Using Neural Network for Execution of Programmed Pulse Width Modulation (PPWM) Method

M. Tarafdar Haque, and A. Taheri

Abstract—Application of neural networks in execution of programmed pulse width modulation (PPWM) of a voltage source inverter (VSI) is studied in this paper. Using the proposed method it is possible to cancel out the desired harmonics in output of VSI in addition to control the magnitude of fundamental harmonic, contionously. By checking the non-trained values and a performance index, the most appropriate neural network is proposed. It is shown that neural networks may solve the custom difficulties of practical utilization of PPWM such as large size of memory, complex digital circuits and controlling the magnitude of output voltage in a discrete manner.

Keywords—Neural Network, Inverter, PPWM.

I. INTRODUCTION

The inherent ability and recent progresses of digital signal processors have attracted a great deal of attention to use them in controlling of voltage source inverters (VSIs). These digital systems can implement the numerical computation with considerable speed and accuracy. Among different control methods of inverters, there are some pulse width modulation (PWM) methods such as equal area PWM (EAPWM), approximated EAPWM (AEAPWM) and Programmed PWM (PPWM) which need some numerical computations [1-2].

The PPWM has the ability of elimination of undesired harmonics, up to the specified order and to control the magnitude of fundamental harmonic at the output of VSI. The execution of PPWM needs solving a set of nonlinear equations. These equations should be solved by numerical methods which have inherent difficulties such as divergence of response, convergence to wrong results, consuming time and requiring large memory size. Neural networks can be used to solve these problems especially in on line applications [3-4].

In this paper a feed-forward neural network with back propagation training algorithm is used for execution of PPWM method. The results verify the ability of neural networks in controlling of switching pattern of VSIs. Inherent advantages of neural networks such as very fast and parallel calculations, noise and fault tolerant performance and potential of being easily implemented in dedicate analog or digital hardware chips, make the neural networks as a practical candidate for implementation of PPWM in controlling of VSIs [5,6].

The authors are with Department of Electrical and Computer Engineering, Tabriz University, Tabriz, Iran (e-mail: tarafedar@tabrizu.ac.ir).

II. THE REVIEW OF PPWM METHOD

Fig. 1 shows the power circuit topology of a single-phase VSI. The VSI basically consists of a DC voltage source and four controlled switches such as transistor and IGBT. The switching pattern of an inverter is the most important factor in generation of a controlled sinusoidal waveform. The most popular switching patterns are those based on PWM. The PWM methods are used in different manners, but all of them try to reduce the undesired harmonics at the output of inverter. These PWM methods focus on the elimination of low order harmonics that is because of generally higher magnitudes of lower order harmonics compared with higher orders.

The PPWM is one of the famous PWM based methods, which may be used to eliminate or control the magnitude of harmonics at the output of VSIs. The numbers of harmonics that can be controlled by PPWM, depends on maximum switching frequency. The advantages of PPWM over the well known sinusoidal PWM (SPWM) are as follows:

1) It is possible to execute the over modulation in PPWM with more efficiency.
2) In a specific power, the switching frequency of PPWM is almost 50% of SPWM.
3) The lower switching frequency results in a lower power loss by PPWM at the converter.
4) Because of better quality of output waveform in PPWM compared with SPWM, it is possible to use smaller filters at the input and output of inverter.
5) By elimination of low order harmonics, the resonance condition with output filter would not occurs.

Fig. 2 shows a sample voltage waveform in the output of a VSI using PPWM method. This voltage waveform has four

Fig. 1 Power circuit topology of a single-phase VSI

Fig. 2 Sample voltage waveform in the output of a VSI using PPWM method.

The Fourier series of this odd and one-fourth symmetric waveform is as follows:
\[ v(t) = \sum_{n=0}^{\infty} \left[ a_n \cos(n \omega_0 t) + b_n \sin(n \omega_0 t) \right] \]

where:
\[ a_n = \frac{4}{n \pi} \times \left[ -1 - 2 \sum_{k=0}^{n-1} (-1)^k \cos(n \alpha_k) \right], \quad n=1,3,5,... \]
\[ b_n = 0 \]

In Eq. (2), \( N \) is the number of switching instants between zero to \( \pi/2 \) radians. Obviously, it is possible to control the magnitude of \( N \) number of harmonics using eq. (2). According to Eq. (2), by choosing \( a_n = 0 \), the \( n \)’th order harmonics will be eliminated. Meanwhile, it is possible to control the magnitude of fundamental harmonic, \( a_1 \). This results in \( N \) nonlinear equations with \( N \) number of unknown parameters in each one, which are firing angles of switches of VSI. In above mentioned equations the magnitude of fundamental harmonic, \( a_1 \) is normalized using the magnitude of DC side voltage of VSI as the base voltage. In this way, the magnitude of \( a_1 \) may vary between zero to one per unit, while one-unit equals DC side voltage of VSI.

