The Comparisons of Average Outgoing Quality Limit between the MCSP-2-C and MCSP-C

P. Guayjarerpanishkand, T. Mayureesawan

Abstract—This paper presents a comparison of average outgoing quality limit of the MCSP-2-C plan with MCSP-C when MCSP-2-C has been developed from MCSP-C. The parameters used in MCSP-2-C are: \( i \) (the clearance number), \( c \) (the acceptance number), \( m \) (the number of conforming units to be found before allowing \( c \) non-conforming units in the sampling inspection), \( f_1 \) and \( f_2 \) (the sampling frequency at level 1 and 2, respectively). The average outgoing quality limit (AOQL) values from two plans were compared and we found that for all sets of \( i, r \), and \( c \) values, MCSP-2-C gives higher values than MCSP-C. For all sets of \( i, r \), and \( c \) values, the average outgoing quality values of MCSP-C and MCSP-2-C are similar when \( p \) is low or high but is difference when \( p \) is moderate.

Keywords—average outgoing quality, average outgoing quality limit, continuous sampling plan.

I. INTRODUCTION

A continuous sampling plan (CSP) is a sampling inspection plan for inspecting individual product units on a continuous basis. CSP involves alternating between two phases of inspection, i.e. 100% screening and sampling inspection. The original continuous sampling plan was the single-level continuous sampling plan that was presented by Dodge [1], namely CSP-1. This plan is the simplest and most famous and was used to develop other plans such as CSP-2 and CSP-3 by Dodge and Torrey [2], CSP-M by Lieberman and Solomon [3], TCSP-1 by Govindaraju and Balamurali [4], MLP-T-2 by Balamurali and Kalyanasundaram [5], CSP-C by Govindaraju and Kandnsamy [6] and MCSP-C by Balamurali and Subramani [7]. A review of various CSPs available in many statistical quality control textbooks for example Grant [8], Stephens [9], and Montgomery [10].

The MCSP-2-C plan is a two-level continuous sampling plan that has been developed as a single-level continuous sampling plan based on MCSP-C by Guayjarerpanishk and Mayureesawan [11]. MCSP-2-C has been proposed to reduce inspection or extended restart 100% inspection in the MCSP-C plan process. The operating procedure of the MCSP-2-C plan starts at 100% inspection, inspected one by one consecutively in the order of production. When \( i \) successive units are found to be conforming then discontinue 100% inspection and start sampling inspection at level 1 which inspects only a fraction \( f_1 \) of the units selected at random. If a non-conforming unit is found within the first \( m \) sampled conforming units then starts sampling inspection at level 2, which inspects only a fraction \( f_2 \) until a total of \( c+1 \) non-conforming sampled units have been found then revert to a 100% inspection.

II. DESIGN AND THEORY OF THE MCSP-2-C PLAN

A. The Operating Procedure of the MCSP-2-C

The MCSP-2-C uses five parameters (\( i, c, m, f_1 \), and \( f_2 \)) for inspection of the units being produced on the production line, which are defined by:

\[
\begin{align*}
  i & = \text{the clearance number,} \\
  c & = \text{the acceptance number,} \\
  m & = \text{the number of conforming units to be found before allowing} \ c \text{ non-conforming units in the sampling inspection,} \\
  f_1 & = \text{the sampling frequency at level 1 or} f_1 = 1/r, \\
  f_2 & = \text{the} \ c+1 \text{ sampling frequency at level 2 or} f_2 = 2 f_1.
\end{align*}
\]

The operating procedure of the MCSP-2-C plan is as follows:

1. **Step i.** Start with 100% inspection of units in the order of production. When \( i \) successive units are found conforming, discontinue 100% inspection and start sampling inspection at level 1.

2. **Step ii.** During the sampling inspection at level 1, inspect only a fraction \( f_1 \) of the units, selecting individual units one at a time in the order of production in such a way as to ensure an unbiased sample.

3. **Step iii.** If \( c \) non-conforming units are found after the first \( m \) sampled units have been found conforming then continue sampling at level 1 until \( c+1 \) non-conforming sampled unit have been found, and then revert immediately to 100% inspection.

4. **Step iv.** If a non-conforming unit is found within the first \( m \) sampled conforming units then start sampling inspection at level 2, inspect only a fraction \( f_2 \) until \( c+1 \) non-conforming sampled units have been found then return to Step i.

5. **Step v.** Replace or correct all the non-conforming units found with conforming units.
B. The Performance Measures of the MCSP-2-C

A derivation of these performance measures assumed that the production process is under statistical control and based on the Markov Chain formulation.

