Comparing the Performance of the Particle Swarm Optimization and the Genetic Algorithm on the Geometry Design of Longitudinal Fin

Hassan Azarkish, Said Farahat, and S. Masoud H. Sarvari

Abstract—In the present work, the performance of the particle swarm optimization and the genetic algorithm compared as a typical geometry design problem. The design maximizes the heat transfer rate from a given fin volume. The analysis assumes that a linear temperature distribution along the fin. The fin profile generated using the B-spline curves and controlled by the change of control point coordinates. An inverse method applied to find the appropriate fin geometry yield the linear temperature distribution along the fin corresponds to optimum design. The numbers of the populations, the count of iterations and time to convergence measure efficiency. Results show that the particle swarm optimization is most efficient for geometry optimization.

Keywords—Genetic Algorithm, Geometry Optimization, longitudinal Fin, Particle Swarm Optimization

I. INTRODUCTION

Optimum geometry design of systems has been widely attracted in the field of engineering. There have been many optimization methods for optimizing the objective function to achieve desirable plan or systems. Gradient based methods such as conjugate gradient and Levenberg–Marquardt or stochastic and population based optimization methods such as genetic algorithm and particle swarm optimization [1, 2]. Each method has some advantages and disadvantages. Thus there has been some controversy recently about the performance of these algorithms.

Fin profile optimization is one of classical conjugated heat transfer problems. Azarkish et al. used the B-spline curves and modified genetic algorithm for optimized the convective-radiative single fin profile [3] and a fin array [4]. In this method, the effect of variation of convective heat transfer coefficient, variable conductivity along the fin, the effect of radiation and the length of arc could be modeled easily without needing to evaluation of gradients and fall to local optimum. However, the number of objective function evaluations in this method is more than gradient-based methods.

Moreover, the conduction mechanism is not very sensitive under the differential change of the shape. Thus, numerical evaluation of the sensitivity matrix and gradients are more difficult. Therefore, for simple objective functions such as one dimensional fin profile optimization this method is acceptable, however the computational cost would be dramatically increased for more complex problems such as two dimensional geometry optimization. In conclude, it seems that a low computational cost optimization method without needing to calculate of gradients could be suitable for these types of problems. In the present work, the performance of particle swarm optimization for the geometry optimization has been investigated and compare with the performance of the genetic algorithm. A single convective-radiative fin is considered as subject. Azarkish et al. [3], show that the optimum temperature distribution along the fin was linear in absence of volumetric heat generation. Therefore the aim of inverse problem is find an appropriated fin profile to achieve the leaner temperature distribution along the fin. Application of both particle swarm optimization and genetic algorithm for this problem has been investigated. First, the best value of constant parameters in the particle swarm optimization is determined. The effect of variation of these parameters on the convergence rate has been investigated. Finally, the necessary number of population for good convergence and corresponding number of iterations and convergence time are compared between two optimization algorithms mentioned above.

II. DIRECT PROBLEM

Consider a longitudinal fin with variable cross sectional area at the base temperature $T_b$ which is extended into a quiescent fluid of temperature $T_f$ and surrounded by an enclosure of temperature $T_{sur}$. The surface of fin is considered as diffuse and gray. The heat losses from the boundaries are assumed to be due to the radiation to the surrounding and the natural convection to the ambient (Fig. 1). The radiation heat transfer between the base and fin surface, also between the different elements of fin surface are neglected. The width of fin is assumed to be very thin, in such a way that the temperature distribution (and conduction heat transfer rate) may be regarded one dimensional along the x-axis. The energy equation and the boundary condition in this situation can be presented as:
Fig. 1 Schematic shape and orientations of the longitudinal fin with variable cross sectional area

\[
\frac{d}{dx} \left( y(x) \frac{dT}{dx} \right) = \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \left[ \frac{h}{k} (T - T_a) + \frac{\sigma \epsilon}{k} (T^4 - T_a^4) \right] 
\]  \hspace{1cm} (1)

\[ T \bigg|_{x=0} = T_0 \]  \hspace{1cm} (2)

\[ \frac{dT}{dy} \bigg|_{y=L} = 0 \]  \hspace{1cm} (3)

