Gravitino Dark Matter in (nearly) SLagy $D3/D7$ μ-Split SUSY

Mansi Dhuria and Aalok Misra

Abstract—In the context of large volume Big Divisor (nearly) SLagy $D3/D7$ μ-Split SUSY [1], after an explicit identification of first generation of SM leptons and quarks with fermionic superpartners of four Wilson line moduli, we discuss the identification of gravitino as a potential dark matter candidate by explicitly calculating the decay life times of gravitino (LSP) to be greater than age of universe and lifetimes of decays of the co-NLSPs (the first generation squark/slepton and a neutralino) to the LSP (the gravitino) to be very small to respect BBN constraints. Interested in non-thermal production mechanism of gravitino, we evaluate the relic abundance of gravitino LSP in terms of that of the co-NLSP's by evaluating their (co-)annihilation cross sections and hence show that the former satisfies the requirement for a potential Dark Matter candidate. We also show that it is possible to obtain a 125 GeV light Higgs in our setup.

Keywords—Split Supersymmetry, Large Volume Swiss-Cheese Calabi-Yau's, Dark Matter, (N)LSP decays, relic abundance.

I. Introduction

OVER the past few years, producing realistic model satisfying both cosmological as well as phenomenological requirements from string compactifications has proven to be a daunting challenge. To get the phenomenological implications of B(eyond) S(tandard) M(odel)s, they must be invoked with a particular SUSY breaking mechanism and hence scale of SUSY breaking. The recently proposed split-SUSY model (based on high SUSY breaking scale) inspired by the need of fine-tuning to obtain a very small cosmological constant, is emerging out to be quite interesting from the point of view of phenomenology because of the fact that heavy scalars mostly appearing as virtual particles in most of the particle decay studies, help to resolve many diverse issues of both particle physics and cosmology [2]. Gravitino, a spin $3/2$ particle and the supersymmetric partner of the graviton in local SUSY, which acquires a mass from the spontaneous breaking of supersymmetry, in gravity mediated SUSY breaking theories, generally, appears to be lightest among all superpartners and can have impact on the issue of dark matter. Because of their interactions suppressed by M_p, they are decoupled from thermal plasma very early in the universe and their abundance might overclose the universe known as famous 'cosmological gravitino problem', the resolution of which is quite natural because abundance gets diluted as universe experiences through inflationary phase. Therefore, cosmologically relevant gravitino abundance is then recreated in the reheating phase after inflation by inelastic $2 \rightarrow 2$ scattering and $1 \rightarrow 2$-decay processes of particles from the thermal bath where abundance varies linearly with the reheating temperature T_R. However, in string/M theory-inspired models, this 'standard' way of production of Dark Matter (DM) particles is significantly altered because of the presence of moduli as decay of modului increases the entropy which sufficiently decreases the relic abundance of gravitino. Therefore, sizable amount of gravitinos can be produced by (non-thermal) production of gravitinos (LSP) formed by decays of moduli and can dominate the thermal production of gravitinos in the early plasma, discussed in [3]. Even in particle physics models, sufficient amount of relic density of LSP can be produced by decays of N(ext-to) L(ightest) S(upersymmetric) P(article) and has been studied in literature [4]. Motivated by this approach, we had studied the abundance of gravitino produced from decay of right set of moduli (corresponding to Co-NLSP's) in type IIB large volume μ split SUSY set up in [1].

In this paper, we summarize our study in [1] of decay width of gravitino appearing as the L(ightest) S(upersymmetric) P(article) and sleptons/squarks as N(ext-to) L(ightest) S(upersymmetric) P(article)s with (Bino/Wino-type)gaugino-dominant neutralino, emphasizing their impact on the issue of dark matter in the context of large volume μ split SUSY set up in an improvement of [5]. In addition to getting a light Higgs of $125 GeV$ and a long life time of gluino, we show that the presence of non-zero R-parity violating couplings and high squark masses help to reduce the decay width and hence lifetime of the gravitino (LSP) becomes very large (of the order or greater than age of the Universe) satisfying the requirement of potential dark matter candidate whereas life times of co-NLSP's are small enough not to dsiturb the beautiful predictions of BBN. Relying on the non thermal production occurring through decays of sleptons/neutralino existing as co-NLSP's in our set up, we evaluate the relic abundance of gravitino, the right amount of which helps to resolve cosmological gravitino problem while very heavy moduli masses($>>$ 10TeV) in the set-up automatically resolves the cosmological moduli problem.

II. The Model

Let us first briefly describe our large volume $D3/D7$ Swiss-Cheese setup of [1]. Type IIB compactified on the orientifold of a Swiss-Cheese Calabi-Yau in the L(arge) V(olume) S(cenarios) limit that includes non-(perturbative) α' corrections and non-perturbative instanton-corrections in superpotential [8]. For studying phenomenological issues, we

Aalok Misra and Mansi Dhuria are with the Department of Physics, Indian Institute of Technology, Roorkee - 247 667, Uttarakhand, India; e-mail: aalokfph@iitr.ernet.in, mansidph@iitr.ernet.in

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included a single mobile spacetime filling $D3$-brane and stacks of D7-branes wrapping the "big" divisor. Interested in generating the possibility of distinct non-abelian gauge groups by turning on different fluxes on the world volume of "big" divisor, therefore requires to construct four involutively odd harmonic distribution one-forms A_I localized on the sub-locus of Σ_B. The most non-trivial example of involutions which are meaningful only at large volumes is mirror symmetry implemented as three T-dualities in [6]. Interestingly, we found in [1] that the toroidal three-cycle supporting harmonic one-forms

$$C_3 : |z_1| \sim \mathcal{V}^{\frac{1}{36}},\ |z_2| \sim \mathcal{V}^{\frac{1}{36}},\ |z_3| \sim \mathcal{V}^{\frac{1}{6}}$$

(the Calabi-Yau can be thought as a T^3(swept out by ($argz_1$,$argz_2$, $argz_3$)-fibration over a large base $(|z_1|, |z_2|, |z_3|)$; precisely apt for application of mirror symmetry as three T-dualities a la S(trominger) Y(au) Z(aslow) [6]) is almost a s(pecial) Lag(rangian) sub-manifold because it satisfies the requirement that $f^*J \approx 0,\ f^*\Omega = e^{i\theta}\mathrm{vol}(C_3)$, where $f : C_3 \to CY_3$ [7]). Therefore, as discussed in [8], the harmonic distribution one-forms can be constructed by integrating:

$$dA_I = \left(P_{\Sigma_B}(z_{1,2,3})\right)^I dz_1 \wedge dz_2 \text{with } (I = 1, 2, 3, 4), \quad (1)$$

such that A_I is harmonic only on Σ_B and not at any other generic locus in the Calabi-Yau manifold and distribution one-forms on Σ_B localized along the $D3$-brane can be written as:

$$A_I \sim \delta\left(|z_3| - \mathcal{V}^{\frac{1}{6}}\right)\delta\left(|z_1| - \mathcal{V}^{\frac{1}{36}}\right)\delta\left(|z_2| - \mathcal{V}^{\frac{1}{36}}\right) \times \left[\omega_I(z_1, z_2)dz_1 + \tilde{\omega}_I(z_1, z_2)dz_2\right]. \quad (2)$$

