Cascade Kalman Filter Configuration for Low Cost IMU/GPS Integration in Car Navigation Like Robot

Othman Maklouf, Abdurazag Ghila and Ahmed Abdulla

Abstract—This paper introduces a low cost INS/GPS algorithm for land vehicle navigation application. The data fusion process is done with an extended Kalman filter in cascade configuration mode. In order to perform numerical simulations, MATLAB software has been developed. Loosely coupled configuration is considered. The results obtained in this work demonstrate that a low-cost INS/GPS navigation system is partially capable of meeting the performance requirements for land vehicle navigation. The relative effectiveness of the kalman filter implementation in integrated GPS/INS navigation algorithm is highlighted. The paper also provides experimental results; field test using a car is carried out.

Keywords—GPS, INS, IMU, Kalman filter.

I. INTRODUCTION

NAVIGATION comprises the methods and technologies to determine the time varying position and attitude of a moving object by measurement. Position, velocity, and attitude, when presented as time variable functions are called navigation states because they contain all necessary navigation information to geo-reference the moving object at that moment of time [1]. A navigation sensor measures quantity related to one or more elements of the navigation state such as Global Positioning System (GPS). A Combination of sensors capable of determining all navigation states makes up a navigation system such as Inertial Navigation System (INS). Inertial navigation is the determination of the position of a vehicle through the implementation of inertial sensors. It is based on the principle that an object will remain in uniform motion unless disturbed by an external force. This force in turn generates acceleration on the object. If this acceleration can be measured and then mathematically integrated, then the change in velocity and position of the object with respect to an initial condition can be determined.

Rotational motion of the body with respect to the inertial reference frame may be sensed using gyroscopic sensors and used to determine the orientation of the accelerometers at all times. Given this information, it is possible to transform the accelerations into the computation frame before the integration process takes place. At each time-step of the system’s clock, the navigation computer time integrates this quantity to get the body’s velocity vector. The velocity vector is then time integrated, yielding the position vector. Hence, inertial navigation is the process whereby the measurements provided by gyroscopes and accelerometers are used to determine the position of the vehicle in which they are installed. By combining the two sets of measurements, it is possible to define the translational motion of the vehicle within the inertial reference frame and to calculate its position within that frame.

II. INERTIAL NAVIGATION SYSTEM

The basic principle of an INS is based on the integration of accelerations observed by the accelerometers on board the moving platform. The system accomplishes this task through appropriate processing of the data obtained from the specific force and angular velocity measurements. Thus, an appropriately initialized inertial navigation system is capable of continuous determination of vehicle position, velocity and attitude without the use of the external information [1].

A major advantage of using inertial units is that given the acceleration and angular rotation rate data in three dimensions, the velocity and position of the vehicle can be evaluated in any navigation frame. For land vehicles, a further advantage is that unlike wheel encoders, an inertial unit is not affected by wheel slip. However, the errors caused by bias, scale factors and non-linearity in the sensor readings cause an accumulation in navigation errors with time and furthermore inaccurate readings are caused by the misalignment of the unit’s axes with respect to the local navigation frame. This misalignment blurs the distinction between the acceleration measured by the vehicles motion and that due to gravity, thus causing inaccurate velocity and position evaluation. Since an inertial unit is a dead reckoning sensor, any error in a previous evaluation will be carried onto the next evaluation, thus as time progresses the navigation solution drifts [2].

