FEA for Transient Responses of an S-Shaped Force Transducer with a Viscoelastic Absorber Using a Nonlinear Complex Spring

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Abstract—To compute dynamic characteristics of nonlinear viscoelastic springs with elastic structures having huge degree-of-freedom, Yamaguchi proposed a new fast numerical method using finite element method [1]-[2]. In this method, restoring forces of the springs are expressed using power series of their elongation. In the expression, nonlinear hysteresis damping is introduced. In this expression, nonlinear complex spring constants are introduced. Finite element for the nonlinear spring having complex coefficients is expressed and is connected to the elastic structures modeled by linear solid finite element. Further, to save computational time, the discrete equations in physical coordinate are transformed into the nonlinear ordinary coupled equations using normal coordinate corresponding to linear natural modes. In this report, the proposed method is applied to simulation for impact responses of a viscoelastic shock absorber with an elastic structure (an S-shaped structure) by colliding with a concentrated mass. The concentrated mass has initial velocities and collides with the shock absorber. Accelerations of the elastic structure and the concentrated mass are measured using Levitation Mass Method proposed by Fujii [3]. The calculated accelerations from the proposed FEM, corresponds to the experimental ones. Method proposed by Fujii [3]. The calculated accelerations from the proposed FEM, corresponds to the experimental ones. Moreover, using this method, we also investigate dynamic errors of the S-shaped force transducer due to elastic mode in the S-shaped structure.

Keywords—Transient response, Finite Element analysis, Numerical analysis. Viscoelastic shock absorber, Force transducer.

I. INTRODUCTION

To reduce impacts from precision instruments and so on, viscoelastic shock absorbers are used. These viscoelastic absorbers sometimes have nonlinearity between their restoring forces and deformations under relatively large load. Moreover, their restoring forces sometimes have nonlinearity in hysteresis. Therefore, it is of importance to clarify nonlinear dynamic properties of viscoelastic absorbers with elastic structures under impact load.

This paper deals with numerical analysis of impact responses for a viscoelastic shock absorber connected with an elastic structure (an S-shaped structure) using a fast finite element method proposed by Yamaguchi [1]-[2]. In this analysis, the viscoelastic absorber is modeled by using a nonlinear complex expression, nonlinear complex spring constants are introduced for not only the linear component but also nonlinear components of the restoring force. Finite element for the nonlinear complex spring is expressed and is connected to an elastic structure modeled by linear solid finite elements.

The discrete equations of this system in physical coordinate are transformed into the nonlinear ordinary coupled equations using normal coordinate corresponding to linear natural modes. The transformed equations are integrated numerically in extremely small degree-of-freedom.

In this paper, we apply our proposed FEM to examine dynamic errors in an S-shaped force transducer when transient responses of a viscoelastic shock absorber are measured. The transient responses are obtained when a concentrated mass (i.e. a rigid levitated block) is collided with the absorber. The elastic structure in this study is an S-shaped structure, which is a part of the force transducer as shown in Fig.1.

To investigate the dynamic errors in the transducer, the reference force is measured using Levitation Mass Method proposed by Fujii [3]. The experimental dynamic errors [3] are compared with the calculated ones from our proposed FEM.

II. EXPERIMENTAL SYSTEM [3]

Fig. 1 shows the outline of the experimental system carried out by Fujii [3]. To evaluate mechanical responses of a viscoelastic shock absorber, an S-shaped force transducer is connected with the absorber. A mass levitated by linear pneumatic bearing is collided with the absorber in y direction, and transient responses are measured using the force transducer. The S-shaped structure in the force transducer has two thin portions as twin beams near the central hole as shown in Fig.2. These two beams play a role of a pair of parallel springs in y direction. When external load is applied in y direction, strains are measured using strain gauges fabricated in the S-shaped structure. Using the strains, forces $F_{\text{max}}$ of the transient responses are measured.

