The Effect of Response Feedback on Performance of Active Controlled Nonlinear Frames

M. Mohebbi and K. Shakeri

Abstract—The effect of different combinations of response feedback on the performance of active control system on nonlinear frames has been studied in this paper. To this end different feedback combinations including displacement, velocity, acceleration and full response feedback have been utilized in controlling the response of an eight story bilinear hysteretic frame which has been subjected to a white noise excitation and controlled by eight actuators which could fully control the frame. For active control of nonlinear frame Newmark nonlinear instantaneous optimal control algorithm has been used which a diagonal matrix has been selected for weighting matrices in performance index. For optimal design of active control system while the objective has been to reduce the maximum drift to below the yielding level, Distributed Genetic Algorithm (DGA) has been used to determine the proper set of weighting matrices. The criteria to assess the effect of each combination of response feedback have been the minimum required control force to reduce the maximum drift to below the yielding drift. The results of numerical simulation show that the performance of active control system is dependent on the type of response feedback where the velocity feedback is more effective in designing optimal control system in comparison with displacement and acceleration feedback. Also using full feedback of response in controller design leads to minimum control force amongst other combinations. Also the distributed genetic algorithm shows acceptable convergence speed in solving the optimization problem of designing active control systems.

Keywords—Active control, Distributed genetic algorithms, Response feedback, Weighting matrices.

I. INTRODUCTION

ACTIVE control of structures has been studied theoretically and tested in laboratories and also installed in prototype full scale structures through some past years[1]-[3]. Through these studies different mechanism and algorithms have been developed for active control of linear and nonlinear structures. In Most of the algorithms proposed for active control of structures such as classical optimal control and instantaneous optimal control [4] a time- dependent quadratic performance index has been defined which includes different combination of response (displacement, velocity and acceleration) feedback and control force where uses positive semi-definite weighting matrices for response related matrices and positive definite matrix for control force related matrix. In the previous researches different combinations of response feedback such as velocity and displacement [5], velocity and acceleration [6] and full response feedback (displacement, velocity and acceleration) [7]-[8] have been used in performance index function. Chang and Yang [5] used the velocity and displacement of the system in the performance index of instantaneous optimal control algorithm for active control of linear structures and concluded that regarding the maximum required control force using the velocity feedback in performance index is more efficient than the displacement feedback. Since in the practical application of active control systems the measurement of the response which are to be used in performance index has high effect in reducing time delay ,therefore the methods which use the feedback which are easy to be measured have high importance. Yang and Li [6] regarding that the measurement of acceleration and velocity of the structural response are easier, proposed a new instantaneous optimal control law which uses the acceleration and velocity responses in performance index and concluded that the proposed algorithm works as good as other methods use the displacement and velocity response in the performance index . In the instantaneous optimal control the control force is determined by minimizing the performance index, consequently the control force is obtained as a function of the same feedback used in performance index function. Hence, for linear structures using displacement, velocity and acceleration feedback in defining the performance index results modifying stiffness, damping and mass matrices respectively. To modify the mass, damping and stiffness matrices of the structure Bahar et al. [7] proposed an instantaneous optimal control which uses the displacement, velocity and acceleration response in the performance index and studied elementary the effect of some weighting matrices on the performance of active tuned mass damper on linear frames which the numerical simulation was based on try and error. However the effect of different combinations of response feedback on the performance of active control systems on nonlinear frames has not been studied perfectly. On the other hand in active control algorithms which use performance index to determine control force, the control law includes response and control force.

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K. Shakeri is with the Faculty of Engineering, University of Mohaghegh Ardabili, Ardabil, Iran. (e-mail: shakeri@uma.ac.ir).
related weighting matrices. Theoretically, by adjusting the weights it is possible to mitigate the response to below any desired level. In the previous studies different methods have been used for selecting the weighting matrices. Yang et al. [9] to guarantee the stability of the controlled structures have proposed a systematic way of assigning the weighting matrices by using the Lyapunov direct method. Following the proposed method by Yang et al. [9] it is possible to define the response related weighting matrices as a function of mass and stiffness matrices which has been used by Chang and Yang [5] for active control of linear structures. However defining these weights as a function of stiffness function is valid only for linear structures. Such techniques of determining the weights are not systematic and the mitigation of response to a specified desired level generally requires extensive numerical analysis with the additional drawback that there is no guarantee that a set of proper values can be obtained [5], [7].