A. Physical implementation

The first type of INS developed was a gimbaled system the accelerometers are mounted on a gimbaled. In a gimbaled system the accelerometer triad is rigidly mounted on the inner gimbal of three gyros, Figure 1(b). The inner gimbal is isolated from the vehicle rotations and its attitude remains constant in a desired orientation in space during the motion of the system. The gyroscopes on the stable platform are used to sense any rotation of the platform, and their outputs are used in servo feedback loops with gimbal pivot torque actuators to control the gimbals such that the platform remains stable. These systems are very accurate, because the sensors can be designed for very precise measurements in a small measurement range. In contrary, a strap-down inertial navigation system uses orthogonal accelerometers and gyro triads rigidly fixed to the axes of the moving vehicle, Figure 1(a). The angular motion of the system is continuously measured using the rate
sensors. The accelerometers do not remain stable in space, but follow the motion of the vehicle. A strap-down system is a major hardware simplification of the old gimbaled systems. The accelerometers and gyros are mounted in body coordinates and are not mechanically moved. Instead, a software solution is used to keep track of the orientation of the IMU (and vehicle) and rotate the measurements from the body frame to the navigational frame. This method overcomes the problems encountered with the gimbaled system, and most importantly reduces the size, cost, power consumption, and complexity of the system.

B. Error sources

This section will provide a quick overview of some difficulties present in inertial navigation. This will provide a better understanding for the difficulties encountered with the IMU.

1) Bias: A sensor bias is always defined by two components: A deterministic component called bias offset which refers to the offset of the measurement provided by the sensor from the true input; and a stochastic component called bias drift which refers to the rate at which the error in an inertial sensor accumulates with time. The bias offset is deterministic and can be quantified by calibration while the bias drift is random in nature and should be treated as a stochastic process.

2) The scale factor: The scale factor is the relationship between the output signal and the true physical quantity being measured and it is usually expressed in parts per million (ppm). The scale factor is deterministic in nature and can be quantified or determined through lab calibration. The variation of the scale factor with the variation of the exerted acceleration/angular rate or temperature represents the scale factor stability and is usually called the non-linear part of the scale factor error.

3) Output stability: The output stability of a sensor defines the run-to-run or switch-on-to-switch-on variation of the gyro-drift/accelerometer-bias as well as in-run variation of gyro-drift/accelerometer-bias. The run-to-run stability can be evaluated from the scatter in the mean output for each run for a number of runs given that the sensor is turned off then on again between each two successive runs. The in-run stability of a sensor is deduced from the average scatter of the measured drift in the output about the mean value during a single run.

4) Thermal sensitivity: Thermal sensitivity refers to the range of variation of the sensor performance characteristics, particularly bias and scale factor errors, with a change in temperature. A bias or scale factor correlation with temperature variation can be defined graphically or numerically (using a mathematical expression) through intensive lab thermal testing. Such correlations can be stored on a computer for online use to provide compensation for temperature variation, provided a thermal sensor is supplied with the sensor.

III. GLOBAL POSITIONING SYSTEM

The Global Positioning System is a satellite-based navigation system that was developed by the U.S. Department of Defense in the early 1970s. Initially developed as a military system, it was later made available to civilians, and is now a dual-use system that can be accessed by both military and civilian users. The GPS consists basically of three segments: the space segment, the control segment, and the user segment. The space segment consists of 24 satellites arranged in 6 orbital planes with an inclination angle of 55° relative to the Earth equator, as shown in Figure 2. The satellites have approximately an average orbit radius of 20200 km and complete one orbit in 11 hours and 58 minutes. The control segment monitors the health of the orbiting satellites and uploads navigation data. It consists of a system of tracking stations located around the world, including six monitor stations, four ground antennas, and a master control station. The user segment consists of receivers specifically designed to receive, decode, and process the GPS satellite signals.

Under good conditions GPS will be able to provide continuous and accurate positioning to the user at all time. But unfortunately good conditions will not always occur as the signal from the satellites can be blocked by (e.g. Mountains and high buildings). Further as the electromagnetic signal travels from the satellites to the Earth it can be influenced by magnetic fields, areas with high amount of free electrons and moisture air that cause the signal to travel slower than expected (speed of light in vacuum). At the Earth the signal can be extended by reflections from e.g. (glass), the clocks onboard the satellites and in the receivers can be unsynchronized and therefore cause more errors on the signal. Hence any sophisticated urban navigation system cannot depend on GPS
as a stand-alone system. Instead one can integrate two (or more) different navigation systems.