Let \( p \) be the probability of non-conforming units and \( q = 1-p \), the following formulas for performance measures may be obtained:

The average number of units inspected in a 100% screening sequence following the finding of a non-conforming unit, \( u \):

\[
u = \frac{1-q}{pq}
\]

(1)

The average number of units passed under the sampling inspection, \( v \):

\[
v = \frac{f_1 f_2 (1 + cq^m) + q f_1 (c+1) f_1 (1 - q^m)}{pq f_1 f_2}
\]

(2)

The average cycle length, ACL:

\[
ACL = \frac{f_1 f_2 (1 - q^i) + q f_1 f_2 (1 + cq^m) + (c+1) q f_1 (1 - q^m)}{pq f_1 f_2}
\]

(3)

The average fraction inspected, AFI:

\[
AFI = \frac{f_1 f_2}{f_1 f_2 (1 - q^i) + q f_1 f_2 (1 + cq^m) + (c+1) q f_1 (1 - q^m)} + \frac{(c+1) q f_1 f_2}{f_1 f_2 (1 - q^i) + q f_1 f_2 (1 + cq^m) + q f_1 (c+1) (1 - q^m)} - \frac{q f_1 f_2}{f_1 f_2 (1 - q^i) + q f_1 f_2 (1 + cq^m) + q f_1 (c+1) (1 - q^m)}
\]

(4)

The average outgoing quality, AOQ:

\[
AOQ = \frac{pq (1 - q^i) (1 - f_1) f_2}{f_1 f_2 (1 - q^i) + q f_1 f_2 (1 + cq^m) + q f_1 (c+1) (1 - q^m)} + \frac{pq q f_1 f_2}{f_1 f_2 (1 - q^i) + q f_1 f_2 (1 + cq^m) + q f_1 (c+1) (1 - q^m)} + \frac{pq f_1 (1 - q^m)}{f_1 f_2 (1 - q^i) + q f_1 f_2 (1 + cq^m) + q f_1 (c+1) (1 - q^m)}
\]

(5)

The average outgoing quality limit, AOQL:

\[
AOQL = \frac{Max (AOQ)}{p}
\]

(6)

The average outgoing quality limit, AOQL:

\[
AOQL = \frac{Max (AOQ)}{p}
\]

III. RESULTS

A. The Comparisons of Average Outgoing Quality Limit

In Table I, the AOQL values of MCSP-2-C and MCSP-C and the percentage differences of the AOQL values between MCSP-2-C and MCSP-C for all sets of \( i, r, \) and \( c \) values are shown. We observed that the AOQL values of the two plans are different with the AOQL values of MCSP-2-C higher than the AOQL values of MCSP-C for all sets of \( i, r, \) and \( c \) values.

In Table II, the values of \( q \) and \( r \) are shown in Fig 1 to 3. We found that when \( i \) changes from 10 to 15, 20, 30, 40 and 50, respectively, the \( \% \text{Diff}_{AOQL} \) values are slightly different at the same level of \( r \) and \( c \). When \( r \) changes from 4 to 10, the \( \% \text{Diff}_{AOQL} \) values are greater at the same level of \( i \) and \( c \). When \( c \) changes from 2 to 3, the \( \% \text{Diff}_{AOQL} \) values are similar or different for all sets of \( i, r, \) and \( c \) values.

B. The Values of \( p \)

In this section, the AOQ values of MCSP-C and MCSP-2-C at \( c = 2 \) for all sets of \( p \) for each set of \( i \) and \( r \) are shown in Fig 4 to 7. We saw that for all sets of \( i \) and \( r \) at \( c = 2 \), for the low level of \( p \), the AOQ values of MCSP-2-C are a little lower than MCSP-C. However at the high level of \( p \), the AOQ values of MCSP-2-C are a little higher than MCSP-C and the AOQ values of MCSP-2-C are greater than the AOQ values of MCSP-C when \( p \) is at a moderate level. For all sets of \( r \), the difference of the AOQ values between MCSP-C and MCSP-2-C are relatively small when the value of \( i \) increases. For all sets of \( i \) the difference of the AOQ values between MCSP-C and MCSP-2-C are relatively large when \( r \) increases.

In Table II, the values of \( p \) for the AOQ values of MCSP-C and MCSP-2-C are similar or different for all sets of \( i, r, \) and \( c \) values are shown. We found that the AOQ values of MCSP-C and MCSP-2-C are similar at the low or high level of \( p \) but the AOQ values of MCSP-C and MCSP-2-C are different at the moderate level of \( p \).

