Performance Analysis of a Series of Adaptive Filters in Non-Stationary Environment for Noise Cancelling

Anam Rafique, and Syed Sohail Ahmed

Abstract—One of the essential components of much of DSP application is noise cancellation. Changes in real time signals are quite rapid and swift. In noise cancellation, a reference signal which is an approximation of noise signal (that corrupts the original information signal) is obtained and then subtracted from the noise bearing signal to obtain a noise free signal. This approximation of noise signal is obtained through adaptive filters which are self adjusting. As the changes in real time signals are abrupt, this needs adaptive algorithm that converges fast and is stable. Least mean square (LMS) and normalized LMS (NLMS) are two widely used algorithms because of their plainness in calculations and implementation. But their convergence rates are small. Adaptive averaging filters (AFA) are also used because they have high convergence, but they are less stable. This paper provides the comparative study of LMS and Normalized NLMS, AFA and new enhanced average adaptive (Average NLMS-ANLMS) filters for noise cancelling application using speech signals.

Keywords—AFA, ANLMS, LMS, NLMS.

I. INTRODUCTION

The classical noise cancellation method uses a reference input signal (correlated noise signal) which is passed through the adaptive filter to make it equal to the noise that is added to original information bearing signal. Then this filtered signal is subtracted from noise corrupted information signal. This makes the corrupted signal a noise free signal. An adaptive filter self adjusts its weights through an optimized algorithm. The very first adaptive filter named wiener filter was developed by wiener. After that in 1960 LMS was developed by Hoff and widrow. It has low convergence rate. Nagumo and Noda developed NLMS which is better than LMS from convergence point of view [4]. Both of these filters have small computational and implementation complexity [2]–[3]. Later on AFA was developed which has high tape weight learning rate but very small stability. In this paper a new adaptive filter is proposed which has high convergence as compared to LMS and NLMS and is more stable than AFA. This paper proposes the use of ANLMS filter in noise cancellation application.

Section II briefly explains the adaptive noise cancelling theory. Section III discusses the backgrounds of LMS, NLMS, AFA and new enhanced filter. In section IV new enhanced ANLMS filter is discussed. In section IV simulation outputs of LMS and NLMS, AFA and ANLMS are reviewed. At the end in section IV conclusion is derived.

II. ADAPTIVE NOISE CANCELLATION (ANC) THEORY

Fig. 1 explains the concept of an ANC system. An information bearing signal $A(n)$ that is degraded with noise $u_o(n)$. Information signal becomes $S(n)=A(n)+u_o(n)$. This degraded signal $S(n)$ is primary input to the ANC system. An additional noise signal $u_o(n)$ that has some correlation with the noise added to the information signal, acts as a reference signal to ANC system [3]–[5]. In very first step $u_o(n)$ is filtered through adaptive filter which produces an estimate $y(n)$ of noise $u_o(n)$ added to original signal. After this The estimation signal $y(n)$ is subtracted from $S(n)$ i.e. $e(n) = S(n) − y(n)$ [5]. Doing this the signal obtained signal $e(n)$ is a noise free signal.

III. BACKGROUND OF LMS, NLMS AND AFA ALGORITHMS

A. Least Mean Square Solution

A very simple approach in noise cancelling approach is use of LMS algorithm. LMS algorithm belongs to a class of steepest descent algorithms [1]–[2]. Following equations shows the details of LMS filter.

1. Weights evaluation –

$$w(n+1) = w(n) + μe(n)$$

(1)

2. Filtering function –

$$Y(n) = w(n) * u(n)$$

(2)
3. Error estimation (where error is the desired output)–
\[ e(n) = S(n) - y(n) \]  \hspace{1cm} (3)

From these equations it is clear that at each iteration, the information of most recent values \( S(n) \), \( u(n) \), \( w(n) \) and \( e(n) \) are required. \( \mu \) is the step size that depends on the power spectral density of the reference input \( u(n) \) and filter length \( M \). It defines that how long step we take along error function on each iteration. \( \mu \) is a preset constant factor in LMS filter [6]–[7]. Larger step size makes the algorithm to converge early. But too large step size makes the algorithm diverging.

B. Normalized Least Mean Square Solution

In LMS filters tape weight learning is a function of \( u(n) \), if \( u(n) \) is large, then the problem of noise gradient amplification rises. To recompense this problem, a new algorithm Normalized least mean square was developed [1]–[6]. NLMS algorithm is based on the minimal disturbance principle, where the learning of tap weights, from one iteration to next, takes place in a nominal a way [4]. Following equation shows the tap weight learning of NLMS algorithm:
\[ w(n + 1) = w(n) + \frac{\mu}{||u(n)||^2} u(n). e(n) \]  \hspace{1cm} (4)

In comparison to LMS, the NLMS has varying step size that makes the NLMS to converge more quickly [3].