IV. MODELING OF CAR LIKE ROBOT

This section discusses the mathematical model used for the vehicle. Development of a rigorous model for a land vehicle velocity requires information about the steering angle, front and rear slip angle, tire pressure, angular speed of the wheels, suspension system, etc. Since it is very difficult to acquire such information, So that using some of these parameters based constraints hold under the assumption that there is no slippage that the vehicle consists of front and rear virtual wheels is more reliable [3].

A. System Modeling

The first step of the derivation is to create the kinematic model by employing the nonholonomic constraints. These constraints hold under the assumption that there is no slippage at the wheel. A nonholonomic constraint is one that is not integrable. The constraints related to an automobile are those of the vehicle’s velocity. As a result, the general form of the nonholonomic constraint is

\[ \dot{u} \sin \theta - \dot{w} \cos \theta = 0 \]  

(1)

Where \( \dot{u} \) and \( \dot{w} \) are the velocities of a wheel within a given \((u, w)\) coordinate system, and \( \theta \) is the angle of the wheel with respect to the \( x \)-axis. For small angles of steering, the car can be modeled as a bicycle, as shown in Figure 3. Denote \((x, y)\) as being the position of the center of gravity, \( \theta \) as the orientation of the vehicle with respect to the \( x \)-axis, and \( \phi \) as the steering angle between the front wheel and the body axis.

\[ \dot{x} = v_u \cos \theta - v_w \sin \theta \]  

(2)

\[ \dot{y} = v_u \sin \theta - v_w \cos \theta \]  

(3)

where \( v_u \) and \( v_w \) are the velocities of the center of gravity along the \( u \) and \( w \) axes respectively. The dynamic equations can be derived with a couple added assumptions. These added assumptions are that there is no friction force between the wheels and the vehicle, and that the rear wheels are locked to be in the same orientation as the vehicle. Other assumptions that there is no slip at the wheel, and that the driving force, based on the radius of the wheel and the drive torque, can be modeled as acting at the center of the rear wheels. With the slippage assumption comes a pair of forces, one acting at each wheel, and perpendicular to that wheel. The forces involved in this derivation are shown in Figure 3 [3].

\[
\begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta} \\
\dot{v}_u \\
\dot{v}_w \\
\dot{\phi}
\end{pmatrix} =
\begin{pmatrix}
\cos \theta - \frac{b}{\tau_x} \tan \phi \sin \theta & v_u \\
\sin \theta + \frac{b}{\tau_x} \tan \phi \cos \theta & v_u \\
\tan \phi & 0 \\
v_u (b^2 + 2) \tan \phi + \frac{\tau_x^2 (\cos \phi)^2}{\tau_y} F_D \\
-\frac{\tau_x \phi + C_s u_2}{\tau_x}
\end{pmatrix}
\]  

(4)

Where \( F_D \) is the driving force, applied at the rear axle, along the \( u \)-axis, and \( F_F \) and \( F_R \) are the resultant lateral forces on the front and rear tires respectively, \( R_w \) is the radius of the wheel and \( N_w \) and \( N_m \) are the number of teeth on the gears connecting the axle and motor respectively, and \( u_1 \) and \( u_2 \) are the input voltages [3].

B. 2D simulation of Strapdown Inertial Navigation

For a vehicle moving in 2D space, it is necessary to monitor both the translational motion in two directions and the change in the direction of vehicle (i.e. rotational motion). Two accelerometers are required to detect the acceleration in two directions. One gyroscope is required to detect the direction of the vehicle (rotational motion) in a direction perpendicular to the plane of motion. Strapdown systems mathematically transform the output of the accelerometers attached to the body into the navigation coordinate system before performing the mathematical integration. These systems use the output of the gyroscope attached to the body to continuously update the transformation necessary to convert from body coordinate to navigation. The derivation of the transformation matrix is explained below [4].