Writing $A_I(z_1, z_2) = \omega_I(z_1, z_2)dz_1 + \tilde{\omega}_I(z_1, z_2)dz_2$ where $\omega(-z_1, z_2) = \omega(z_1, z_2), \tilde{\omega}(-z_1, z_2) = -\tilde{\omega}(z_1, z_2)$ and $\partial_1\tilde{\omega} = -\partial_2\omega$, one obtains:

$$\begin{aligned} A_1(z_1, z_2, z_3 \sim \mathcal{V}^{\frac{1}{6}}) &\sim -z_1^{18}z_2^{19}dz_1 + z_1^{19}z_2^{18}dz_2,\\ A_2(z_1, z_2, z_3 \sim \mathcal{V}^{\frac{1}{6}}) &\sim -z_1^{18}z_2dz_1 + z_2^{18}z_1dz_2,\\ A_3(z_1, z_2, z_3 \sim \mathcal{V}^{\frac{1}{6}}) &\sim -z_1^{18}z_2^{37}dz_1 - z_2^{18}z_1^{37}dz_1,\\ A_4(z_1, z_2, z_3 \sim \mathcal{V}^{\frac{1}{6}}) &\sim -z_1^{36}z_2^{37}dz_1 + z_2^{36}z_1^{37}dz_2. \end{aligned} \quad (3)$$

The $\mathcal{N} = 1$ chiral co-ordinates with the inclusion of mobile D3-brane position moduli $z_{1,2}$ and D7-branes Wilson line moduli a_I will be appropriate generalizations of [9]) for multiple $D7$-branes (See [1]). The quadratic contribution arising due to Wilson line moduli contribution is of the form: $i\kappa_4^2\mu_7C^B_{I\bar{J}}a^I\bar{a}^{\bar{J}}$ with $C^B_{I\bar{J}} = \int_{\Sigma_B} i^*\omega \wedge A^I \wedge \bar{A}^{\bar{J}}$, where $\omega \in H^{(1,1)}_+(\Sigma_B)$. We calculate the intersection matrices $C^B_{I\bar{J}}$ by constructing harmonic one forms using equation (2). Also, coefficient of quadratic term

$$(\omega_\alpha)_{i\bar{j}}\, z^i \left(\bar{z}^{\bar{j}} - \frac{i}{2}\left(\mathcal{P}_{\tilde{a}}\right)^{\bar{j}}{}_l\, \bar{z}^{\tilde{a}} z^l\right)$$

arising in T_B due to inclusion of position moduli z_i can be shown to be $\mathcal{O}(1)$ by calculating $(\omega_B)_{i\bar{j}} \sim (\omega_S)_{i\bar{j}} \sim \mathcal{O}(1)$ near $z_{1,2} \sim \frac{\mathcal{V}^{\frac{1}{36}}}{\sqrt{2}}$ (See [1]). Therefore one can argue ([8], [1]) that near

$$\begin{aligned} |z_{1,2}| \sim \mathcal{V}^{\frac{1}{36}}M_p, |z_3| \sim \mathcal{V}^{\frac{1}{6}}M_p, |a_1| \sim \mathcal{V}^{-\frac{2}{9}}M_p,\\ |a_2| \sim \mathcal{V}^{-\frac{1}{3}}M_p, |a_3| \sim \mathcal{V}^{-\frac{13}{18}}M_p, |a_4| \sim \mathcal{V}^{-\frac{11}{9}}M_p, \zeta^A = 0;\\ \mathcal{G}^a \sim \frac{\pi}{\mathcal{O}(1)k^a(\sim\mathcal{O}(10))}M_p, \end{aligned}$$

one obtains a local meta-stable dS-like minimum corresponding to the positive minimum of the potential $e^K G^{T_S\bar{T}_S}|D_{T_S}W|^2$ stabilizing

$$\mathrm{vol}\,(\Sigma_B) \sim \Re e(\sigma_B) \sim \mathcal{V}^{\frac{2}{3}}, \mathrm{vol}\,(\Sigma_S) \sim \Re e\sigma_S \sim \mathcal{V}^{\frac{1}{18}}$$

such that

$$\Re e(T)_S \sim \mathcal{V}^{\frac{1}{18}}$$

and in the dilute flux approximation, gauge couplings corresponding to the gauge theories living on stacks of $D7$ branes wrapping the "big" divisor Σ_B will given by: $g^{-2}_{YM} = Re(T_B) \sim \mathcal{V}^{\frac{1}{18}} \sim O(1)$ (justified by the partial cancelation between between Σ_B and $C_{I\bar{J}}a_I\bar{a}_{\bar{J}}$ i.e ($Vol(\Sigma_B)+C_{I\bar{J}}a_I\bar{a}_{\bar{J}}+h.c. \sim \mathcal{V}^{\frac{1}{18}}$).

The Kähler potential relevant to all the calculations in this paper (without being careful about $\mathcal{O}(1)$ constant factors) is given as under:

$$\begin{aligned} K\sim -2ln\Bigg(\Bigg[T_B + \bar{T}_B - \Bigg(\mu_3 l^2\left\{|z_1|^2 + |z_2|^2 + z_1\bar{z}_2 + z_2\bar{z}_1\right\}\\ +\mathcal{V}^{\frac{10}{9}}|a_1|^2 + \mathcal{V}^{\frac{11}{18}}\left(a_1\bar{a}_2 + h.c.\right) + \mathcal{V}^{\frac{1}{9}}|a_2|^2 + \mathcal{V}^{\frac{29}{18}}\left(a_1\bar{a}_3 + h.c.\right)\\ +\mathcal{V}^{\frac{10}{9}}\left(a_2\bar{a}_3 + h.c.\right) + \mathcal{V}^{\frac{19}{9}}|a_3|^2 + \mathcal{V}^{\frac{19}{9}}\left(a_1\bar{a}_4 + a_4\bar{a}_1\right) +\\ \mathcal{V}^{\frac{29}{18}}\left(a_2\bar{a}_4 + a_4\bar{a}_2\right) + \mathcal{V}^{\frac{47}{18}}\left(a_3\bar{a}_4 + a_4\bar{a}_3\right) + \mathcal{V}^{\frac{28}{9}}|a_4|^2\Bigg)\Bigg]^{3/2} -\\ \left(T_S + \bar{T}_S - \mu_3 l^2\left\{|z_1|^2 + |z_2|^2 + z_1\bar{z}_2 + z_2\bar{z}_1\right\}\right)^{3/2} + \sum n^0_\beta(...)\Bigg) \end{aligned}$$

and ED3 generated non-perturbative superpotential we will be using is given by:

$$W \sim \left(1 + z_1^{18} + z_2^{18} + \left(3\phi_0 z_1^6 z_2^6 - z_1^{18} - z_2^{18}\right)^{\frac{2}{3}} - 3\phi_0 z_1^6 z_2^6\right)^{n^s} \times e^{-n^s vol(\Sigma_S) - \mu_3(\alpha_S z_1^2 + \beta_S z_2^2 + \gamma_S z_1 z_2)}.$$

The evaluation of "physical"/normalized Yukawa couplings, soft SUSY breaking parameters and various 3-point vertices needs the matrix generated from the mixed double derivative of the Kähler potential to be a diagonalized matrix. After diagonalization the corresponding eigenvectors of the same are given by:

$$\begin{aligned} \mathcal{A}_4 &\sim a_4 + \mathcal{V}^{-\frac{3}{5}}a_3 + \mathcal{V}^{-\frac{6}{5}}a_1 + \mathcal{V}^{-\frac{9}{5}}a_2 + \mathcal{V}^{-2}\left(z_1 + z_2\right);\\ \mathcal{A}_3 &\sim -a_3 + \mathcal{V}^{-\frac{3}{5}}a_4 - \mathcal{V}^{-\frac{3}{5}}a_1 - \mathcal{V}^{-\frac{7}{5}}a_2 + \mathcal{V}^{-\frac{8}{5}}\left(z_1 + z_2\right);\\ \mathcal{A}_1 &\sim a_1 - \mathcal{V}^{-\frac{3}{5}}a_3 + \mathcal{V}^{-1}a_2 - \mathcal{V}^{-\frac{6}{5}}a_4 + \mathcal{V}^{-\frac{6}{5}}\left(z_1 + z_2\right);\\ \mathcal{A}_2 &\sim -a_2 - \mathcal{V}^{-1}a_1 + \mathcal{V}^{-\frac{7}{5}}a_3 - \mathcal{V}^{-\frac{3}{5}}\left(z_1 + z_2\right);\\ \mathcal{Z}_2 &\sim -\frac{(z_1+z_2)}{\sqrt{2}} - \mathcal{V}^{-\frac{6}{5}}a_1 + \mathcal{V}^{-\frac{3}{5}}a_2 + \mathcal{V}^{-\frac{8}{5}}a_3 + \mathcal{V}^{-2}a_4;\\ \mathcal{Z}_1 &\sim \frac{(z_1-z_2)}{\sqrt{2}} - \mathcal{V}^{-\frac{6}{5}}a_1 + \mathcal{V}^{-\frac{3}{5}}a_2 + \mathcal{V}^{-\frac{8}{5}}a_3 + \mathcal{V}^{-2}a_4, \end{aligned}$$