Another alternative to determine the weighting matrices in controller design is by using optimization techniques such as Genetic Algorithm (GA) [8], [10] which has the significant advantage that the method is systematic and also that smaller control forces are required. Hence in this paper for each combination of response feedback to design the optimal controller, following the method proposed by Joghataie and Mohebbi [10] the weighting matrices have been determined through solving an optimization method using distributed genetic algorithm (DGA).

In the following sections, first the equations and algorithm for Newmark nonlinear instantaneous optimal control algorithm will be explained briefly. Next a brief explanation of the Distributed GA (DGA) will be presented followed by an example and conclusions.

II. NEWMARK NONLINEAR OPTIMAL CONTROL ALGORITHM

In this paper for active control of nonlinear n-DOF structure, the DGA based nonlinear optimal control [10] has been used which a brief of the method is explained in this section.

The equation of motion of a controlled nonlinear n-DOF structure with m actuators at times (k-1) $\Delta t$ and (k)$\Delta t$ can be written as:

$$M \ddot{X}_{k+1} + F_{D_{k+1}} + F_{S_{k+1}} = M \ddot{X}_{g_{k+1}} + D_{u_{k+1}}$$  \hspace{1cm} (1)

$$M \ddot{X}_k + F_{D_k} + F_{S_k} = M \ddot{X}_{g_k} + D_{u_k}$$  \hspace{1cm} (2)

Where t= time, $\ddot{X}_{g}$=ground acceleration, $X$, $\dot{X}$ and $\ddot{X}$ are displacement, velocity and acceleration vectors respectively, $M = n \times n$ mass matrix, $F_D$=vector of damping forces which is a function of velocity, $F_S$=vector of restoring forces which is a function of displacement, $D = n \times m$ location matrix of actuators, $e = [-1,-1,\ldots,-1]^T = n$-dimensional ground acceleration transformation vector, $u(t)$ = m-dimensional control force vector, $k$ = integration time step.

Subtracting (1) from (2):

$$M \Delta \ddot{X}(t) + C^* \Delta X(t) + K^* \Delta X(t) = \Delta P(t)$$  \hspace{1cm} (3a)

where

$$\Delta \ddot{X}(t) = \ddot{X}_k - \ddot{X}_{k-1}$$  \hspace{1cm} (3b)

$$\Delta X(t) = X_k - X_{k-1}$$  \hspace{1cm} (3c)

$$\Delta X(t) = \frac{1}{2} \left( \int_t^{t+\Delta t} \ddot{X}(s) \, ds \right)$$  \hspace{1cm} (3d)

$$\Delta P(t) = P_k - P_{k-1}$$  \hspace{1cm} (3e)

$$P_k = Me\ddot{X}_{g_k} + Du_k$$  \hspace{1cm} (3f)

$$P_{k+1} = Me\ddot{X}_{g_{k+1}} + Du_{k+1}$$  \hspace{1cm} (3g)

Also $C^*$ and $K^*$ are tangential damping and stiffness matrices respectively.

Based on Newmark method [11], by solving the set of (3a) to (3g) the response of a nonlinear structure can be obtained as follows:

$$X_k = X_{k-1} + \Delta X_k$$  \hspace{1cm} (4a)

$$X_k = (1 - a_3)X_{k-1} - a_6 \ddot{X}_{k-1} + a_4 \Delta X_k$$  \hspace{1cm} (4b)

$$\ddot{X}_k = (1 - a_3)\ddot{X}_{k-1} - a_2 \dddot{X}_{k-1} + a_1 \Delta \dddot{X}_k$$  \hspace{1cm} (4c)

$$\Delta X_k = K_n^* \Delta F_k$$  \hspace{1cm} (4d)

$$K_n^* = a_4 M + a_6 C_{k-1}^* + K_{k-1}^*$$  \hspace{1cm} (5)

$$\Delta F_k = (P_k - P_{k-1}) + M (a_1 \dddot{X}_{k-1} + a_3 \dddot{X}_{k-1})$$

$$+ C_{k-1}^* \left( a_2 \dddot{X}_{k-1} + a_6 \dddot{X}_{k-1} \right)$$  \hspace{1cm} (6)

$K_n^*$ varies at each time step.

$$a_1 = \frac{1}{\Delta t}; \ a_2 = \frac{1}{\Delta t}; \ a_3 = \frac{1}{2\Delta t};$$  \hspace{1cm} (7a,b,c)

$$a_4 = \gamma \Delta t; \ a_5 = \frac{\gamma}{\Delta t}; \ a_6 = \Delta t \left( \frac{\gamma}{2\Delta t} - 1 \right);$$  \hspace{1cm} (7d,e,f)

where $\gamma$, $\Delta t$ are Newmark parameters [11].

