Fuzzy PID Controller with Coupled Rules for a Nonlinear Quarter Car Model

Şaban Çetin, and Özgür Demir

Abstract—In this study, Fuzzy PID Control scheme is designed for an active suspension system. The main goal of an active suspension system for using in a vehicle model is reducing body deflections and handling high comfort for a passenger car. The present system was modelled as a two-degree-of-freedom (2-DOF) nonlinear vehicle model.

Keywords—Active suspension system, Fuzzy PID controller, a nonlinear quarter car model.

I. INTRODUCTION

In the control literature, there are several types of control systems which use Fuzzy Logic as an essential system component. In this study direct action type fuzzy logic controller applied to the 2-DOF nonlinear car model. The direct action fuzzy PID controller is placed within the feedback control loop and manages the PID actions through fuzzy algorithm.

The main role of a car suspension system is to improve car ride comfort and to provide better handling characteristics for riding on different road conditions. Ride comfort is improved by the reduction of the body acceleration. Because of good road handling and high passenger comfort active suspension systems have been suggested by many researchers for suppression of vehicle vibration [1-4].

In order to analyze the behaviour of the quarter-car model several nonlinear mathematical models have been investigated [3, 5-9]. To deal with the real world nonlinearities, many control methods have been studied. Fuzzy logic controllers are very suitable since the model has nonlinearities. Fuzzy logic autopilot is an intelligent controller which is combined fuzzy logic theory with control theory. The most advantageous property is that FLC do not rely on the exact mathematical models of the systems. After Mamdani’s case study about fuzzy logic theory, many different fuzzy systems have been used to reduce the vehicle vibration [1, 2, 4, 10-12,15,16].

In this paper a nonlinear quarter car model vibration is controlled by two input fuzzy logic structure with coupled rules which is combined fuzzy controller with PID. Its structure is different from typical fuzzy logic controller. The paper is organized as follows. In Section 2, mathematical model of nonlinear quarter car model is represented. Section 3 shows the general structure of control scheme. Section 4 discusses the simulation results. Finally section 5 provides conclusions.

II. MATHEMATICAL MODEL OF NONLINEAR QUARTER-CAR MODEL

A two degree of freedom sprung/unsprung mass nonlinear quarter car model which is shown in Fig. 1 is used in this paper.

The model consists of a body and a wheel which are connected with a suspension system. The suspension system is consisted of spring, damper, nonlinear elements. The nonlinearities that were studied in this paper are cubic stiffness, nonlinear damping and coulomb friction damping in the suspension. An actuator is also connected between the car body and the wheel and the wheel spring is in contact with the road profile. The dynamical differential equations of this nonlinear quarter car model can be given as:

\[ \begin{align*}
\frac{d^2 x_b}{dt^2} &= -\frac{k_w}{m_b} x_b - \frac{c_b}{m_b} \frac{d x_b}{dt} - \frac{k_w}{m_w} x_w + \frac{f_r}{m_b} \\
\frac{d^2 x_w}{dt^2} &= -\frac{k_w}{m_w} x_w + \frac{f_r}{m_w}
\end{align*} \]
where \( m_b \) represent 1/4 vehicle body mass and \( m_w \) is the wheel mass. The linear stiffness of the suspension vehicle body spring and vehicle tire are \( k_b \) and \( k_w \) respectively, \( c_b \) is the linear damping constant; \( k_b^n \) and \( c_b^n \) are the nonlinear elements of suspension system. \( \mu \) is the friction coefficient and \( f_{\text{susp}} = 0.1 m_u g \) [9]. \( x_b \) and \( x_w \) are the vertical displacements of the vehicle body and wheel, respectively. ‘\( u \)’ is the suspension control force.

### III. FUZZY LOGIC CONTROL SCHEME

Well-known proportional-integral-derivative (PID) controller is the most widely used in industrial application because of its simple structure. On the other hand conventional PID controllers with fixed gains do not yield reasonable performance over a wide range of operating conditions and systems (time-delayed systems, nonlinear systems, etc.). Control techniques which based on fuzzy logic and modified PID controllers are alternatives to conventional control methods.

Fuzzy PID controller used in this paper is based on two input FLC structure with coupled rules [13,14]. The overall structure of used controller is shown in Fig. 2.

![Fig. 2 Block diagram of the two input Fuzzy PID structure](image)

FLC has two inputs and one output. These are error (\( e \)), error change (\( de \)) and control signal, respectively. Linguistic variables which implies inputs and output have been classified as: NB, NM, NS, ZE, PS, PM, PB. Inputs and output are all normalized in the interval of [-1, 1] as shown in Fig. 3.