and the effective Yukawa couplings can be calculated using $\hat{Y}^{\rm eff}_{C_iC_jC_k} \equiv \frac{e^{\frac{K}{2}}Y^{\rm eff}_{C_iC_jC_k}}{\sqrt{K_{C_i\bar{C}_i}K_{C_j\bar{C}_j}K_{C_k\bar{C}_k}}}, C_i$ being an open string modulus which for us is $\delta\mathcal{Z}_{1,2}, \delta\mathcal{A}_{1,2,3,4}$. where $Y^{\rm eff}_{\mathcal{Z}_i\mathcal{A}_I\mathcal{A}_J}$ is given by $\mathcal{O}(\mathcal{Z}_i)$-coefficient in the mass term $e^{\frac{K}{2}}\mathcal{D}_{\bar{\mathcal{A}}_I}D_{\bar{\mathcal{A}}_J}\bar{W}\bar{\chi}^{\mathcal{A}_I}\chi^{\mathcal{A}_J}$ in the $\mathcal{N} = 1$ SUGRA action of [13]. By estimating in the large volume limit, all possible Yukawa couplings corresponding to four Wilson line moduli and showing that the RG-flow of the effective physical Yukawa's change almost by $\mathcal{O}(1)$ under an RG flow from the string scale down to the EW scale [1], we see that for $\mathcal{V} \sim 10^5, \langle\mathcal{Z}_i\rangle \sim 246GeV$:

- $$\frac{\mathcal{O}(\mathcal{Z}_i) \text{ term in } e^{\frac{K}{2}}\mathcal{D}_{\mathcal{A}_1}D_{\mathcal{A}_3}W}{\sqrt{K_{\mathcal{Z}_i\bar{\mathcal{Z}}_i}K_{\mathcal{A}_1\bar{\mathcal{A}}_1}K_{\mathcal{A}_3\bar{\mathcal{A}}_3}}} \equiv \hat{Y}^{\rm eff}_{\mathcal{Z}_i\mathcal{A}_1\mathcal{A}_3} \sim 10^{-3}\times\mathcal{V}^{-\frac{4}{9}}, \tag{4}$$

giving $\langle\mathcal{Z}_i\rangle\hat{Y}_{\mathcal{Z}_1\mathcal{A}_1\mathcal{A}_3} \sim MeV$ - about the mass of the electron!

- $$\frac{\mathcal{O}(\mathcal{Z}_i) \text{ term in } e^{\frac{K}{2}}\mathcal{D}_{\mathcal{A}_2}D_{\mathcal{A}_4}W}{\sqrt{K_{\mathcal{Z}_i\bar{\mathcal{Z}}_i}K_{\mathcal{A}_2\bar{\mathcal{A}}_2}K_{\mathcal{A}_4\bar{\mathcal{A}}_4}}} \equiv \hat{Y}^{\rm eff}_{\mathcal{Z}_i\mathcal{A}_2\mathcal{A}_4} \sim 10^{-\frac{5}{2}}\times\mathcal{V}^{-\frac{4}{9}}, \tag{5}$$

giving $\langle z_i\rangle\hat{Y}_{\mathcal{Z}_i\mathcal{A}_2\mathcal{A}_4} \sim 10MeV$ - close to the mass of the up quark! The above shows that fermionic superpartners of $\mathcal{A}_1$ and $\mathcal{A}_3$ correspond respectively to first generation of left-handed $SU(2)$ and right-handed $U(1)$ leptons while fermionic superpartners of $\mathcal{A}_2$ and $\mathcal{A}_4$ correspond respectively to left-handed $SU(2)$ and right-handed U(1) quarks.

III. Mass Scales

The gravitino mass

$$m_{3/2} = e^{\frac{K}{2}}WM_p \sim \mathcal{V}^{-\frac{n^s}{2}-1}M_p,$$

in the context of gravity mediation and for $n^s = 2$ and $\mathcal{V} = 10^5$, turns out to be around 10^8 GeV. Hereafter, using diagonal metric matrix and calculating F-terms corresponding to bulk moduli $T_{B,S}, \mathcal{G}^a$, soft SUSY breaking parameter i.e position moduli mass (to be identified with Higgs) $\mathcal{Z}_i$ come out to be (see [1])

$$m_{\mathcal{Z}_i} \sim \mathcal{V}^{\frac{59}{72}}m_{3/2}$$

and Wilson line moduli masses(to be identified with scalar masses), come out to be very heavy :

$$m_{\mathcal{A}_1} \sim \sqrt{\mathcal{V}}m_{3/2}$$

for all Wilson line moduli at string scale and changes only by $\mathcal{O}(1)$ under the RG solution down to EW scale as shown in [11], giving one of the signatures of μ-split SUSY. In addition to these, SUSY breaking trilinear couplings $A_{\mathcal{I}\mathcal{J}\mathcal{K}}$ and supersymmetric Higgsino mass parameter $\hat{\mu}_{\mathcal{Z}_1\mathcal{Z}_2}$ also turns out to be very large of the order

$$\hat{\mu}_{\mathcal{Z}_1\mathcal{Z}_2} \sim \mathcal{V}^{\frac{37}{36}}m_{3/2}.$$

Now, to realize split SUSY as in [2], we calculated in [1] the mass of light Higgs formed by linear combination of two Higgs doublets by first calculating the masses of same which after soft supersymmetry breaking are given by $(m^2_{Z_i}+\hat{\mu}^2_{Z_i})^{1/2}$ and thereafter using RG solution to Higgs mass discussed in [5], one can get the contribution of Higgs doublets as well as the Higgsino mass parameter $\hat{\mu}_{Z_i}$ at EW scale. The Higgs mass matrix is given as

$$\begin{pmatrix} m^2_{H_1} & \hat{\mu}B \\ \hat{\mu}B & m^2_{H_2} \end{pmatrix} \sim \begin{pmatrix} m^2_{H_1} & \xi\hat{\mu}^2 \\ \xi\hat{\mu}^2 & m^2_{H_2} \end{pmatrix}.$$

Assuming non-universality w.r.t. to both $D3$-brane position moduli masses ($m_{Z_{1,2}}$) given by δ_1 and

$$\hat{\mu}B \sim \xi\hat{\mu}_{Z_iZ_j}(\xi \sim O(1)),$$

considering a small fine tuning i.e $(0.03+\delta_1)m_0^2 \sim -0.06S_0$ (S_0 is hypercharge weighted sum of squared soft scalar mass having value around m_0^2) and

$$\xi \sim 2 + \frac{1}{8}\frac{m^2_{EW}}{m_0^2},$$

we obtain one light Higgs (corresponding to the negative sign of the square root) of order 125 GeV and one heavy Higgs (corresponding to the positive sign of the square root) whereas the squared Higgsino mass parameter $\hat{\mu}_{\mathcal{Z}_1\mathcal{Z}_2}$ then turns out to be heavy with a value, at the EW scale of around $\mathcal{V}m_{3/2}$, thus showing the possibility of realizing μ split SUSY scenario in the context of LVS $D3-D7$ set up.