A. Performance Index and Control Force

In the instantaneous optimal control, the performance index at time step $k$ includes feedback of the system response and control force. To assess the effect of displacement, velocity and acceleration response on the performance of control system it has been decided to use full feedback of the system response and control force in the performance index as:

$$J_k = \frac{1}{2} \left( \frac{1}{2} Q_1 Q X_k + \frac{1}{2} Q_2 \dot{X}_k + \frac{1}{2} Q_3 \dddot{X}_k + u_i^T R u_i \right)$$  \hspace{1cm} (8)

where $Q_1$, $Q_2$ and $Q_3$ are $n \times n$ positive semi-definite weighting matrices corresponding to the penalty for large displacements, velocities and accelerations, and $R$ is a $m \times m$ positive definite matrix representing the cost for applying large forces [4].

For determination of active control force, $u_k$, the DGA based nonlinear optimal control method proposed by Joghataie and Mohebbi [10], has been used as follows:

$$u_k = -R^{-1}D^* K_n^* \left( Q_1 X_k + a_4 Q_2 X_k + a_6 Q_3 \dddot{X}_k \right)$$  \hspace{1cm} (9)

where superscript (–T) means transpose of inverse matrix.
According to (9) it is obvious that the control force is dependent to feedback of response and weighting matrices. Hence in this paper it has been decided to assess the effect of selecting different feedback on the control system performance.

Equation (10) gives the control force as a function of the weighting matrices $R$, $Q_1$, $Q_2$ and $Q_3$ as design variables. According to the method proposed by Joghataie and Mohebbi [10] these weights have been considered as design variables and determined by solving an optimization problem. The objective has been defined as minimizing the maximum required control force for reducing the maximum drift to a desired level.

### III. DISTRIBUTED GENETIC ALGORITHM (DGA)

Genetic algorithm (GA) is a computational method starts with a generation of individuals and changes the current set towards producing a fitter generation of design points, through some transformation operations. There are three genetic algorithm operators [12]: selection, cross over and mutation. In every generation, a set of chromosomes is selected for mating based on their relative fitness. The fitters are given more chance of passing their genes into the next generation. This process of natural selection is operated by selection. The selected individuals are then chosen randomly through cross over to produce offspring. In order to maintain the variability of the population, mutation should be performed in certain individuals. At the final stage the individual which has the best fitness is chosen as a design point.

When the number of variables and individuals in an optimization problem is large, using traditional GA to obtain the best answer may need high number of generations. Also in some cases it is cumbersome to determine the optimum point at all. In such problems it is better to divide the chromosomes into $N_{sub}$ subpopulations of smaller size, when a traditional GA is executed on each subpopulation separately. This process is called distributed genetic algorithm (DGA). A smaller number of individuals in DGA lead to quicker convergence and higher searching capability as compared to the conventional GAs [13]-[14]. In the DGA some individuals migrate from one subpopulation to the others periodically according to specified rules of migration such as the ring topology, neighborhood migration and unrestricted migration.

To maintain the size of the original population, the new chromosomes have to be reinserted into the old population. An insertion rate, $\eta$, determines the number of newly produced chromosomes $N_{inc}$ inserted in the old population. $N_{elites}$ of the best chromosomes are selected as elites of the current generation to go to the next generation without modification. The rest of the chromosomes in the population are replaced by inserted newborns.