![Fig. 3 Membership functions of inputs (a,b) and output (c)](image)

Real interval of variables is obtained by using scaling factors which are \( S_e \), \( S_{de} \) and \( S_u \). The fuzzy control rule is in the form of: IF \( e = E_i \) and \( de = dE_j \) THAN \( U_{PD} = U_{PID}(i, j) \). These rules are written in a rule base look-up table which is shown in Fig. 4. The rule base structure is Mamdani type. The two input fuzzy controller with coupled rules formed by combining both PI and PD actions.

The final fuzzy PID controller signal can be given as:

\[
U_{PID}(t) = S_u \left[ K_{pi} \sum_{i=0}^{n} U_{PD}(i) + K_{pd} U_{PD}(t) \right]
\]  

(2)

The linguistic labels used to describe the Fuzzy sets were ‘Negative Big’ (NB), ‘Negative Medium’ (NM), ‘Negative Small’ (NS), ‘Zero’ (ZE), ‘Positive Small’ (PS), ‘Positive Medium’ (PM), ‘Positive Big’ (PB). It is possible to assign the set of decision rules as shown in Table I. The fuzzy rules are extracted from fundamental knowledge and human experience about the process. These rules contain the input/output relationships that define the control strategy. Each control
input has seven fuzzy sets so that there are at most 49 fuzzy rules.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>EXAMPLE OF DECISION TABLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>de/e</td>
<td>NB</td>
</tr>
<tr>
<td>NB</td>
<td>NB</td>
</tr>
<tr>
<td>NM</td>
<td>NB</td>
</tr>
<tr>
<td>NS</td>
<td>NB</td>
</tr>
<tr>
<td>Z</td>
<td>NM</td>
</tr>
<tr>
<td>PS</td>
<td>NM</td>
</tr>
<tr>
<td>PM</td>
<td>NS</td>
</tr>
<tr>
<td>PB</td>
<td>ZE</td>
</tr>
</tbody>
</table>

IV. SIMULATIONS AND DISCUSSION

In order to verify the effectiveness of the used controller, the following numerical simulations are performed with MATLAB-Simulink. The mathematical model of the vehicle suspension system as defined in equation (1) and the system parameters are listed in Table II.

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>PARAMETERS OF THE MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body mass - ( m_b )</td>
<td>295 kg</td>
</tr>
<tr>
<td>Damping of suspension - ( c_b )</td>
<td>1000 N.s/m</td>
</tr>
<tr>
<td>Wheel mass - ( m_w )</td>
<td>45 kg</td>
</tr>
<tr>
<td>Nonlinear stiffness constant - ( k_b^n )</td>
<td>( 16 \times 10^5 ) N/m³</td>
</tr>
<tr>
<td>Stiffness of the body - ( k_b )</td>
<td>160 000 N/m</td>
</tr>
<tr>
<td>Nonlinear damping constant - ( c_b^n )</td>
<td>600 N.(s/m)³</td>
</tr>
<tr>
<td>Stiffness of the tire - ( k_w )</td>
<td>160 000 N/m</td>
</tr>
<tr>
<td>Friction coefficient - ( \mu )</td>
<td>1</td>
</tr>
</tbody>
</table>

In the simulation, the performance of the fuzzy PID was compared with passive suspension system. Simulation is carried out with an upward bump function. Comparisons between active and passive case are evaluated in terms of body displacement, body acceleration and suspension deflection. The results of the simulations are reported in Figs. 4-5-6.

In Fig. 4 comparison of the deflection of vehicle body is presented. The acceleration of vertical motion is given in Fig. 5. As it can be seen from Fig. 5, Fuzzy PID Controller achieved to reduce vertical body motion. Fuzzy Logic controller provides a good decreasing velocity of vertical motion. Fig. 6 compares the suspension deflection responses. In these figures, we observed that the stability of vertical motion is increased by Fuzzy Logic Controller.
V. CONCLUSION

In this paper, the two input fuzzy PID controller structure with coupled rules has presented design active controllers for two-degrees of freedom nonlinear quarter car model. The performance of controller has also been discussed. The aim of this controller is reduce vehicle body motion and to ensure the comfort of passengers. Simulation results show that the two input fuzzy PID controller structure is able to good tracking performance in the nonlinear quarter car model so that ride comfort can be guaranteed.

REFERENCES