In fact, in addition to being able to generate the mass-scales relevant to (in this paper, first generation) quarks/leptons, using the RG-flow arguments of [10], one can also show that the Weinberg-type dimension-five Majorana-mass generating operator: $\mathcal{O}(\langle z_i\rangle^2)$ coefficient in $\frac{e^{\frac{K}{2}}\frac{\partial^2 W}{\partial\mathcal{A}_1^2}}{\sqrt{K^2_{\mathcal{Z}_i\bar{\mathcal{Z}}_i}K^2_{\mathcal{A}_1\bar{\mathcal{A}}_1}}}\left(\bar{\chi}^{\mathcal{A}_1}_L\mathcal{Z}_i\right)^2$ or in fact $\frac{e^{\frac{K}{2}}\mathcal{D}_{\bar{\mathcal{A}}_1}D_{\bar{\mathcal{A}}_1}\bar{W}}{\sqrt{K^2_{\mathcal{Z}_i\bar{\mathcal{Z}}_i}K^2_{\mathcal{A}_1\bar{\mathcal{A}}_1}}}\left(\bar{\chi}^{\mathcal{A}_1}_L\mathcal{Z}_i\right)^2$ produces the correct first-generation neutrino mass scale of slightly less than $1eV$ for $\langle z_i\rangle \sim \mathcal{O}(1)\mathcal{V}^{\frac{1}{36}}$.

In addition to this it was shown in [1], decay life time of gluino comes out to be high (in the range $10^{-5}s(\tilde{g}\rightarrow q\bar{q}\tilde{\chi}^0_3), 10^2 s(\tilde{g}\rightarrow g_\mu\tilde{\chi}^0_3), 10^4 s(\tilde{g}\rightarrow q\bar{q}\psi_\mu), 10^{11}s(\tilde{g}\rightarrow g_\mu\psi_\mu)$ sec), thus showing the stability of Gluino and hence giving another concrete evidence of μ split SUSY in our set up.

IV. Life time of N(LSP) decay channels

The very important constraint that the hadronic/electromagnetic energy released from decay products of next-to-lightest supersymmetric particle (NLSP) must not alter the observed abundance of light elements in the universe essentially fixed by average lifetime around $\tau \sim 10^2 sec$ referred to as the B(ig) B(ang) N(ucleosynthesis) constraint, is satisfied by NLSP candidates if decay of same occurs before BBN era [12]. In addition to this, taking R-parity violating couplings into account, the (lightest) neutralino might decay into leptons/quarks rather than gravitino and hence elude the relic abundance of gravitino coming from decay of neutralino (Co-NLSP) if life time for the former decay is less than the latter; via explicit calculations, we ensure that this does not happen. For the same one needs to calculate the decay widths of all important 2- and 3-body decay channels for which we will be using the following terms (written out in four-component notation or their two-component analogs and utilizing/generalizing results of [9]) in the $\mathcal{N}=1$ gauged supergravity action of Wess and Bagger [13] with the understanding that $m_{\rm moduli/modulini} << m_{\rm KK}\left(\sim \frac{M_s}{\mathcal{V}^{\frac{1}{6}}}\Big|_{\mathcal{V}\sim 10^{5/6}} \sim 10^{14}GeV\right), M_s = \frac{M_p}{\sqrt{\mathcal{V}}}\Big|_{\mathcal{V}\sim 10^{5/6}} \sim 10^{15}GeV$, and that for multiple $D7$-branes, the non-abelian gauged isometry group[1], corresponding to the killing vector $6i\kappa_4^2\mu_7\left(2\pi\alpha'\right)Q_B\partial_{T_B}, Q_B = \left(2\pi\alpha'\right)\int_{\Sigma_B} i^*\omega_B \wedge P_-\tilde{f}$ arising due to the elimination of of the two-form axions $D_B^{(2)}$ in favor of the zero-form axions ρ_B under the KK-reduction of the ten-dimensional four-form axion [9] (which results in a modification of the covariant derivative of T_B by an additive shift given by $6i\kappa_4^2\mu_7\left(2\pi\alpha'\right)Tr(Q_B A_\mu)$) can be identified with the SM group (i.e. A_μ is the SM-like adjoint-valued gauge field [13]):

$$\begin{aligned}
\mathcal{L} = {} & g_{YM} g_{T_B\bar{\mathcal{J}}} Tr\left(X^{T_B}\bar{\chi}_L^{\bar{\mathcal{J}}}\lambda_{\tilde{g},\ R}\right) \\
& + i g_{\mathcal{I}\bar{\mathcal{J}}} Tr\left(\bar{\chi}_L^{\bar{\mathcal{I}}}\left[\not{\partial}\chi_L^{\mathcal{I}} + \Gamma^i_{Mj}\not{\partial}a^M\chi_L^{\mathcal{J}} + \frac{1}{4}\left(\partial_{a_M}K\not{\partial}a_M - \text{c.c.}\right)\chi_L^{\mathcal{I}}\right]\right) \\
& + \frac{e^{\frac{K}{2}}}{2}\left(\mathcal{D}_{\bar{\mathcal{I}}}D_{\mathcal{J}}\bar{W}\right)Tr\left(\chi_L^{\mathcal{I}}\chi_R^{\mathcal{J}}\right) \\
& + g_{T_B\bar{T}_B} Tr\left[\left(\partial_\mu T_B - A_\mu X^{T_B}\right)\left(\partial^\mu T_B - A^\mu X^{T_B}\right)^\dagger\right] \\
& + g_{T_B\mathcal{J}} Tr\left(X^{T_B}A_\mu\bar{\chi}_L^{\mathcal{J}}\gamma^\nu\gamma^\mu\psi_{\nu,\ R}\right) \\
& + \bar{\psi}_{L,\ \mu}\sigma^{\rho\lambda}\gamma^\mu\lambda_{\tilde{g},\ L}F_{\rho\lambda} + \bar{\psi}_{L,\ \mu}\sigma^{\rho\lambda}\gamma^\mu\lambda_{\tilde{g},\ L}W^+_\rho W^-_\lambda \\
& + Tr\left[\bar{\lambda}_{\tilde{g},\ L}\not{A}\left(6\kappa_4^2\mu_7(2\pi\alpha')Q_B K + \frac{12\kappa_4^2\mu_7(2\pi\alpha')Q_B v^B}{\mathcal{V}}\right)\lambda_{\tilde{g},\ L}\right] \\
& + \frac{e^K G^{T_B\bar{T}_B}}{\kappa_4^2}6i\kappa_4^2(2\pi\alpha')Tr\left[Q_B A^\mu\partial_\mu\left(\kappa_4^2\mu_7(2\pi\alpha')^2 C^{I\bar{J}}a_I\bar{a}_{\bar{J}}\right)\right] \\
& + \text{h.c.}.
\end{aligned}$$

The gaugino mass obtainable from bulk F-terms comes out to be $\mathcal{V}^{\frac{2}{3}}m_{3/2}$ [1]. The smallest eigenvalue of neutralino corresponding to eigenvector

$$\tilde{\chi}_3^0 \sim -\lambda^0 + \tilde{f}\left(\tilde{H}_1^0 + \tilde{H}_2^0\right)$$