### IV. WEIGHTING MATRIX ARRANGEMENTS

In this paper for active tendon mechanism which applies the control force on each floor of the frame (Fig. 1(b)), for elementary study and to focus on the main objective of this paper a simple arrangement for weighting matrices $Q_1$, $Q_2$ and $Q_3$ has been considered as follows which satisfies the necessary conditions for weighting matrices.

$$Q_1=q_1[I]_{n \times n} \quad (10a)$$
$$Q_2=q_2[I]_{n \times n} \quad (10b)$$
$$Q_3=q_3[I]_{n \times n} \quad (10c)$$

where $[I]_{n \times n}$ is the unit matrix of size $n \times n$. In this case there are only 3 variables in the optimization problem. For this study $R$ has been a diagonal matrix with equal elements as follows:

$$R=r[I]_{n \times m}, \quad m=\text{the number of actuators.} \quad (11)$$

### V. NUMERICAL EXAMPLE

In this paper the eight-story shear frame [15] shown in Fig. 1(a) and mitigation of its vibrations by active controlling using 8 actuators (Fig. 1(b)) which applies control force on each floor has been studied.

The bilinear hysteretic material behavior with positive post-yielding stiffness and full hysteresis loops, as shown in Fig. 2, has been assumed for the structure. It has been assumed that the unloading occurs with the initial stiffness. In Fig. 2 the elastic (initial) stiffness and post elastic (post-yield) stiffness have been $K_0=3.404 \times 10^5$ kN/m and $K_p=3.404 \times 10^4$ kN/m respectively. The floor mass has been 345.6 tons and the linear viscous damping coefficient $c$ is 734.3 kN.sec/m which corresponds to the 0.5% damping ratio of the first vibration mode of the structure. All the floors have the same stiffness and also a floor yields when its inter-story drift is $Y_{yielding} = 2.4$ cm.

The uncontrolled structure has been subjected under a white noise, $W(t)$, ground acceleration with PGA=0.4g as shown in Fig. 3 and the maximum drift and total acceleration of the floors are reported in Table I. The structure has experienced nonlinear deformation at stories 1, 2 and 3 and the maximum drift has exceeded the yielding drift $Y_{yielding} = 2.4$ cm. Also the first story has experienced the maximum drift amongst all the floors, which has been $Y_{max(uncon)} = 4.75$ cm.
A. Arrangements of Response Feedback

To assess the effect of different kinds of response feedback on the active control system performance, seven sets of feedback, A-1 to A-4, including different combinations of displacement, velocity, and acceleration, have been considered as follows:

\[ A-1) \quad Q_1 \neq 0, Q_2 = 0, Q_3 = 0 \quad \text{(Displacement feedback)} \]  
\[ A-2) \quad Q_1 = 0, Q_2 \neq 0, Q_3 = 0 \quad \text{(Velocity feedback)} \]  
\[ A-3) \quad Q_1 = 0, Q_2 = 0, Q_3 \neq 0 \quad \text{(Acceleration feedback)} \]  
\[ A-4) \quad Q_1 \neq 0, Q_2 \neq 0, Q_3 \neq 0 \quad \text{(Full Feedback)} \]

where the weighting matrices \( Q_1, Q_2, \) and \( Q_3 \) can be chosen as defined in (10a-c).

B. Design Optimal Controller

For each arrangement of response feedback A-1 to A-4 by using the weighting matrices \( Q_1, Q_2, \) and \( Q_3 \) it has been desired to design the controllers to reduce the drift up to below the yielding level, \( Y_{\text{max}} = 2.4 \text{ cm} \), when a ground white noise acceleration with PGA = 0.4g has been applied and \( r = 5 \times 10^{-7} \) has been selected in (11).

For the arrangements A-1 to A-3 which have only one variable \( q_1, q_2, \) or \( q_3 \) as defined in (10a-c), the variable and the required control force have been determined by trial and error so that the maximum drift = \( Y_{\text{max}} = 2.4 \text{ cm} \) has been obtained.