To verify precision of the measured force $F_{\text{max}}$, Fujii also measured the velocity $v_1$ of the levitated mass using an interferometer as shown in Fig.1. By differentiating the measured velocity $v_1$, acceleration $a_1$ of the levitated mass is obtained. Further, the reference force $F_{\text{max}} = M a_1$ is identified.
using the acceleration $a_i$ and mass $M_i = 2.6526 \text{kg}$ of the levitated mass. Fujii compared between the reference force and mass $M_i = 2.6526 \text{kg}$ of the levitated mass. Fujii compared between the reference force and mass $M_i = 2.6526 \text{kg}$ of the levitated mass. Fujii compared between the reference force and mass $M_i = 2.6526 \text{kg}$ of the levitated mass. Fujii compared between the reference force and mass $M_i = 2.6526 \text{kg}$ of the levitated mass. Fujii compared between the reference force and mass $M_i = 2.6526 \text{kg}$ of the levitated mass. Fujii compared between the reference force

Fujii compared between the reference force $F_{\text{trans}}$ and the force $F_{\text{trans}}$ obtained from the S-shaped transducer as shown in Fig.3. And he showed that there exist small dynamic errors in the measured force $F_{\text{trans}}$ of the S-shaped force transducer. Fig.4 shows a time history of the discrepancy $\Delta F = F_{\text{trans}} - F_{\text{meas}}$ between the measured force $F_{\text{trans}}$ by the transducer and the reference force $F_{\text{meas}}$. Further, Fujii found out that the discrepancy $\Delta F = F_{\text{trans}} - F_{\text{meas}}$ corresponds to the force calculated by $M_2 \ddot{a}_2$. $M_2$ is the acceleration of the S-shaped structure and $M_2$ is the half of the mass for the S-shaped structure. Using $\Delta F = M_2 \ddot{a}_2$, Fujii proposed a correction method of the measured force $F_{\text{trans}}$. To obtain correction force $\Delta F$, he added an accelerometer to the S-shaped structure as shown Fig.1. The measured acceleration $\ddot{a}_2$ is utilized to correct the measured force $F_{\text{trans}}$ in real time.

In this paper, to investigate the correction force $\Delta F$ (i.e. the discrepancy between $F_{\text{trans}}$ and $F_{\text{meas}}$) in detail, numerical simulations for this experimental system are performed.

We also use the experimental results to verify our proposed numerical method using FEA (i.e. Finite Element Analysis) with a nonlinear complex spring.

The detail conditions of this experimental system are added as follows. The experimental system has the levitated mass (i.e. levitated mass).
a levitated rigid block), which can smoothly travel along a guide way in y direction due to a pneumatic linear bearing as shown in Fig.1. The block is levitated due to air film (thickness is 8 (μm)) at the interfaces between the block and the guide. Pressure at the interfaces is self-controlled by orifice effect to maintain the thickness of the air film. Owing to this system, the block can move toward the y-direction under extremely low friction. An extension rod is connected with the levitated mass. These parts have a role of moving parts with initial velocity \( v_0 \) given by a hand. The moving parts collide with the small strip of the viscoelastic shock absorber attached with one end of the S-shaped structure. This experimental system is given for the nodal force of the spring at the point \( y \). The levitated block is connected at the nodal point \( y \) direction to the concentrated mass (i.e. the levitated mass). An initial velocity \( v_0 = -180 \) (mm/sec) in y-direction is given to the concentrated mass (i.e. the levitated mass).

A. Discretized Equation for the Viscoelastic Absorber

A small strip of the viscoelastic absorber is modeled by using a concentrated nonlinear spring with nonlinear hysteresis. To represent the nonlinear hysteresis, we propose to introduce a nonlinear complex spring. As shown in Fig.5, it is assumed that the nonlinear complex spring with viscoelasticity has principal elastic axis in y direction. We denote displacement as \( U_y \) at the nodal point \( y \). Nonlinear function using power series is given for the nodal force of the spring at the point \( y \). Therefore, restoring force of the spring is expressed as \( R_y = \gamma_1(U_y - U_y) + \gamma_2(U_y - U_y)^2 + \gamma_3(U_y - U_y)^3 \) using the relative displacement \( U_y - U_y \). Further, linear hysteresis damping is introduced as \( \gamma_1 = \delta_{11}(1 + j \eta_s) \), \( \gamma_2 = \delta_{22}(1 + j \eta_s) \), \( \gamma_3 = \delta_{33}(1 + j \eta_s) \) in the same manner. \( \gamma_1 \), \( \gamma_2 \), and \( \gamma_3 \) are nonlinear components of material loss factor for the concentrated spring, respectively. For the nonlinear complex spring, nonlinear spring constants have complex quantity to represent changes of hysteresis due to the deformation of the spring. These relations can be rewritten in the matrix form as:

\[
[r] = [\gamma_1][U_y] + [\gamma_2],
\]

\[
[\gamma_1] =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & \gamma_1 & 0 & 0 & -\gamma_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\gamma_1 & 0 & 0 & \gamma_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

where \( \gamma_1 = [0, R_y, 0, 0, R_y, 0] \) is nodal force vector at the nodes \( \alpha \) and \( \beta \). \( [U_y] = [U_\alpha, U_\alpha, U_\alpha, U_\alpha, U_\beta, U_\beta] \) is nodal displacement vector at the nodes. \( \gamma_2 \) is complex stiffness matrix involving only linear term of the restoring force. \( \gamma_2 \) is vector containing nonlinear terms of the restoring force.

For the later computations, we use nonlinear complex spring constants as \( R_\alpha = 1.37 \times 10^4 \) (N/m), \( R_\beta = 0.00 \) (N/m²), \( \gamma_1 = 2.35 \times 10^{11} \) (N/m³), \( \eta_s = 0.300 \), \( \eta_1 = 0.000 \), \( \eta_3 = 0.300 \).

B. Discretized Equations of the S-Shaped Force Transducer Filled with the Gel

We assumed that equations of motion for the elastic structure of the S-shaped force transducer are expressed under infinitesimal deformation using a conventional three dimensional finite element method. To add damping effects, a viscoelastic gel is filled in the central hole of the force transducer as shown in Fig.2. Thus, we also create the three dimensional finite element method. To add damping effects, a viscoelastic gel is filled in the central hole of the S-shaped force transducer. The gel is filled in the central hole of the S-shaped structure to obtain damping effect as shown in Fig.2.

III. Numerical Procedure

Fig. 5 represents the FEM model for the S-shaped structure in the force transducer to evaluate. In this figure, the S-shaped structure is modeled as an elastic body using finite elements, and the levitated mass is modeled by a concentrated mass. The viscoelastic shock absorber is modeled by a nonlinear spring with nonlinear damping. Origin of this model is set on the position where the levitated mass begins to contact with the viscoelastic absorber. An initial velocity \( v_0 \) is nodal force vector at the nodes \( \alpha \) and \( \beta \). \( [U_y] = [U_\alpha, U_\alpha, U_\alpha, U_\alpha, U_\alpha, U_\beta] \) is nodal displacement vector at the nodes. \( [\gamma_1] \) is complex stiffness matrix involving only linear term of the restoring force. \( \gamma_2 \) is vector containing nonlinear terms of the restoring force.

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\[
[M_s][\ddot{u}_s] + [K_s][u_s] = \{f_s\}
\]
where, \([M_i], [K_i], \{f_i\}\) and \([u_i]\) are mass matrix, complex stiffness matrix, nodal force vector and displacement vector for the S-shaped transducer and the gel, respectively. Isoparametric hexahedral elements with non-conforming modes are mainly used for the three dimensional finite elements.

C. Discrete Equation for Combined System between the S-Shaped Force Transducer with the Gel and the Viscoelastic Absorber with the Levitated Block

The restoring force \(\{r\}\) in Eq. (1) is added to the nodal force at the ends of the nonlinear complex spring on the nodes \(\alpha\) and \(\beta\). On the node \(\alpha\), the nonlinear complex spring (i.e. the viscoelastic absorber) is connected with the S-shaped force transducer. On the node \(\beta\), the spring is attached with the levitated mass modeled as a concentrated mass. And then the following expression in global system can be obtained:

\[
[M]\{\ddot{u}\} + [K]\{u\} + \{d\} = \{f\} \quad (3)
\]

where, \([u], [M], [K] \) and \([f]\) are displacement vector, mass matrix, complex stiffness matrix and external force vector in global system, respectively. \([\ddot{u}]\) is modified from \([\dddot{u}]\) to have the identical vector size to degree-of-freedom of the Eq.(3).