(where $\tilde{H}_{1,2}^0$ are the Higgsinos) formed by solving neutralino mass matrix for the four-Wilson-line-moduli setup similar to [5] comes out to almost same as gaugino mass [1]. The dominant decay channels (See Figs. 1 - 4) of gaugino/neutralino into gravitino include

$$\tilde{B}/\chi_3^0 \to \psi_\mu\gamma, \psi_\mu Z, \tilde{B} \to \psi_\mu u\bar{u}, \tilde{W} \to \psi_\mu u\bar{u}$$

and R-parity violating (Fig. 7)

$$\chi_3^0 \to u\bar{d}e^-$$

decay while dominant decay modes of sleptons into gravitino's (See Figs. 5 and 6) are

$$\tilde{l} \to l'\psi_\mu V \text{ and } \tilde{l}/\tilde{q} \to l/q\psi_\mu.$$

Utilizing the general expression of decay width for each different channel (See [14]), life time of $\tilde{B} \to \psi_\mu\gamma$ was shown in [1] to be extremely small for $m_{\tilde{B}} \sim m_{\tilde{g}} \sim \mathcal{V}^{\frac{2}{3}}m_{\frac{3}{2}}$ and $m_{3/2} \sim \mathcal{V}^{-2}M_p$ ($n^s = 2$), life time of $\tilde{B} \to \psi_\mu Z$ to be around $10^{-30}s$, for wino decay $\tilde{W}^0 \to \psi_\mu W^+W^-$, around $10^{-66}s$ and for three body decays $\tilde{B} \to Z\psi_\mu u\bar{u}$ and $W \to \tilde{q}\psi_\mu u\bar{u}$, it would be $10^{-13}s$ and $10^{-15}s$ respectively. In case of sleptons, using extensively the analytical expressions given

[1]As explained in [9], one of the two Pecci-Quinn/shift symmetries along the RR two-form axions c^a and the zero-form axion ρ_B gets gauged due to the dualization of the Green-Schwarz term $\int_{\mathbf{R}^{1,3}} dD_B^{(2)} \wedge A$ coming from the KK reduction of the Chern-Simons term on $\Sigma_B \cup \sigma(\Sigma_B)$ - $D_B^{(2)}$ being an RR two-form axion. In the presence of fluxes for multiple $D7$-brane fluxes, the aforementioned Green-Schwarz is expected to be modified to $Tr\left(Q_B\int_{\mathbf{R}^{1,3}} dD_B^{(2)} \wedge A\right)$, which after dualization in turn modifies the covariant derivative of T_B and hence the killing isometry.

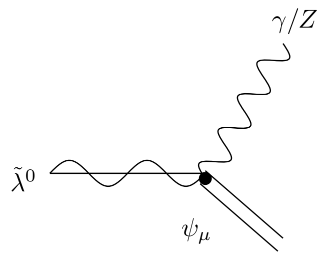

Fig.1 Two-body gaugino-decay diagramas

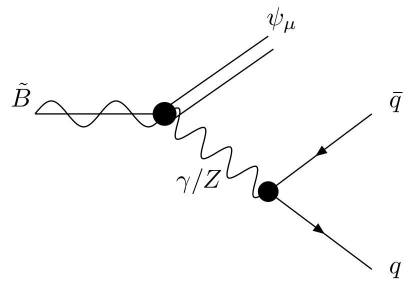

Fig.2 Three-body gaugino-decay diagram

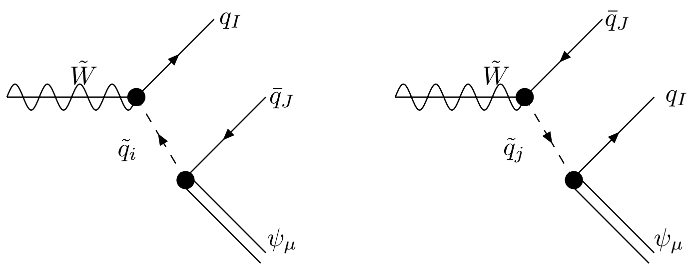

Fig.3 Three-body gaugino decays into the gravitino

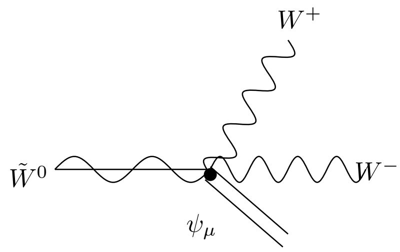

Fig. 4 Contact-vertex three-body decay diagram

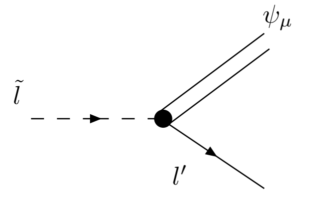

Fig. 5 Two-Body Slepton/Squark Decay

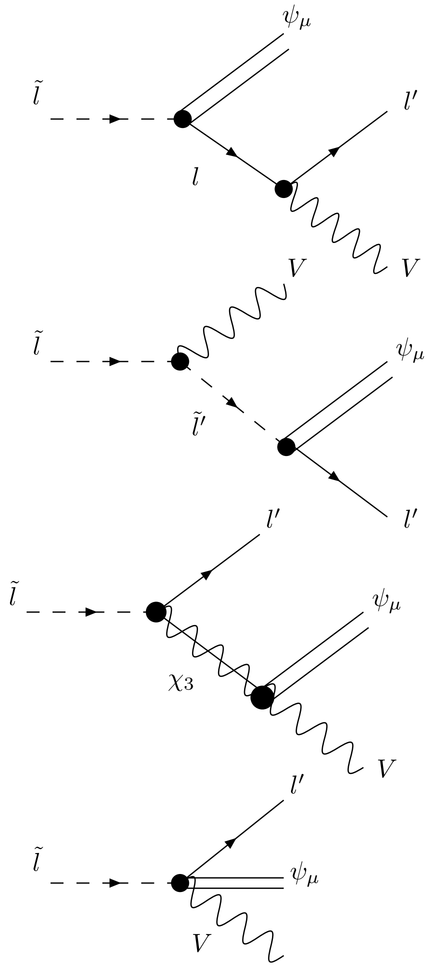

Fig. 6 Three-body slepton decays

in the references in [1], life time of $\tilde{l} \to l'\psi_\mu V$ comes out to be around 10^{-28}s and for $\tilde{l}/\tilde{q} \to l/q\psi_\mu$, around $10^{-25.5}$s.

Since life time of all aforementioned co-NLSP decay channels is smaller than 10^2 sec (onset of BBN era), one is justified to argue that NLSP decays into gravitino do not disturb the cosmological BBN constraint. Thereafter, we calculated the decay channels corresponding to R-parity violating neutralino three-body decays to ordinary particles in [1], i.e., $\chi_3^0 \to u\bar{d}e^-$ - life time comes around 10 sec - more than the lifetime of neutralino decays into the gravitino thereby ensuring that the gravitino relic abundance is not spoilt.

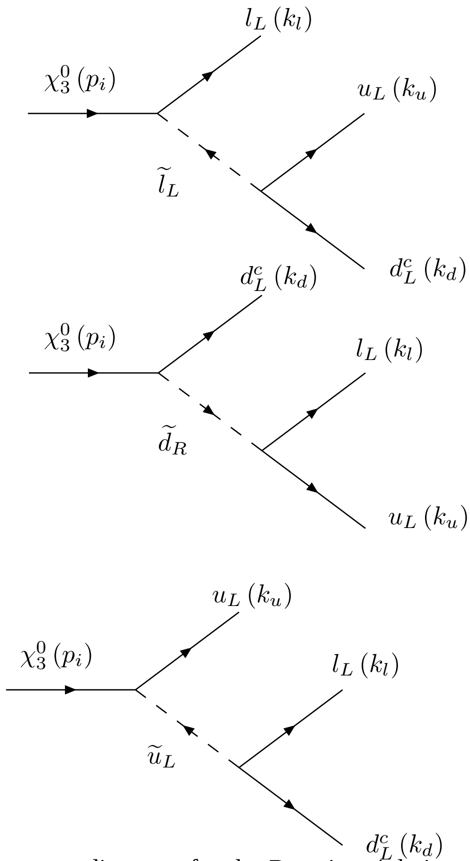

Fig. 7 Feynman diagrams for the R-parity violating decays of Neutralino χ_3^0.