### Table I

<table>
<thead>
<tr>
<th>( i ), ( r ), ( c )</th>
<th>MCSP-2-C</th>
<th>MCSP-C</th>
<th>( % \text{Diff}_{AOQL} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10 ), ( 4 )</td>
<td>0.02</td>
<td>0.01</td>
<td>100</td>
</tr>
<tr>
<td>( 15 ), ( 8 )</td>
<td>0.03</td>
<td>0.02</td>
<td>150</td>
</tr>
<tr>
<td>( 20 ), ( 10 )</td>
<td>0.04</td>
<td>0.03</td>
<td>200</td>
</tr>
<tr>
<td>( 30 ), ( 16 )</td>
<td>0.06</td>
<td>0.05</td>
<td>300</td>
</tr>
<tr>
<td>( 40 ), ( 23 )</td>
<td>0.09</td>
<td>0.08</td>
<td>400</td>
</tr>
<tr>
<td>( 50 ), ( 31 )</td>
<td>0.12</td>
<td>0.11</td>
<td>500</td>
</tr>
</tbody>
</table>

[Table I: The AOQL values of MCSP-2-C and MCSP-C and the percentage differences of the AOQL values between MCSP-2-C and MCSP-C]

Full details of the derivation of these performance measures can be found in Guayjarenpanish and Mayureesawan [11].
Table II
THE VALUES OF P FOR THE AOQL VALUES OF MCSP-C AND MCSP-2-C ARE SIMILAR OR DIFFERENT

<table>
<thead>
<tr>
<th>i</th>
<th>r</th>
<th>c</th>
<th>similar</th>
<th>different</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4</td>
<td>all</td>
<td>0.130 - 0.560</td>
<td>0.131 - 0.559</td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td>all</td>
<td>0.090 - 0.410</td>
<td>0.091 - 0.409</td>
</tr>
<tr>
<td>20</td>
<td>4</td>
<td>all</td>
<td>0.070 - 0.315</td>
<td>0.071 - 0.314</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>all</td>
<td>0.045 - 0.215</td>
<td>0.046 - 0.214</td>
</tr>
<tr>
<td>40</td>
<td>4</td>
<td>all</td>
<td>0.035 - 0.160</td>
<td>0.036 - 0.159</td>
</tr>
<tr>
<td>50</td>
<td>4</td>
<td>all</td>
<td>0.030 - 0.125</td>
<td>0.031 - 0.124</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>all</td>
<td>0.110 - 0.620</td>
<td>0.111 - 0.619</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
<td>all</td>
<td>0.075 - 0.465</td>
<td>0.076 - 0.464</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>all</td>
<td>0.060 - 0.365</td>
<td>0.061 - 0.364</td>
</tr>
<tr>
<td>30</td>
<td>10</td>
<td>all</td>
<td>0.040 - 0.255</td>
<td>0.041 - 0.254</td>
</tr>
<tr>
<td>40</td>
<td>10</td>
<td>all</td>
<td>0.030 - 0.190</td>
<td>0.031 - 0.189</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>all</td>
<td>0.025 - 0.155</td>
<td>0.026 - 0.154</td>
</tr>
</tbody>
</table>

Fig. 1 The percentage differences of the AOQL values (%Diff_AOQL) between MCSP-2-C and MCSP-C for all sets of i.

Fig. 2 The percentage differences of the AOQL values (%Diff_AOQL) between MCSP-2-C and MCSP-C for all sets of r.

Fig. 3 The percentage differences of the AOQL values (%Diff_AOQL) between MCSP-2-C and MCSP-C for all sets of c.

Fig. 4 The AOQ values of MCSP-C and MCSP-2-C at level of c = 2 for r = 4 where i = 10, 15 and 20.

Fig. 5 The AOQ values of MCSP-C and MCSP-2-C at level of c = 2 for r = 4 where i = 30, 40 and 50.
The average outgoing quality limit (AOQL) is one of the performance measures which is the primary index for choosing the continuous sampling plans. So when considering the results of the AOQL comparisons, the operators may choose to use MCSP-C because this plan gives a lower number of non-conforming units that passed inspection and an easier operating process of inspection than MCSP-2-C. If sampling plans give high values of AOQL then they give low number of units inspected. In case the operators want to reduce the number of units inspected, they may choose the MCSP-2-C plan. We also observed that for values of $i$, there was a small effect on the differences of the AOQL values between MCSP-2-C and MCSP-C. However, for values of $r$, there was a great influence on the differences of the AOQL values. For values of $c$, there was no effect on the differences of the AOQL values when $r = 4$ but there was influence when $r = 10$.

When considering the low or high level of $p$, the two plans give similar $AOQL$ values and the operators can choose MCSP-C or MCSP-2-C. At the moderate level of $p$, MCSP-C gives lower values of $AOQL$ than MCSP-2-C, so they may choose MCSP-C. For values of $i$ and $r$, there are also effects on the levels of $p$ values for choosing the sampling plan.

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