Where \( y(x) \) is the half thickness of fin (fin profile), \( k \) is the fin thermal conductivity of fin and \( h \) is the local convective heat transfer coefficient calculated by the following correlation [3]:

\[
h = \frac{8k \Pr^{1/3}}{336 \left( \Pr + \frac{5}{9} \right)} \left( \frac{g \beta}{v} \right)^{1/4} \left( T(x) - T_a \right)^{1/4} 
\]  \hspace{1cm} (4)

This non-linear equation is solving with the finite volume method [5] to obtain the temperature distribution along the fin. A detailed description of direct problem was briefly explained elsewhere by the authors [3, 4].

### III. INVERSE PROBLEM

In the present work, the inverse problem is considered instead of direct optimum geometry design of single fin. Fin profiles generated by B-spline curves [6] and controlled by moving the coordinates of control points in \( x, y \) directions (Fig. 2). The number of control points is considered to be \( N_{CP} = 4 \). The first control point is placed at the base of fin (\( x = 0 \)), that can move freely along the y-axis. Therefore this control point represents the thickness of the fin base. Conversely, the last control point is placed on the fin axis of symmetry (\( y = 0 \)), which can move freely along the x-axis in such a way that its position specifies the fin length. Other control points can move in xy-plane and therefore, their degree of freedom is equal to 2.

The number of design variables is equal to 6 and defined as:

\[ \text{eps} < x_1 \leq L_{\text{max}} \]
\[ \text{eps} < y_1, y_2, y_3 \leq y_{\text{max}} \]
\[ 0 \leq \alpha_1, \alpha_2 \leq 1 \]  \hspace{1cm} (5)

Where \( \text{eps} \) is an arbitrary small value, \( L_{\text{max}} \) is maximum length of fin and \( y_{\text{max}} \) is maximum amount of half thickness of the fin base. The position of control points defined as:

\[ \text{P}_i : (0, y_i) \]
\[ \text{P}_i : (\alpha_i x_i, y_i) \]  \hspace{1cm} (6)

Therefore, the design variables are defined as the position of control points. Each chromosome in the genetic algorithm or each particle in particle swarm optimization represents the set of control points correspond to a fin profile. In order to evaluated the fitness of each set of control points, Direct problem (Eq(1)) is solved, the temperature distributions and the volume of fin are obtained and compared with ideal temperature distributions and the given volume. Thus, two error functions have been introduced for predict the fitness of each profile:

\[ E_1 = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{V_{\text{ideal},i} - V_i}{V_{\text{ideal},i}} \right) \]  \hspace{1cm} (7)

\[ E_2 = 100 \left| \frac{V - V_{\text{allow}}}{V_{\text{allow}}} \right| \]  \hspace{1cm} (8)

Where \( n \) is the number of control volumes, \( T_i \) is the temperature of each control volume obtained by solving the direct problem and \( V_{\text{allow}} \) is the allowable volume of the fin.
T_{ideal,i} \) is the ideal temperature of each control volume defined as:

\[
T_{Ideal} = T_a - (T_b - T_a) \left( \frac{x}{L} \right)
\]  
(9)

If \( E_1 \rightarrow 0 \) the temperature distribution along the fin become linear as Eq(9). \( E_2 \rightarrow 0 \) satisfied to have a given volume \( V_{allow}. \) \( E_1 \) and \( E_2 \) are positive functions, therefore, the aim of inverse problem is minimize \( E_1 + E_2. \)