The viable dark matter particle should have life time of the order or greater than the age of the universe. Unlike assuming R-parity to be conserved and hence stability of LSP, we first calculate the contribution of possible trilinear R-parity violating couplings λ_{ijk}, λ'_{ijk} and λ''_{ijk}:

$$W_{R\!\!\!/_p} = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c + \mu_i H L_i$$

in the effective $\mathcal{N} = 1$ gauged supergravity action [1]. Evaluating the same, we explicitly calculate life time for gravitino which comes out in the range $10^{20} - 10^{21}$ sec (of the order or greater than the age of the universe).

Similarly, the evaluation of two-body decays $(\psi_\mu \to \gamma\nu_e, Z\nu_e, \nu_e h)$ also give the life time respectively of the order 10^{21} sec for $\psi_\mu \to \gamma\nu_e, Z\nu_e$ and 10^{17} sec for $\psi_\mu \to \nu_e h$.

V. Relic Abundance of Gravitino

For gravitino to be an appropriate potential dark matter candidate, the contribution of gravitinos to the energy density of the universe must not exceed the closure limit, i.e.$\Omega = \rho_G/\rho_c < 1$. If the gravitino LSP produced by decay of Co-NLSP's is to account for all the gravitinos, the relic abundance of gravitino is given as

$$\Omega_{\tilde{G}} h^2 = \Omega_{\chi_3^0} h^2 \times \frac{m_{\frac{3}{2}}}{m_{\chi_3^0}} \tag{6}$$

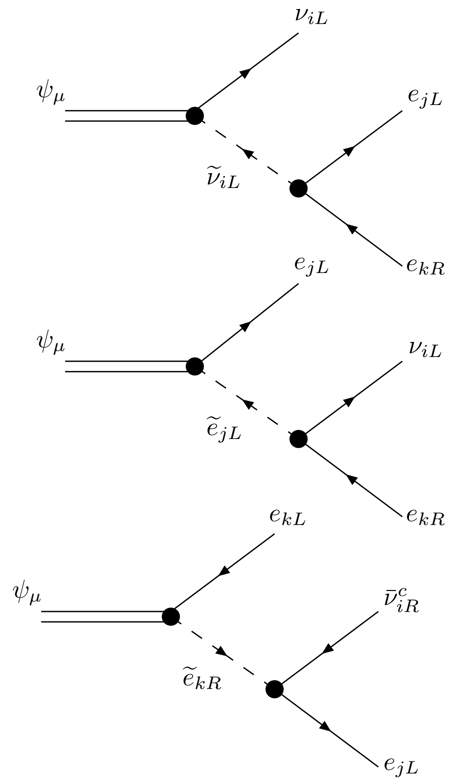

Fig. 8 Three-body gravitino decays involving $R\!\!\!/_p$ λ_{ijk} coupling

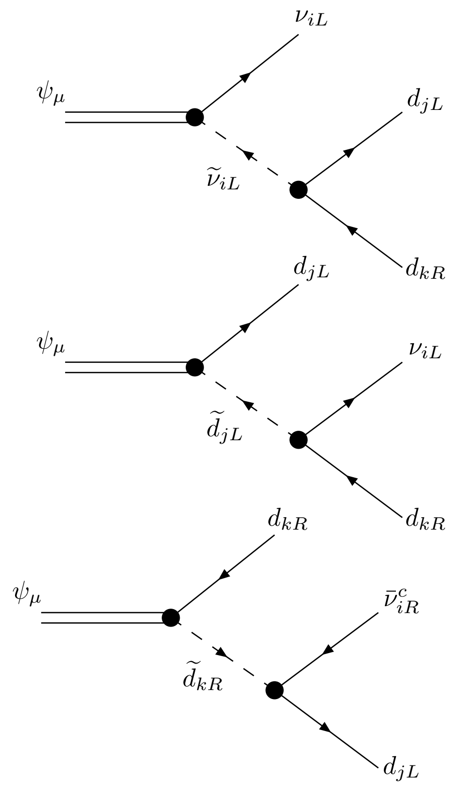

Fig. 9 Three-body gravitino decays involving $R\!\!\!/_p$ λ'_{ijk} coupling

if Co-NLSP's freeze out with appropriate thermal relic density ($\Omega_{\chi_3^0}$) before decaying and then eventually decay into the gravitino [4]. The freeze out condition depends on thermal cross-section $\sigma v_{\text{Møl}}$ of such particles which in partial wave expansion approach, is given as:

$$\langle \sigma v_{\text{Møl}} \rangle \equiv a + bx + \mathcal{O}(x^2)$$

where analytical expression of a and b are given for each annihilation channel in [15]. To evaluate these for important annihilation channels possible in our set up (Figs. 14 - 16):

$$\chi_3^0\chi_3^0 \to hh,\ \chi_3^0\chi_3^0 \to ZZ,\ \chi_3^0\chi_3^0 \to ff$$

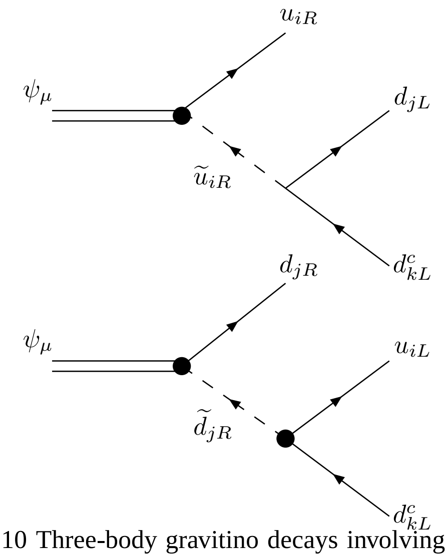

Fig. 10 Three-body gravitino decays involving $R\!\!\!/_p$ λ''_{ijk} coupling

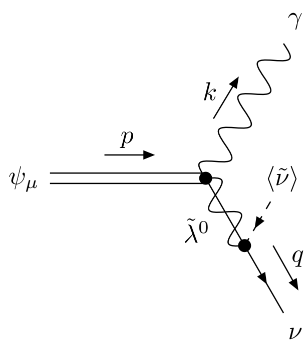

Fig. 11 Two-body gravitino decay: $\psi_\mu \to \nu + \gamma$

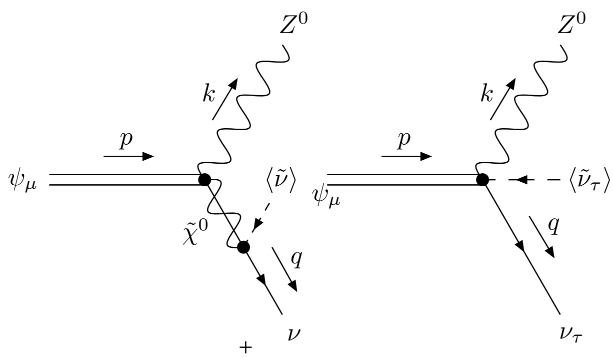

Fig. 12 Two-body gravitino decay: $\psi_\mu \to Z^0 + \nu$

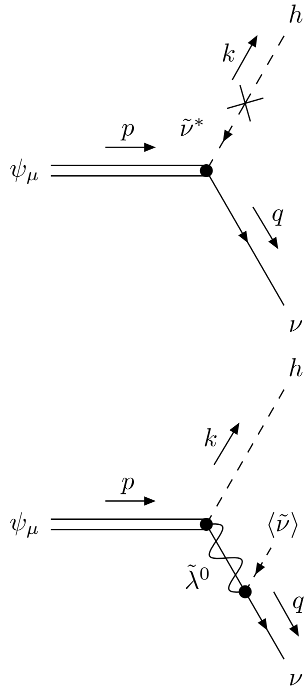

Fig. 13 Two-body gravitino decay: $\psi_\mu \to h + \nu$

in case of neutralino annihilation and (Figs. 17 - 19)

$$\widetilde{\ell}_a\widetilde{\ell}_b^* \to ZZ,\ \widetilde{\ell}_a\widetilde{\ell}_b^* \to Zh,\ \widetilde{\ell}_a\widetilde{\ell}_b^* \to hh,\ \widetilde{\ell}_a\widetilde{\ell}_b^* \to \gamma\gamma,$$
$$\widetilde{\ell}_a\widetilde{\ell}_b^* \to \gamma h,\ \widetilde{\ell}_a\widetilde{\ell}_b^* \to ll$$

in case of slepton annihilation, we had calculated in [1] the volume suppression factors corresponding to each interaction vertex making use of $\mathcal{N} = 1$ gauged supergravity action. Utilizing the same and thereafter solving for partial wave coefficient of each channel, we found in [1]:

$$a_{\chi_3^0\chi_3^0\to f_1f_2} = \widetilde{a}_{hh} + \widetilde{a}_{ZZ} + \widetilde{a}_{f\bar{f}} \equiv O(10)^{-29}GeV^{-2}$$

and

$$b_{\chi_3^0\chi_3^0\to f_1f_2} = \widetilde{b}_{hh} + \widetilde{b}_{ZZ} + \widetilde{b}_{f\bar{f}} \equiv O(10)^{-10}GeV^{-2}.$$

