explicative power of the statistical models has been outlined. As this study is innovative with respect to the Italian experience, it is quite reasonable setting the research on models which have a major explicative power such as the statistical ones. With this goal in mind it is analysed the hourly electricity consumption of the North zone as defined by the Italian electricity market (GME - Gestore Mercati Energetici). Comparing different models, this research intends to give a judge on the punctual forecasting accuracy even with respect to the percentage deviation between observed and forecasted values. This measure represents the base on which it is calculated the set of penalty and payments which derive form wrong planning of the electricity consumes. The statistical distribution of these deviations identifies what is called the volumetric risk of the company. The analysis considers the exponential smoothing technique and a seasonal ARIMA model: these models are identified on a specified data set while the hourly forecast is set for one day up to six day ahead. The estimation is updated at the end of each forecasting session by moving the estimation in advance on a period equal to the previous forecasting. An important element of the present analysis is the distinction between test set and hold out set which definitively permits to test the forecast accuracy of the models presented in this work. In addition, the parameters of the models are updated after every forecasting session. The data set comes from the information systems of RFI S.p.A. i.e. the railway company which supplies with the electricity at national level. This paper is organised as follow: in Section A the data are presented, in Section B and C the models are implemented while the conclusions and main results are reported in Section V.

A. Presentation of the data set

The following analysis concerns with the forecasting of the hourly load curve of the North zone as defined by the GME in 2008. The amount of electricity consumed by the railway system is directly related to the railway time table. As it is clearly shown below, the load curve is a seasonal type, strongly dependent from regional transports during the working days (weekly seasonality). In addition, a daily seasonality must be taken into account because of the transport movement from Monday to Friday (daily seasonality). Such seasonality shapes the load curve in a particular way so that we have heavy consumption when people go to and come back from work (from 6 to 8 a.m. and from 17 to 19 p.m.). On the other hand freight transport is much more important during night hours and from Monday to Friday. Consumption dynamics at the week end is pretty much the same as the working days but with substantial lower level of electricity consumption. This is mainly due to the obvious reduction of regional transport services (i.e. commuter transport) on Saturday and Sunday which identify what we can call commuter transport. Figure 1 shows the dynamics of a typical weekly hourly load curve (LOADN) in megawatt hours. This data set is a selection from the sample used in the next Section.

![Figure 1: Weekly hourly load curve of the Italian railways - North Zone of the GME](image)

The forecasting problem is set up by estimating the model on the test set and then checking the robustness and accuracy of it on the hold out set. Because of the data set and the kind of forecast (very short term) it has been decided to choose a sample with some degree of stability (from Monday, 3 March 2008 to Monday, 31 March 2008). The models have been estimated on a constant sample of 23 days for a total of 552 observations and have been used for a day ahead forecasting on a week period. Figure 2 reports the test set and hold out set of the North zone in megawatt hours corrected for the presence of outliers. The outliers have been corrected considering the arithmetic mean of the previous two weeks for the same day and hour (the label on the x axis is purely indicative).

![Figure 2: Hourly load curve of the Italian railway- North Zone of the GME](image)

B. The Holt-Winters method

The Holt-Winters method chosen for forecasting the hourly load curve considers the features of the data set which presents, in the period under analysis, constant weekly seasonality. It is so reasonable the application of the Holt-Winters model with linear trend and additive seasonality. The formula for determining the smoothed value of the load curve $y_t$ (LOADN at the time $t$) is reported below:
\[ \hat{y}_{t+\delta} = a + bk + c_{t+\delta} \]

where \( a, b \) and \( c \) are respectively the intercept, trend and seasonal factor of the model. The system of recursive equations is structured as follow:

1. \[ a(t) = \alpha(y_t - c_t(t-s)) + (1-\alpha)(a(t-1) + b(t-1)) \]
2. \[ b(t) = \beta(a(t) - a(t-1)) + (1-\beta)b(t-1) \]
3. \[ c_t(t) = \gamma(y_t - a(t+1)) + (1-\gamma)c(t-s) \]

With \( \alpha > 0, \beta < 1, \gamma < 1 \). The forecast is then determined by the expression below:

\[ \hat{y}_{t+\delta} = a(T) + b(T)k + c_{T+\delta-s} \]

The seasonal parameter has been set to 168 to take into account the features of the hourly load curve. Figure 3, which compares observed and forecasted values, shows the results of the simulation process. The period considered is from 26 up to 31, March 2008. The graphical analysis confirms the good forecast accuracy of the model proposed through the period under analysis. The Root Mean Squared Error is equal to 18 while the Mean Percentage Error is equal to 0,1%.