C. Adaptive Average Filter

Applications where the convergence rate is of importance the LMS and NLMS are not much efficient [3]. Adaptive filter with averaging can be used for such applications. In AFA algorithm the weights and product of \( u(n) \)*\( e(n) \) are averaged [3].
\[ w(n+1) = \frac{1}{n} \sum_{k=1}^{n} w(k) + 1/n \sum_{k=1}^{n} u(n)e(n) \]
where \( 1/2 < \gamma < 1 \).

But when the stability is major concern, AFA algorithm is not a good choice because it is not much stable as compared to LMS and NLMS.

IV. ENHANCED AVERAGE NLMS FILTER

In order to have a stable filter with averaging we developed an enhanced NLMS filter with averaging called Average NLMS filter (ANLMS). In ANLMS algorithm the weights and \( u(n) \)*\( e(n) \) are averaged where the adjustment applied to \( n+1 \) iteration is normalized with respect to the reference input vector.

The algorithm of this filter combines both the high convergence rate from averaging and high stability of NLMS filter while providing the high SNR.
\[ w(n + 1) = \frac{1}{n} \sum_{k=1}^{n} w(k) + 1/n \]

\[ \sum_{k=1}^{n} \frac{\mu}{||u(n)||^2} u(n). e(n) \]  \hspace{1cm} (5)
where \( \mu = 1 \) and \( 0 < \gamma < 0.5 \).

V. SIMULATIONS AND RESULTS

In this section software simulation of LMS, NLMS, AFA and ANLMS filters for the noise cancellation are discussed. For all simulations, an audio signal “hello” is used as the original information bearing signal which is corrupted by noise. We used white Gaussian noise as a reference noise and a correlated white Gaussian noise that corrupts the original information bearing signal. Using different \( u(n) \) and \( u_o(n) \) makes the environment non-stationary. For each algorithm, the filtered out signal, weights of signals are plotted and the SNR is calculated. The best parameter choices used for the simulations are listed in the table.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>PARAMETERS FOR ADAPTIVE FILTERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu ) for LMS</td>
<td>( \mu ) for NLMS</td>
</tr>
<tr>
<td>.002</td>
<td>.45</td>
</tr>
</tbody>
</table>

Following table shows the regular improvement in SNR of the signal filtered by LMS, NLMA, AFA and ANLMS filter.

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>SIGNAL TO NOISE RATIOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm</td>
<td>Signal to Noise Ratio (db)</td>
</tr>
<tr>
<td>LMS</td>
<td>7.9419</td>
</tr>
<tr>
<td>NLMS</td>
<td>22.1614</td>
</tr>
<tr>
<td>AFA</td>
<td>25.0788</td>
</tr>
<tr>
<td>ANLMS</td>
<td>29.0982</td>
</tr>
</tbody>
</table>

The SNR results show that the noise cancelling capability of LMS is lowest and ANLMS has the highest.

The graphical results are shown in the figures below; Fig. 2 shows the original, noise, reference and noise corrupted signals.

Fig. 2 Original, noise, Reference and Noise Corrupted Signal
Fig. 3 to 6 shows the filtered output and the corresponding filter weights for different algorithms. Fig. 7 shows the comparison of different filter weights. From fig. 3 to fig. 7 it is clear that the LMS has the least performance ratio and NLMS is next to it. Performance of AFA is better than LMS and NLMS and high convergence rate. But fig. 7 shows that the filter weights of AFA are not much stable although they have highest learning rate of tap weights. On the other hand ANLMS has higher convergence rate than LMS and NLMS and is stable as compared to AFA.

In order to get the optimum results the range of $\gamma$ for ANLMS is specified through experimental results. Fig. 8 and 9 shows that for $\gamma=0.05$ the result is optimum. As its value is increased there is lower SNR for the retained signal. For $\gamma>0.5$ the retained signal is not clear. so $0<\gamma<0.5$. 
TABLE III
DIFFERENT VALUES OF GAMMA FOR ANLMS FILTER

<table>
<thead>
<tr>
<th>γ</th>
<th>SNR (db)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05</td>
<td>29.0982</td>
</tr>
<tr>
<td>.1</td>
<td>25.7036</td>
</tr>
<tr>
<td>.3</td>
<td>10.1516</td>
</tr>
<tr>
<td>.6</td>
<td>2.9628</td>
</tr>
<tr>
<td>.00005</td>
<td>30.5950</td>
</tr>
</tbody>
</table>

Fig. 8 Filtered signal through ANLMS for different gamma values

Fig. 9 Time evaluation of ANLMS filter taps for different gamma values

VI. CONCLUSION

In this paper a series of adaptive filters LMS, NLMA, AFA and ANLMS filter algorithm are used in noise cancellation system. For each filter the filtered output, time evaluation of filter taps and SNR of the signal are calculated and then a comparison based on SNR, convergence and stability of algorithm is done. The simulation results show that the convergence of AFA is highest but it is not much stable as compared to other filter. In AFA there is tradeoff between stability and convergence. On the other hand the new proposed algorithm ANLMS is most stable and has higher convergence than NLMS and LMS. So ANLMS is better than LMS, NLMS and AFA.

REFERENCES