Solving these simultaneous equations results in the firing angles \( \alpha_k \), \( k=1,2,..., N \). The Newton - Raphson method is used to solve these equations. It is possible to find all of the switching instants in a period using one-fourth symmetry of output voltage waveform. Fig. 3 shows the variations of \( \alpha_1 \) to \( \alpha_4 \) to eliminate the 5th, 7th and 11th order of harmonics versus the variations of \( a_1 \). Table I shows five rows of the magnitudes of \( \alpha_1 \) to \( \alpha_4 \) in radians to eliminate the 5th, 7th and 11th orders of harmonics versus the variations of \( a_1 \).

![Fig. 2 Switching pattern in a half cycle for elimination of 5th, 7th and 11th orders harmonics](image)

### III. FEEDFORWARD METHOD AND BACKPROPAGATION ALGORITHM

Figure 4 shows the basic structure of a three-layer feedforward neural network. This neural network has one input, some hidden and four output neurons, respectively. The net value of the \( j \)’th neuron in layer 1, for \( j=1,2,..., k \) is:

\[ S_j = \sum_{i=1}^{k} w_{ij} o_i + b_j = \mathbf{W}^T \mathbf{F} + b_j \]

where the \( b_j \) is the bias of the \( j \)th neuron, \( w_{ij} \) is the connection weight from neuron \( i \) to the neuron \( j \) and \( o_i \) is the activation of neuron \( i \). The net value \( S_j \) is processed by an activation function \( f(.) \).

\[ o_j = f(S_j) \]

It is possible using different activation function such as sigmoid function as eq. (6).

\[ o_j = \frac{1}{1 + e^{-S_j}} \]

The back propagation algorithm is used to train the neural network, where the connection weights are updated using the delta rule as eq. (7) to (10).

### TABLE I

<table>
<thead>
<tr>
<th>Pattern No</th>
<th>Magnitude of ( a_1 )</th>
<th>Firing Angle ( \alpha_1 )</th>
<th>Firing Angle ( \alpha_2 )</th>
<th>Firing Angle ( \alpha_3 )</th>
<th>Firing Angle ( \alpha_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0625</td>
<td>0.3482193</td>
<td>0.6993044</td>
<td>1.0462540</td>
<td>1.396867</td>
</tr>
<tr>
<td>2</td>
<td>0.01250</td>
<td>0.3473707</td>
<td>0.7004752</td>
<td>1.0453130</td>
<td>1.397473</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>75</td>
<td>0.46875</td>
<td>0.2807636</td>
<td>0.7844183</td>
<td>0.9832738</td>
<td>1.447214</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>158</td>
<td>0.98750</td>
<td>0.1923240</td>
<td>0.9343935</td>
<td>0.9777231</td>
<td>1.512386</td>
</tr>
<tr>
<td>159</td>
<td>0.99375</td>
<td>0.1909101</td>
<td>0.9405243</td>
<td>0.9820038</td>
<td>1.513185</td>
</tr>
</tbody>
</table>
Fig. 4 Basic structure of a multi-layer feed-forward neural network

\[
\delta_j = o_j (1 - o_j) (y_j - o_j) \text{, for neuron } j \text{ of output layer} \quad (7)
\]

\[
\delta_j = o_j (1 - o_j) \sum \delta_k w_{jk} \text{, for neuron } j \text{ of hidden layers} \quad (8)
\]

\[
\Delta w_{jk} (i + 1) = \eta \delta_j o_k + \alpha \Delta w_{jk} (i) \quad (9)
\]

\[
w_{jk} (i + 1) = w_{jk}^p + \Delta w_{jk} (i + 1) \quad (10)
\]

Where \( \delta \) is learning rate and \( \alpha \) is the momentum term.