Similarly

$$a_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to f_1f_2} = \widetilde{a}_{ZZ}+\widetilde{a}_{hZ}+\widetilde{a}_{h\gamma}+\widetilde{a}_{\gamma\gamma}+\widetilde{a}_{hh}+\widetilde{a}_{ll} \equiv O(10)^{-9}GeV^{-2}$$

and

$$b_{\widetilde{\ell}_a\widetilde{\ell}_b^*\to f_1f_2} = \widetilde{b}_{ZZ}+\widetilde{b}_{hZ}+\widetilde{b}_{h\gamma}+\widetilde{b}_{\gamma\gamma}+\widetilde{b}_{hh}+\widetilde{b}_{ll} \equiv O(10)^{-9}GeV^{-2}.$$

Integrating $\langle\sigma v_{\mathrm{M\o l}}\rangle(x)$ in limits from 0 to x_f(value of this comes out to be around $1/33$ by solving numerically the equation

$$x_f^{-1} = \ln\left(\frac{m_\chi}{2\pi^3}\sqrt{\frac{45}{2g_*G_N}}\langle\sigma v_{\mathrm{M\o l}}\rangle(x_f)\,x_f^{1/2}\right),$$

using the analytical expression of relic abundance [16]

$$\Omega_\chi h^2 = \frac{1}{\mu^2\sqrt{g_*}J(x_F)}$$

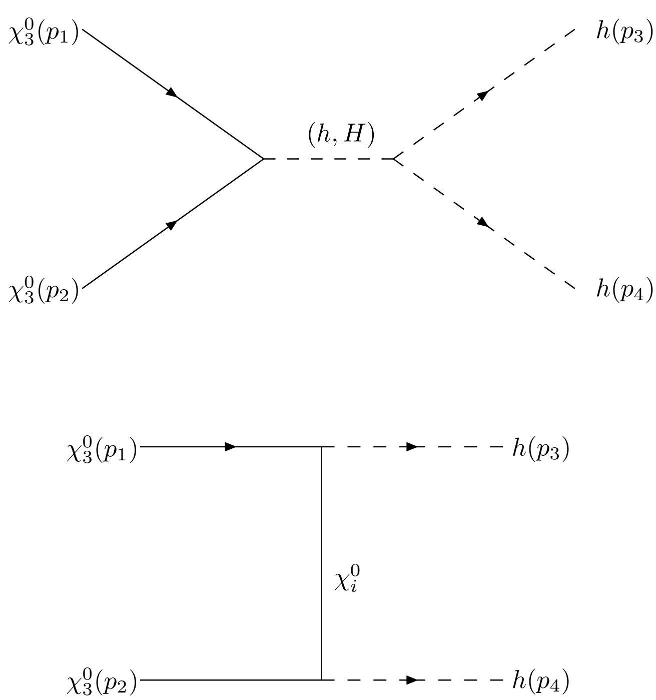

Fig. 14 Feynman diagrams for $\chi_3^0\chi_3^0 \to hh$ via s–channel Higgs exchange and t-channel χ_i^0 exchange.

where $\mu = 1.2 \times 10^5 GeV$, $\sqrt{g_*} = 9$ and

$$J(x_f) \sim a(x_f) + b\frac{{x_f}^2}{2} \sim 10^{-10}\frac{{x_f}^2}{2}GeV^{-2}$$

in case of neutralino annihilation and $10^{-9}x_f GeV^{-2}$ in case of sleptons, relic abundance of gravitino $\Omega_{\tilde{G}}h^2$ comes out to be 0.16 by considering neutralino to be NLSP and 0.001 by considering sleptons to be NLSP.

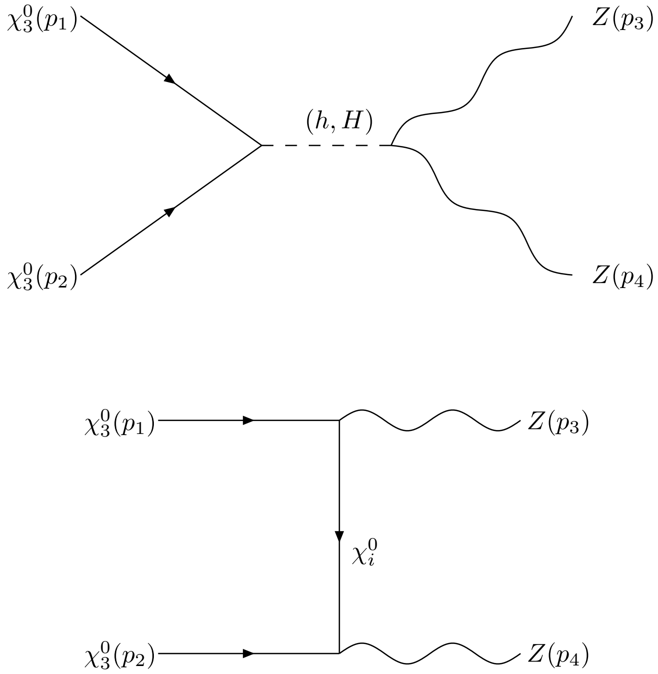

Fig. 15 Feynman diagrams for $\chi_3^0\chi_3^0 \to ZZ$ via s–channel Higgs exchange and t-channel χ_i^0 exchange.

VI. Conclusion

To summarize, in the framework of L(arge) V(olume) "$D3/D7\mu$- split SUSY" scenario including four Wilson line moduli on the world volume of space-time filling $D7$-branes wrapped around the "big divisor" and two position moduli of a mobile space-time filling $D3$-brane restricted to (nearly) a special Lagrangian sub-manifold, we show that fermionic superpartners of $\mathcal{A}_1$ and $\mathcal{A}_3$ get identified, respectively with e_L and e_R, and the fermionic superpartners of $\mathcal{A}_2$ and $\mathcal{A}_4$ get identified, respectively with the first generation quarks: u/d_L and u/d_R The scenario is very appealing on the cosmology side; explicit life times calculation of co-N(LSP) candidates' possible decay channels verify that decay of NLSP into gravitino do not disturb primordial abundance i.e BBN and life time of gravitino comes out to be around the order or greater than the age of the universe and hence satisfies the requirement of an appropriate dark matter candidate in the context of $\mathcal{N} = 1$ gauged supergravity. The numerical estimates of various N(LSP) decay lifetimes are provided in table I. Next, inspired from non-thermal production mechanism of gravitino, the calculated value of relic abundance equal to 0.16 from neutralino annihilation, is almost in agreement with the value suggested by WMAP 7-year CMB anisotropy observation [17].