D. Approximate Computation of Modal Damping

To calculate linear modal damping (i.e. modal loss factor), we consider the following complex eigenvalue problem by neglecting the terms of the non-linear restoring force and the external force.

\[
\sum_{i=1}^{\text{max}} (-\omega_i^2)^2 \left( 1 + j \eta_{\text{tot}i} \right) [M]_i + [K]_i \left( 1 + j \eta_i \right) \{\phi_i\} = \{0\} \quad (4)
\]

In this equation, superscript \((i)\) stands for the \(i\)-th eigenmode. \(\eta_i\) includes \(\eta_{\text{dis}i}, \eta_{\text{mat}i}\) and \(\eta_{\text{g}i}\). \((-\omega_i^2)^2\) is the real part of complex eigenvalue. \(\{\phi_i\}\) is the complex eigenvector and \(\eta_{\text{tot}i}\) is the modal loss factor. Next, we introduce the following \(\beta_e\) using the maximum value \(\eta_{\text{max}}\) among the elements' material loss factors \(\eta_{\text{e}i}\), \((e = 1, 2, 3, \ldots, \text{max})\).

\[
\beta_e = \eta_e / \eta_{\text{max}}, \beta_e \leq 1 \quad (5)
\]

On assumption of \(\eta_{\text{max}} \ll 1\), solutions of Eq.(4) are expanded using a small parameter \(\mu = \eta_{\text{max}}\):

\[
\{\phi_i\} = \{\phi_i\}_0 + \mu \{\phi_i\}_1 + \mu^2 \{\phi_i\}_2 + \ldots, \quad (6)
\]

\[
(\omega_i^2)^2 = (\omega_0^2)^2 + \mu^2 (\omega_1^2)^2 + \mu^4 (\omega_0^2)^2 + \ldots, \quad (7)
\]

\[
\eta_{\text{tot}i} = \eta_{\text{dis}i} + \mu^2 \eta_{\text{mat}i} + \mu^4 \eta_{\text{mat}i} + \ldots, \quad (8)
\]

In these equations, under conditions of \(\beta_e \leq 1\) and \(\eta_{\text{max}} \ll 1\), we can obtain \(\eta_{\text{max}} \beta_e \ll 1\). Thus, \(\mu \beta_e\) is regarded as small parameters like \(\mu\)

In the equations, \(\{\phi_i\}_0, \{\phi_i\}_1, \{\phi_i\}_2 \ldots\) and \((\omega_0^2)^2, (\omega_1^2)^2, \ldots\) and \(\eta_{\text{dis}i}, \eta_{\text{mat}i}, \eta_{\text{mat}i} \ldots\) have real quantities.
F. Nonlinear Impact Response

We give an initial velocity to the levitated mass connected with the viscoelastic shock absorber. And nonlinear impact responses of the S-shaped force transducer with the viscoelastic absorber are calculated by applying Runge-Kutta-Gill method to Eq. (11).

IV. NUMERICAL RESULTS AND DISCUSSION

A. Verification of the Proposed Numerical Method

First, we verify our proposed numerical method using Finite Element Analysis with nonlinear complex spring. Fig. 6 shows time histories of the measured and calculated reference forces $F_{\text{mass}}$ using the levitated mass. These reference forces are obtained by $F_{\text{mass}} = M_{\text{a}}a_{\text{a}}$, where $a_{\text{a}}$ is the measured or calculated acceleration. In this graph, the origin of time is set when the levitated mass begins to collide with the viscoelastic shock absorber connected with the S-shaped structure. From Fig. 6, the calculated reference force is consistent with the measured one. We can say that the reference force can be simulated using our proposed FEA for the transient response of the viscoelastic shock absorber.