IV. EXECUTION OF PPWM METHOD BY NEURAL NETWORKS

In this paper some three-layer feed-forward neural networks are used to train the look up Table I. This table has 159 rows and five columns. Magnitude of \( a_1 \) (with increment steps equal to 0.00625 (pu)) is as input and firing angles \( \alpha_1 \) to \( \alpha_4 \) are as output of proposed neural networks. Only 40 rows including pattern No. 1, 5, 9, 13, 17, etc. are used in training process, while remainder patterns are used to examine the validation of proposed neural networks in generation of correct values in output, for non-trained input values. The number of neurons in hidden layer, \( n \), is considered to be 3 to 15 for determination of the most suitable neural network structure. Table II shows the values of performance index PI, and validation factor VF, for some of the selected three-layer neural networks with different number of neurons in hidden layer.

The performance index PI, is defined using eq. (11) where, \( N_p \) is the number of patterns (in this work \( N_p=40 \)), \( N \) is the number of neurons in output layer (in this work \( N=4 \)) and \( o_{kp} \) and \( \alpha_{kp} \) are the desired output and the activation values of the \( k \)’th neuron of output layer for the \( p \)’th pattern, respectively. The PI values in Table II are calculated after 5000 time repeating of feed-forward/ back-propagation algorithm for each neural network. The linear activation function is used in training process.

\[
PI = \frac{1}{N_p} \sum_{p=1}^{N_p} \left( \sum_{k=1}^{N} (o_{kp} - \alpha_{kp})^2 \right) \quad (11)
\]

The validation factor VF, is calculated using eq. (12). In this equation \( \alpha_{kp}^* \) is the output value for non-trained input values, of \( k \)’th neuron for \( p \)’th pattern. The non-trained values are pattern No. 2, 3, 4, 6, 7, 8, 10, 11, 12, etc of Table I. The VF values in table 2 are calculated after 5000 times of teaching of each neural network.

\[
VF = \frac{1}{N_p} \sum_{p=1}^{N_p} \left( \sum_{k=1}^{N} (\alpha_{kp}^* - \alpha_{kp})^2 \right) \quad (12)
\]

While PI is a factor to determine the teaching ability of neural networks for trained input values, VF is a factor to determine the teaching ability of neural networks for non-trained input values. However, the goal of PPWM is canceling out the desired harmonics in output voltage waveform but the second aim is controlling of fundamental harmonic \( a_1 \), continuously. So the VF factor is more important than PI to choose the best neural network structure. Considering the values of VF in Table II it is easy to notice that the three layer neural network with 13 neuron in hidden layer is the best neural network to implement of PPWM in voltage source inverters.

Fig. 5 shows the variation of performance index PI, of proposed neural network with 13 neurons in hidden layer versus the number of training. As mentioned before, after 5000 time of training the PI is reached to 1.23e-15.

![Fig. 5 PI versus training numbers](imageURL)

### Table II

<table>
<thead>
<tr>
<th>Hidden layer neurons</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>VF</td>
<td>1.238e-7</td>
<td>6.03e-9</td>
<td>8.11e-10</td>
<td>2.17e-10</td>
<td>1.28e-10</td>
<td>3.42e-10</td>
<td>1.53e-15</td>
<td>1.585e-15</td>
<td>6.34e-8</td>
</tr>
</tbody>
</table>
Fig. 6. shows sum of absolute errors for p'th pattern of table 1, $SAE_p$, between desired and output of above mentioned neural network while the fundamental harmonic $a_1$, varies between zero to one per unit. Equation (13) is used to compute the values of $SAE_p$ for each value of $a_1$.

$$SAE_p = \sum_{k=1}^{4} |o_{kp} - o_{kp}|$$  \hspace{1cm} (13)

This figure shows that near zero and near one per unit there is some more difference between desired and output values of neural network compared with other values of $a_1$. But, it should be noticed that the absolute error in all of the $a_1$ values is less than 1e-4 radians that is very good from practical point of view in reduction of undesired harmonics.

![Fig. 6 PI vs. training time for 1-3-4 network](image)

V. CONCLUSION

In this paper the application of neural networks to execution of PPWM method in producing the switching pattern of a VSI is studied. The training procedure is considered only for canceling out of three harmonics and controlling the magnitude of fundamental frequency, but this method can be used to control more harmonics to achieve better performance. The results show that a three-layer neural network with 13 neurons in hidden layer can produce the best results in execution of PPWM. The trained neural networks may be used to control the magnitude of fundamental harmonic in a wide range from zero to one per unit. It is shown that the neural networks can be used to control the magnitude of $a_1$ in a continuous manner, too. This is a major advantage for utilization of neural networks in execution of PPWM.

REFERENCES