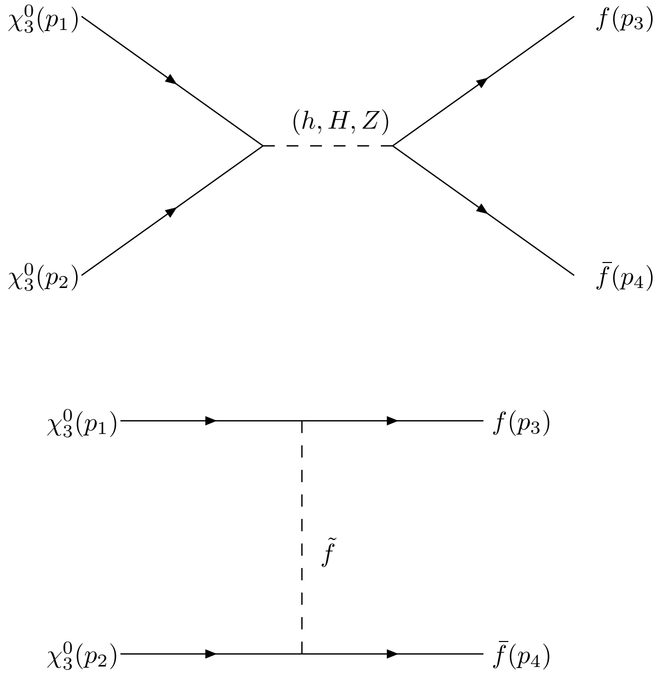

Fig. 16 Feynman diagrams for $\chi_3^0\chi_3^0 \to f\bar{f}$ via s–channel Higgs/Z exchange and t-channel $\tilde{f}$ exchange.

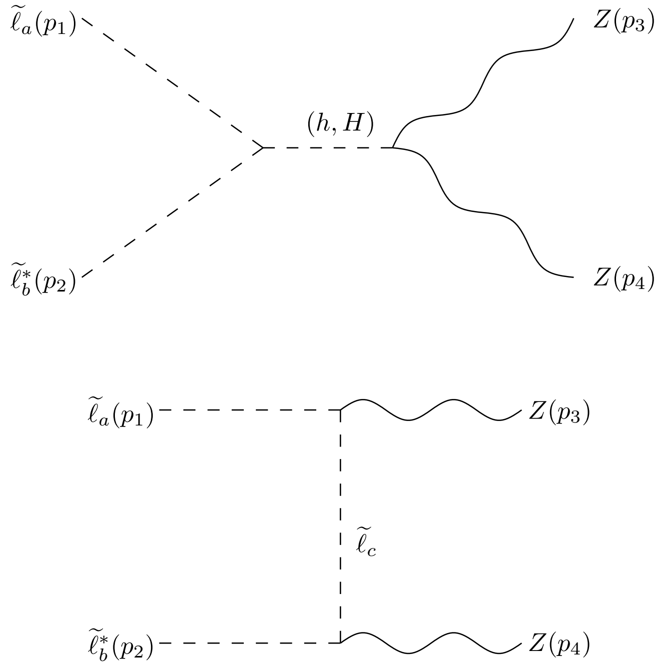

Fig. 17 Feynman diagrams for $\widetilde{\ell}_a\widetilde{\ell}_b^* \to ZZ$ via s–channel Higgs exchange and t-channel $\widetilde{\ell}_c$ exchange.

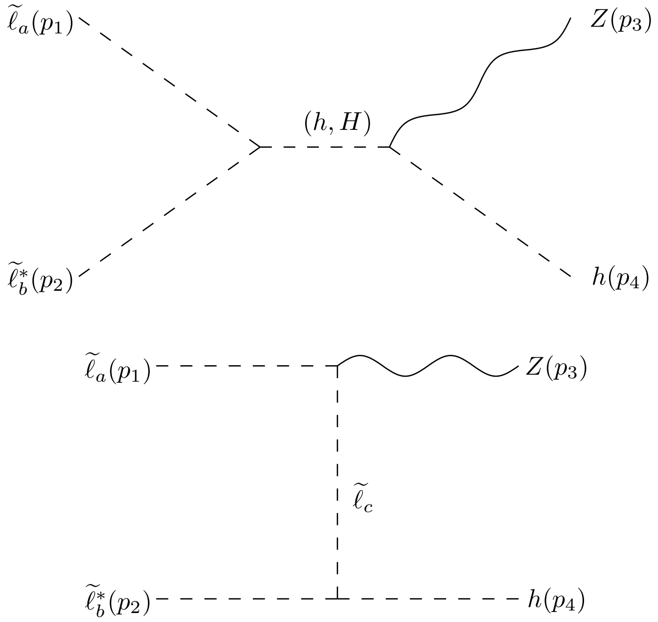

Fig. 18 Feynman diagrams for $\widetilde{\ell}_a\widetilde{\ell}^*_b \to Zh$ via s–channel Higgs exchange and t-channel $\widetilde{\ell}_c$ exchange.

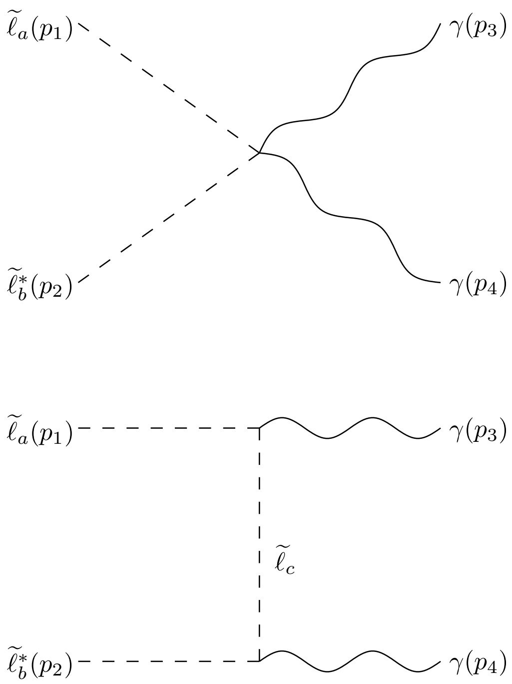

Fig. 19 Feynman diagrams for $\widetilde{\ell}_a\widetilde{\ell}^*_b \to \gamma\gamma$ via point interaction and t-channel $\widetilde{\ell}^*_c$ exchange.

TABLE I
LIFE TIME OF VARIOUS N(LSP) DECAY CHANNELS

Particle decay	Decay Modes	Life Time	Remarks
Neutralino/Gaugino decays	$\tilde{B} \to \psi_\mu Z/\gamma$ $\tilde{W} \to q\psi_\mu u\bar{u}$ $\tilde{B} \to Z\psi_\mu u\bar{u}$	$10^{-30}s$ $10^{-15}s$ $10^{-13}s$	Respect BBN constraint
Slepton decays	$\tilde{l} \to l'\psi_\mu V$ $\tilde{l}/\tilde{q} \to l/q\psi_\mu$	$10^{-28}s$ $10^{-25.5}s$	”
RPV Neutralino decay	$\chi_3^0 \to u\bar{d}e^-$	$10^{1}s$	does not affect gravitino abundance
Gravitino decays	$\psi_\mu \to \nu\gamma, \nu Z$ $\psi_\mu \to h\nu_e$ $\psi_\mu \to l_i l_j e^c_k$ $\psi_\mu \to l_i q_j d^c_k$ $\psi_\mu \to u^c_i d^c_j d^c_k$	$10^{21}s$ $10^{17}s$ $10^{21}s$ $10^{20}s$ $10^{18}s$	Life time greater than age of Universe
Gluino decays	$\tilde{g} \to \chi^o_{\rm n} q_I \bar{q}_J$ $\tilde{g} \to \tilde{\chi}^0_3 g$ $\tilde{g} \to \psi_\mu q_I \bar{q}_J$ $\tilde{g} \to \psi_\mu g$	$10^{-5}s$ $10^{2}s$ $10^{4}s$ $10^{11}s$	stable (from collider point of view)

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