The genetic algorithm and the particle swarm optimization are used to minimize the amount of error functions corresponding to the optimum fin profile. The genetic algorithm is a stochastic search technique that based on the mechanism of genetics and natural selection and it is widely used in the field of engineering optimization problems [7]. On the other hand, Particle swarm optimization is a population-based swarm intelligence algorithm that introduced by Eberhart and Kennedy in 1995. It is starts with a group of particles known as the swarm. Each particle is function of design variables and it is improved through the algorithm by changing the position of particle on the search space. Consider the current position of particles at the moment \( t \) is given by \( X_i(t). \) The new position of particles in the next generation is expressed as:

\[
V_i(t+1) = C_1 \left[V_i(t) + C_2 \quad rand(0,1)[P_i(t) - X_i(t)] + C_3 \quad rand(0,1)[G_i(t) - X_i(t)]\right] + \]

\[
X_i(t+1) = X_i(t) + V_i(t+1)
\]  
(10)
(11)

Where \( P_i(t) \) is the local best position of particle \( i \) at the moment \( t \) and \( G_i(t) \) is the global best position of the swarm. Moreover \( C_1, C_2 \) and \( C_3 \) are the acceleration constants responsible for varying the particle velocity towards \( P_i(t) \) and \( G_i(t), \) respectively. More details about particle swarm optimization are presented in [8].

IV. RESULTS AND DISCUSSION

Use A longitudinal fin is considered at the base temperature \( T_b = 500K \) and \( T_w = 300K. \) The fin is made of aluminum with thermal conductivity of \( k = 210W/mK \) and the surface emissivity of \( \varepsilon = 0.3. \) The fin height is \( H = 40cm \) and the allowable volume is considered to be \( V_{allow} = 160cm^3. \) The \( B \)-spline curve with 4 control points is used to generate the fin profile. The aim of optimization is minimized \( E_1 + E_2 \) to have a linear temperature distribution from \( T_b = 500K \) to \( T_w = 300K \) and also \( V_{allow} = 160cm^3. \) The acceptable error that satisfied these conditions is \( E_1 + E_2 < 0.5. \)

In order to investigate the effects of constant parameters \( C_1, C_2 \) and \( C_3 \) on the convergence rate of particle swarm optimization, the case study is solved with different value of these parameters and the pairs of values \( C_1 = 0.7, C_2 = 1.2 \) and \( C_1 = 1.4 \) is recommended for good convergence. Fig. 3 shows the effect of parameters \( C_1, C_2 \) and \( C_3 \) on the necessary number of iterations for convergence. As shown, parameter \( C_1 \) is more sensible rather than other parameters.

Moreover, the genetic algorithm parameters are considered as crossover rate of 0.4 and mutation rate of 0.02 and with a population size of 100. In order to find the optimum location of control points correspond to linear temperature distributions the inverse problem is solved. The comparison of the temperature distributions obtained by particle swarm optimization, genetic algorithm and ideal temperature distribution is shown in Fig. 4.

As shown, both particle swarm optimization and genetic algorithm could find the appropriate temperature distribution. However, the convergence time and computational cost are very different for two cases mentioned above. Fig. 5 compares the variation of necessary iterations for convergence as function of population number for two kinds of optimization methods. The corresponding convergence time is presented in Fig. 6.
As shown, the number of iterations decreases rapidly on the range of particles between 10 and 25; however the graph shows a slight decrease after 25 particles. Moreover, the minimum correspond convergence time occur on the range of 20 to 25 particles and it is increase after 30 particles. Therefore, the recommended number of particles is 20 to 30 for particle swarm optimization. On the other hand, the recommended number of chromosomes is 60 to 80 for the genetic algorithm. Therefore, the number of populations decreases about 3 times and the convergence time decrease about 4 times in the case of particle swarm optimization rather than the genetic algorithm for this typical problem.

V. CONCLUSION

Particle swarm optimization and the genetic algorithm used for minimized the error functions in the inverse design of convective-radiative fin profile. Value of $C_1 = 0.7$, $C_2 = 1.2$ and $C_3 = 1.4$ recommend as constant parameters for this applications. It was shown the particle swarm optimization was at least 3 times more efficient rather than genetic algorithm. Therefore, particle swarm optimization recommended for geometry optimization especially when the gradient base methods failed.

REFERENCES