Figs. 4(a-c) show the variation of maximum control force versus different value of \( q_1, q_2, \) and \( q_3 \) where the maximum required control force have been found as follows:

\[ A-1) \quad u_{\text{max}} = 819.1 \text{ kN}, \quad Y_{\text{max}} = 2.41 \text{ cm}, \]  
\[ A-2) \quad u_{\text{max}} = 138.5 \text{ kN}, \quad Y_{\text{max}} = 2.40 \text{ cm}, \]  
\[ A-3) \quad u_{\text{max}} = 1332.4 \text{ kN}, \quad Y_{\text{max}} = 2.44 \text{ cm} \]

According to the Figs. 4(a-c) some important results are:

1. For this case study by increasing \( q_2 \) generally the maximum drift decreases monotonically.
2. By increasing \( q_1 \) and \( q_3 \), decreasing the maximum drift is not monotonic while for some values of \( q_1 \) and \( q_3 \) increasing \( q_1 \) and \( q_3 \) leads to increase control force while the maximum drift increases too, so it requires more extensive numerical analysis to find the optimum values of weighting matrices parameters to below the maximum drift to \( Y_{\text{max}} = 2.4 \text{ cm} \).
3. Comparison between arrangements A-1 to A-3 shows that in this case study the velocity feedback (arrangement A-2) has the best effect on the control system performance amongst single response feedback, while the acceleration feedback (arrangement A-3) is not more effective.

For arrangement A-2, Table I shows the maximum drift and total acceleration of the controlled structure as well as the maximum required control force on each story.

According to the results it can be said that the combination of response feedback and using the optimization procedure to determine the weighting matrices parameters could have benefits such as less numerical analysis and smaller control force. So in this paper the arrangement A-4 has been investigated for designing the optimal controller, too.

For arrangement A-4 the weighting matrices have 3 variables so finding the optimal set of variables requires extensive numerical trial and error and there is no guarantee to find the optimal value of variables. Hence it has been decided to use DGA to find the optimal value of weighting matrices parameters where the procedure of DGA has been reported here. For this arrangement while the control force is applied on each floor, to solve the optimization problem for finding
an optimal set of design variables $q_1$, $q_2$ and $q_3$ for DGA procedure 2 subpopulations each with 25 individuals, 25 newborns for each subpopulation, 5 elites, migration interval equal with 20 generation and 20% migration rate have been selected.

For finding the optimal value of variables $Q = (q_1, q_2, q_3)$ by using DGA two subpopulations each with 25 randomly generated vectors of control parameters have been generated as the initial population. The processes of DGA have been continued until convergence has been achieved. Fig. 5 shows the best fitness value, $F(Q^*)$, of chromosomes for generations of 4 runs. All the runs have ended up with the same optimum answer though with different convergence speeds. For the optimum controller to reduce the maximum drift to $Y_{max} = 2.4$ cm, the maximum required control force has been $u_{max} = 108.9$ kN.

Table I shows the maximum drift and total acceleration of the controlled structure as well as the maximum required control force on each story.

Table I

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<th>Generation number</th>
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<th>Run.3</th>
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Fig. 5 The best fitness value of chromosomes in four runs of DGA

According to the results shown in Table I it is clear that the objective of applying active control system to reduce the maximum drift to $Y_{max} = 2.4$ cm and keep the frame in linear domain has been obtained.

Fig. 4 Maximum required control force versus different values of weighting matrices parameters (a) $q_1$; (b) $q_2$; and (c) $q_3$.

Fig. 6 shows the required control force for arrangements $A-2$ as the optimum case of single response feedback and $A-4$ as the full feedback of response. The results show that the full feedback of response, $A-4$, requires the minimum control force amongst the other combinations of response feedback.
VI. CONCLUSION

In this paper the effect of different combinations of response feedback on the performance of active control systems on nonlinear frames has been studied. For active control of nonlinear frames the DGA based nonlinear optimal control has been used where different combinations of feedback of response have been considered in the performance index. Distributed Genetic Algorithms (DGA) has been used to determine the parameters of weighting matrices for each arrangement of feedback. For each combination of response feedback optimal controller has been designed to mitigate the response to the yielding level by using the minimum control force.

For verification, an eight-story shear frame with bilinear nonlinearity and hysteretic behavior under white noise excitation has been considered where the active tendon mechanism applies control force on each floor and optimal controller for each arrangement has been designed. The obtained results have shown that the performance of the controller is significantly sensitive to the type of the response feedback combination. From the numerical analysis it has been concluded that defining performance index as a function of full feedback of response leads to design controllers with smaller control force. From the implementation of active control system point of view it can be said that to reduce the number of sensors, consequently the time delay it is better to use the controller which uses only the velocity feedback which requires smaller measurement time and works relatively as well as the controller uses full feedback response regarding the maximum required control force.

REFERENCES


TABLE I

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Drift, Acc, u_{max} = maximum control force.