Next, we compute dynamic errors of the S-shaped force transducer. Fujii found out in the previous paper [3] that the dynamic errors $\Delta F$ of the transducer have relationship as $\Delta F \geq M_{\text{a}}a_{\text{s}}$. $a_{\text{s}}$ is the measured acceleration from the accelerometer fabricated on the S-shaped structure. And $M_{\text{s}}$ is the half of the mass for the S-shaped structure. Therefore, to investigate the dynamic errors, we simulate the acceleration $a_{\text{s}}$ on the S-shaped structure.

Fig. 7 (b) shows the calculated time history of the acceleration $a_{\text{s}}$ for the S-shaped structure in comparison with an experimental one in Fig. 7 (a). As illustrated in Fig. 8:

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Fig. 6 Comparison of the reference force using the levitated mass between calculated data and measured data [3]

Fig. 7 Comparison of the acceleration at the S-shaped structure between calculated data and measured data [3]

Fig. 8 Elastic mode of the S-shaped structure in y-direction appeared in dynamic errors of the transducer
The calculated curve in Fig. 7 (b) using nonlinear complex spring for the viscoelastic shock absorber is consistent with experimental data in Fig. 7 (a). In Fig. 7, we can find two typical periods both in the experimental and calculated accelerations at the S-shaped structure. The short period 0.0025(S) corresponds to the eigen frequency of the elastic mode for the S-shaped structure in y-direction as shown in Fig.8.

The long period 0.013 (s) is related with the subharmonic component of order 1/7 for this elastic mode. Moreover, the long period is also related with the superharmonic component of order seven for the translation mode of the concentrated mass (i.e. the levitated mass) in y-direction. These relations can be regarded as internal resonance. This is caused by the nonlinearity of the viscoelastic shock absorber.

As can be seen this figure, a half part of the S-shaped structure moves in this eigenmode, while the other half part of the S-shaped structure is at rest. Therefore, in this mode, the dynamic motion of the half part for the S-shaped structure is dominant. This fact leads that the correction method proposed by Fujii [3] using the acceleration $a$ and the half mass $M$ of the S-shaped structure is proper.

Additionally, Fig.9 shows the measured and calculated velocities at the S-shaped structure. Both results also agree well.

### B. Effect of Material Properties for Gel on Dynamic Errors of S-Shaped Force Transducer

Next, we investigate influences of the material properties of the gel filled in the central hole on the dynamic errors of the S-shaped structure. For the previous simulations in Figs. 6-9, we use the material loss factor $\eta_g=0.700 (-)$ and the storage modulus of elasticity $E_g =0.00400 (\text{GPa})$ for the gel. Under this condition, we obtain the modal loss factor $\eta_{tot}(i)=0.0496 (-)$ for the dominant eigen mode on the dynamic errors (i.e. the elastic mode of the S-shaped structure in y-direction) as shown in Fig.8. According to Eq. (9), to increase modal damping $\eta_{tot}(i)$ using the gel, we have to increase not only the material loss factor $\eta_g$ of the gel but also the share of the strain energy of the gel to the total strain energy in this system.

Fig.10 shows the relation between the modal loss factors $\eta_{tot}(i)$ in this system and the storage modulus of elasticity $E_g$ for the gel. In this graph, the constant value $\eta_g =0.700 (-)$ is given for the material loss factor of the gel. Thus, the change of the modal loss factor $\eta_{tot}(i)$ in Fig.10 is caused by the change of the share of...
the strain energy in the gel. As shown in Fig.10, the modal loss factor $\eta_{\text{tot}}^{(i)}$ has the local maximum value $\eta_{\text{tot}}^{(i)}=0.318$ (-) near $E_g=0.0900$ (GPa). Using the local maximum value, we compute a time history of the acceleration $a_x$ for the S-shaped structure as shown in Fig.11. The component of the short period for the mode decreases due to the increment of $\eta_{\text{tot}}^{(i)}$. This leads to less dynamic errors $\Delta F$ in the measured force using the force transducer. Fig.12 shows changes of the reference forces $F_{\text{max}}$ due to change of the modal loss factor from the initial value $\eta_{\text{tot}}^{(i)}=0.0496$ (-) to the local maximum value $\eta_{\text{tot}}^{(i)}=0.318$ (-). There exist significant changes of the reference force $F_{\text{max}}$ due to the increase of the storage modulus of elasticity $E_g$ for the gel. Therefore, the measured force using the force transducer is affected if we give too large storage modulus of elasticity $gE$ for the gel. Thus, this result reveals that it is undesirable to use too large storage modulus of elasticity $E_g$ for precision measurement of the force, even though we obtain less dynamic errors due to the elastic mode in the S-shaped transducer.