As seen from the above figure the two accelerometers are fixed in \( X \) and \( Y \) directions, these directions represent the body coordinates. The measured acceleration will transform to the navigation frame (East and North) using the following transformation matrix.

\[
\begin{pmatrix}
a_E \\
a_N
\end{pmatrix} =
\begin{pmatrix}
\cos A & -\sin A \\
\sin A & \cos A
\end{pmatrix}
\begin{pmatrix}
a_x \\
a_y
\end{pmatrix}
\]  

(5)
provisioned by the accelerometers and denoted by the \( \ddot{v} \) as follows; Firstly, the output of the two accelerometers \( \ddot{v}_b \) denoting the INS mechanization equation can be derived measured with respect to the inertial frame. Considering the rotation matrix which rotates \( a \) in the body frame defined by the accelerometers, and \( R_b^n \) is the rotation matrix which rotates \( a^b \) to the navigation frame.

C. INS mechanization equations in 2D

INS mechanization is the process of determining the navigation states (position, velocity and attitude) from the raw inertial measurements through solving the differential equations describing the system motion. Mechanization is usually expressed by a set of differential equations and typically performed in the local level frame defined by the local east, north and ellipsoid normal. The IMU measurements include one angular rate components provided by the gyroscope and denoted by \( \dot{\theta} \) as well as two linear acceleration components provided by the accelerometers and denoted by the \( 2 \times 1 \) vector \( \ddot{v}_b \). This means that the angular velocity \( \dot{\theta} \) of the body frame is measured with respect to the inertial frame. Considering the block diagram shown in Figure 5, the differential equations describing the INS mechanization equation can be derived as follows; Firstly, the output of the two accelerometers \( \ddot{v}_b \) is transformed from the body frame to the navigation frame (local level frame) using the transformation matrix \( R_b^n \) as given in the following equation

\[
\ddot{v}_l = R_b^n \ddot{v}_b
\]

(7)

where

\[
R_b^n = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\]

(8)

After the transformation of the acceleration components from the body frame to the navigation frame, the velocity components can be derived by integrating the given acceleration in the navigation frame. Consequently the corresponding position can be obtained by double integrating the acceleration in the navigation frame. Also the velocity component of the navigation frame can be directly calculated using the transformation of the velocity component in the body frame using the transformation matrix \( R_b^n \)

\[
v_l = R_b^n \ddot{v}_b
\]

(9)

The acceleration components in the navigation frame can be expressed as

\[
\begin{pmatrix}
a_E \\
a_N
\end{pmatrix} = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
\dot{v}_x \\
\dot{v}_y
\end{pmatrix} - \begin{pmatrix}
0 & \theta \\
-\theta & 0
\end{pmatrix} \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
\dot{v}_x \\
\dot{v}_y
\end{pmatrix}
\]

(10)

D. Simulation and testing of INS for navigation of a car like robot

To understand the mechanization of the strapdown INS in (2-D model), an INS algorithm is carried out under MATLAB/SIMULINK environment. In this computational algorithm the raw measurement data from the IMU is transformed from the body frame to the navigation frame via a transformation matrix designed for this purpose, this transformation matrix is simply a direction cosine matrix given in Eq.( 8), after this transformation is done a double integration are performed to calculate the position, velocity, and attitude in navigation frame.

E. Simulation Results

To understand the mechanization of the strapdown INS in (2-D model), an INS algorithm is carried out under MATLAB/SIMULINK environment. In this computational algorithm the raw measurement data from the IMU is transformed from the body frame to the navigation frame via a transformation matrix designed for this purpose, this transformation matrix is simply a direction cosine matrix given in Eq.( 8), after this transformation is done a double integration are performed to calculate the position, velocity, and attitude in navigation frame.

The results of testing of strapdown INS in navigation of car like robot is shown in Figure 7. It is obvious from the figure that the calculated trajectory using INS algorithm is trying to track the real trajectory of the car, but due to the noisy measurement of the IMU sensors (accelerometers and gyro) this noisy measurements when integrated resulted in high drift in INS trajectory since double integration is needed for position calculation

V. KALMAN FILTER THEORY AND ALGORITHM

An extended kalman filter was developed to estimate the position, velocity and attitude of the system. The full kalman filter equations will not be presented here due to limited space, but an overview of the process is shown below and further information can be found in [5]. The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. As such, the equations for the Kalman filter fall into two groups: time update equations and measurement update equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain
the a priori estimates for the next time step. The measurement update equations are responsible for the feedback, i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate. The time update equations can also be thought of as predictor equations, while the measurement update equations can be thought of as corrector equations. Indeed the final estimation algorithm resembles that of a predictor-corrector algorithm for solving numerical problems.