As shown in the review of the literature, a reasonable way of dealing with these seasonality in presence of high frequency data is by applying the difference operator of order 168. In this way it is possible to eliminate both the seasonality considered. Nevertheless this difference produces a non stationary series. The stationarity can be achieved by applying an additional difference operator of order 1. The autocorrelation and partial autocorrelation function and the application of the Augmented Dickey-Fuller test confirm that the obtained differenced series is a stationary one. Figure 5 reports the series DLOADN obtained from the application of the difference operators to LOADN.

\[
(1-\phi_1B)(1-\Phi_1B_{168})\nabla_{168} \text{LOAD}_t = (1-\theta_1B)c_t
\]

where \( c_t \) is a Brownian motion and \( \text{LOAD}_t \) is the hourly load curve at time \( t \). The coefficients of the model are all significative with very low standard errors while the analysis of the residuals (histogram, autocorrelation and partial autocorrelation function) doesn’t permit to reject the hypothesis of white noise residuals. Figure 6 reports fitted, actual and residual values on the test set obtained from the application of the model (6).
The identified model can be utilized for hourly forecasting on the period 26-31, March 2008. The accuracy of the seasonal ARIMA model identified for forecasting the railways electricity consumption on hourly basis is shown in Figure 7 which compares the observed values (holdout set) with those obtained by means of the ARIMA models estimated in the previous pages. The graphical analysis shows that the forecasts are pretty much the same to those seen for the Holt-Winter’s case. This is coherent with the analogies exiting between the two types of models.

![Figure 7](image)

**Fig. 7 Seasonal ARIMA model: forecast of the Italian railway hourly load observed and forecasted values of the Italian railways hourly load curve**

V. CONCLUSIONS AND MAIN RESULTS

Among the different kinds of undertakings involved in the liberalization process, railways represents, as a whole, one of the biggest electricity consumers, at least in Italy. The goal of the current analysis is mainly directed to the application of two well defined statistical models to a business sector where such type of analysis has never been done before. Therefore, the results must be considered as a first step towards the comprehension of a sector which is very important from a social and economic point of view. The models have been chosen considering the huge variety of solutions proposed in the literature and analysing those which may be more suitable for the railway business. Exponential smoothing in the Holt-Winters version is an adaptive model which can be easily set up at a very reasonable cost. On the other hand, modern time series analysis, which is based on the research of the internal structure of the data needs lots of experience and the forecaster is called to understand the temporal linkages among the observations. As already specified, the electricity hourly consumption considered in the present analysis refers to the North Zone of the GME. Figure 8 shows the results deriving from the application of the models analysed on the period 26-31, March 2008: the graph below reports the observed and forecasted values obtained from the application of the Holt-Winter method and ARIMA(1,1,1)x(1,1,0)168. The plot shows peaks during rush hours; the models specified appear well identified and can reproduce in a very satisfactory way the main features of the series under analysis. The models don’t perform particularly well on Sunday, 30 and Monday, 31. This is probably due to significant changes in the railway services during those days. These variations can alter significantly the structure of the data set because of their randomness. Nevertheless the aspects related to railway supply planning are not the object of the paper which, instead, is dedicated to the possibility of a convenient application of some statistical forecasting techniques. Obviously, a direct application of a model must take into account all the available information coming from company’s departments.

![Figure 8](image)

**Fig. 8 Forecasts of the hourly load curve of the Italian railways**

Table IV below compares the analysed models by means of the most applied out-of-sample accuracy measurements.

<table>
<thead>
<tr>
<th>Time</th>
<th>Mean Error</th>
<th>Mean Absolute Error</th>
<th>Root Mean Squared Error</th>
<th>Mean Percentage Error</th>
<th>Mean Absolute Percentage Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/26/2008</td>
<td>15</td>
<td>18</td>
<td>19</td>
<td>4.4%</td>
<td>4.6%</td>
</tr>
<tr>
<td>03/28/2008</td>
<td>-1.39</td>
<td>10</td>
<td>13</td>
<td>0.2%</td>
<td>3.5%</td>
</tr>
<tr>
<td>03/29/2008</td>
<td>20</td>
<td>17</td>
<td>19</td>
<td>1.8%</td>
<td>2.9%</td>
</tr>
<tr>
<td>03/30/2008</td>
<td>20</td>
<td>12</td>
<td>15</td>
<td>2.2%</td>
<td>2.8%</td>
</tr>
<tr>
<td>03/31/2008</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>0.5%</td>
<td>6.0%</td>
</tr>
<tr>
<td>03/26/2008</td>
<td>15</td>
<td>18</td>
<td>19</td>
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<tr>
<td>03/28/2008</td>
<td>-1.39</td>
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<td>13</td>
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<tr>
<td>03/29/2008</td>
<td>20</td>
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</tr>
<tr>
<td>03/31/2008</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>0.5%</td>
<td>6.0%</td>
</tr>
</tbody>
</table>

All the accuracy measures confirm that the models considered in the analysis perform quite well on all day and along the whole forecasting period. Focusing on mean percentage error and root mean squared error it is also possible concluding that the Holt-Winters model performs better than the ARIMA model presented in this paper. As common knowledge, percentage error on the forecasting period is a very important measure of forecasting accuracy and is often a crucial measure for calculating the financial effect (penalties and payments) deriving from the interaction between demand and supply. As Figure 9 below clearly shows, the distribution of the error is random with mean zero. The standard deviation is 6.6% for the Holt-Winter case and 7.6% for the ARIMA model. It is also worth noticing that the real financial effect of under and over estimations depends heavily on the load curve for each geographical zone so that it...
is not possible to supply with a reliable estimation of the real volumetric risk under which the railway company is subject to.

Fig. 9 Hourly load curve of the Italian railways - Mena Percentage Error on the forecasting period

VI. REFERENCES


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