Fig.13 shows time histories of the reference force $F_{\text{max}}$, when the storage modulus of elasticity $E_g$ changes from $E_g=0.00100$ (GPa) to 0.00900 (GPa). These values of $E_g$ are less than the maximal value $E_g=0.0900$ (GPa) in Fig.12. The corresponding modal loss factor $\eta_{\text{tot}}^{(i)}$ changes from $\eta_{\text{tot}}^{(i)}=0.0277$ (-) to $\eta_{\text{tot}}^{(i)}=0.0928$ (-) as $E_g$ increases. As shown in Fig.13, the curves of the reference force $F_{\text{max}}$ never change under this condition. From these results, there are no influences on the reference force when the storage modulus of elasticity for the gel is less than 0.00900 (GPa). Using the same conditions for the storage modulus of elasticity $E_g$, we also compute the time histories of acceleration $a_x$ for the S-shaped structure as shown in Fig.14. The acceleration $a_x$ decreases as $E_g$ increases from $E_g=0.00100$ (GPa) to 0.00900 (GPa). From these results, it is possible to reduce the dynamic errors due to the acceleration $a_x$ of the S-shaped structure without

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When the material loss factor $\eta_g$ of the gel changes from $\eta_g = 0.0100 (-)$ to $\eta_g = 0.700 (-)$ under the constant storage modulus of elasticity $E_g = 0.00400$ (GPa), Figs.15 and 16 show the calculated reference force $F_{max}$ and the calculated acceleration $a_2$ of the S-shaped structure (i.e. dynamic error), respectively. The corresponding modal loss factor $\eta^i_{tot}$ changes from $\eta^i_{tot} = 0.0144 (-)$ to $\eta^i_{tot} = 0.0496 (-)$. As shown in Fig.16, the dynamic error decreases as the material loss factor $\eta_g$ of the gel increases. On the other hand, the reference force $F_{max}$ does not change though the material loss factor $\eta_g$ of the gel increases as shown in Fig.15. These results imply that the material loss factor $\eta_g$ of the gel can reduce the dynamic error of the force transducer without the influences on the reference force $F_{max}$.

V. CONCLUSION

By applying the numerical method proposed by Yamaguchi, simulation was carried out for impact responses of a viscoelastic shock absorber with an S-shaped elastic structure. This fast computational method using nonlinear finite element method is aimed for nonlinear impact responses of elastic structures connected with nonlinear concentrated springs with nonlinear hysteresis.

The restoring force of the spring is assumed to express as power series of elongation with nonlinear hysteresis damping. In this expression, nonlinear complex spring constants are introduced. Finite element for the nonlinear complex spring is expressed and is connected to elastic structures modeled by linear solid finite elements. Further, the discretized equations in physical coordinate are transformed into the nonlinear ordinary coupled equations using normal coordinate corresponding to linear natural modes. The transformed equations were integrated numerically. This transformation can save computational time drastically.

In this report, the proposed method is applied to simulation for impact responses of a viscoelastic shock absorber with an elastic structure (an S-shaped structure) by colliding with a concentrated mass. The concentrated mass has initial velocities and collides with the shock absorber. Accelerations of the elastic structure and the concentrated mass are measured using Levitation Mass Method proposed by Fujii [3]. The calculated accelerations from the proposed FEM, corresponds to the experimental ones.

Moreover, using this method, we also investigate dynamic errors of the S-shaped force transducer due to elastic mode in the S-shaped structure.

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