The specific equations for the time updates are presented below in

\[
\dot{x}_k = A\hat{x}_{k-1} + Bu_k \\
P_k = AP_{k-1}A^T + Q
\]

Notice how the time update equations project the state and covariance estimates forward from time step \( K - 1 \) to step \( K \). The specific equations for measurement updates are presented as

\[
K_k = P_k^{-1}H^T(H P_k^{-1}H^T + R)^{-1} \\
\dot{x}_k = \dot{x}_{k-1} + K_k(Z_k - H\hat{x}_k) \\
P_k = (I - K_kH)P_k^{-1}
\]

The first task during the measurement update is to compute the Kalman gain \( K \). The next step is to actually measure the process to obtain \( Z_k \), and then to generate an a posteriori state estimate by incorporating the measurement. The final step is to obtain an a posteriori error covariance estimate via Eq.(15). After each time and measurement update pair, the process is repeated with the previous a posteriori estimates used to project or predict the new a priori estimates. This recursive nature is one of the very appealing features of the Kalman filter it makes practical implementations much more feasible. The Kalman filter instead recursively conditions the current estimate on all of the past measurements.

**VI. Hardware**

This section is devoted to provide an overview of the two primaries utilized inertial sensors. The CRISTA IMU and GARMINE GPS specifications are briefly introduced.

**A. Crista IMU**

The Crista Inertial Measurement Unit shown in Figure 8 is a very small three axis inertial sensor that provides high resolution digital rate and acceleration data via serial interfaces from Cloud Cap Technology. [6]. It uses MEMS gyroscopic rate sensors and accelerometers mounted on orthogonal axes to provide 300°/sec rate and 10G acceleration data. It has an in-built GPS pulse per second (PPS) interface which facilitates accurate time synchronization of IMU and GPS data. The Crista IMU does not provide a calculated attitude solution, but rather provides temperature compensated raw rate and acceleration data for the host algorithms. The user controls data update rate and over-sample averaging of output data. The IMU is small in size \( (2.05 \times 1.50 \times 1.00') \) and weighs only 36.8 grams. A GPS Pulse Per Second input signal interface allows time correlation of IMU and GPS data.

**B. Garmin GPS**

The GPS system used in this work is the Garmin 18 PC. It is a low-cost off-the-shelf (Figure 8), outputs the NMEA 0183 (V2.2) protocol (commands: GGA, GSA, GSV, RMC, GLL, VTG) on a RS232 serial interface. The interface speed is 9600 baud and the used datum for the position calculation is WGS84. The receiver is powered over an additional power cable with 5V D.C [7]. The received GPS messages first need to be parsed to extract the desired information. The latitude, longitude, and height above mean sea level are extracted from the GGA message, the PDOP and working mode from the GSA message, and the speed over ground from the RMC message. The altitude, longitude, and height together are converted into rectangular coordinates \( (x, y, z) \) and the speed needs to be converted from knots to meters per second.

**VII. System architecture**

In this section the integration of the GPS and INS using cascade kalman filter in car navigation like robot will be investigated. This will discuss the two dimensional model in which two accelerometers and one rate gyro of the IMU are needed in order to monitor the motion of the vehicle. Simulation for testing of strapdown GPS/INS algorithms based on car modeling like robot is considered here using SIMULINK under MATLAB. Simplified block diagram for this configuration is shown in Figure 9.
A. INS Error Modeling Using Cascade Kalman Filter

The error dynamics equations are obtained by perturbing the kinematic equations. These error equations will be necessary to build the INS/GPS Kalman filters. In cascade kalman filter we use two kalman filters, the first one for estimating the error in position, velocity and the bias associated with each accelerometer, the second one will estimate the error in the heading angle and the drift which associated with the rate gyro. The error model for the first kalman filter can be derived as follow:

\[
\dot{x} = F_x + G_ω
\]

\[
x^T = \begin{bmatrix}
δx & δy & δv_x & δv_y & b_x & b_y \\
\end{bmatrix}
\]

\[
A_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The measurement (update) equation is obtained by comparing the output of the aiding source \(PV_{GPS}\) (GPS position and velocity measurements) to the INS output \(PV_{INS}\) (INS position and velocity measurements). The observation \(Z\) supplied to the Kalman filter is therefore expressed as follows:

\[
Z_1 = PV_{GPS} - PV_{INS}
\]

Therefore, the observation \(Z\) is related to the error state vector \(x\) as follows:

\[
z = Hx + n
\]

\[
Z_1 = \begin{bmatrix}
δx & δy & δv_x & δv_y & b_x & b_y \\
\end{bmatrix} + n
\]

B. Simulation Results

The results of simulating the error model developed for a cascade kalman filter in GPS/INS in car like robot navigation is shown in the following figures. It is clear that from Figure 10 the estimated trajectory using GPS/INS integration is more effective for tracking the real trajectory of the vehicle compared to the INS stand alone trajectory. Figure 11 shows the estimated bias and drift using cascade kalman filter.

VIII. EXPERIMENTAL WORK

The experiments are conducted using a car with the IMU and GPS mount on it. A PC 104 single board computer is connected to both sensors and the data are recorded. The main used hard ware is shown in Figure 12. The data was then taken and analyzed in MATLAB using the proceeding equations. The experimental work is divided into three main parts. The first part is the navigation solution which utilizes the
INS stand alone without inclusion of the Kalman filter or the GPS positional corrections. The second part is the navigation solution using GPS only. The third part will include both the GPS and INS using Kalman filter. This will clearly shows the benefits of integrating the INS and GPS, also the limitation of the IMU can be seen.

A. Stand Alone INS navigation

The recorded data from the crista IMU is consists of the readings from the three accelerometers and the three rate gyros, these readings are shown in Figure 13. Closly looking in the IMU output data reveals that, these data are highly corrupted with noise which is the main feathres of MEMES IMU. The IMU recorded data then processed through INS navigation algorithm, and the resultant estimated car trajectory is shown in Figure 14.

B. Navigation using GPS 18 PC

Data recorded from GPS sensor 18 PC includes the components of the car position in $x$ and $y$ coordinates the velocities along the navigation axis $V_{east}$ and $V_{north}$. Also the altitude
Fig. 15. GPS Sensor 18 PC Output.

Fig. 16. Car trajectory estimated by GPS.

on the ground is recorded. These data are shown in Figure 15. The resultant car trajectory as presented by the GPS is shown in Figure 16.

C. GPS/INS integration using cascade kalman filter results

In this section the benefits of integrating the GPS and INS using cascade kalman filter is shown in Figure 17(a). The GPS and GPS/INS lie right on top of each other. Taking a closer look at this plot, Figure 17(b) show that the two do not really lie exactly on top, but rather the GPS/INS transitions smoothly through the GPS points.

IX. CONCLUSION

This paper presents a cascaded Kalman filter configuration consists of two Kalman filters, this configuration is implemented for the purpose of increasing the accuracy of Low Cost MEMES IMU/GPS integration. This type of configuration enable us to estimate the error in heading of land vehicle using in motion alignment by comparing the measured heading of the GPS and the heading measured by the INS. Simulation and experimental results using this type of configuration has shown the advantages of integrating of two different sensors (GPS and Low Cost IMU) each with their own advantages and drawbacks. The low cost IMU used in this work is not capable of running by itself and providing any reasonable positioning information. GPS provides good results, but is only capable of determining position every second. The two sensors combined have the capability of producing good estimates of position in between the one second updates.

REFERENCES