Gravitino Dark Matter in (nearly) SLagy $D_3/D_7$ $\mu$-Split SUSY

Mansi Dhuria and Aalok Misra

Abstract—In the context of large volume Big Divisor (nearly) SLagy $D_3/D_7$ $\mu$-Split SUSY [1], after an explicit identification of first generation of SM leptons and quarks with fermionic superpartners of four Wilson line moduli, we discuss the identification of gravitino as a potential dark matter candidate by explicitly calculating the decay life times of gravitino (LSP) to be greater than age of universe and lifetimes of decays of the co-NLSPs (the first generation squark/slepton and a neutralino) to the LSP (the gravitino) to be very small to respect BBN constraints. Interested in non-thermal production mechanism of gravitino, we evaluate the relic abundance of gravitino LSP in terms of that of the co-NLSP’s by evaluating their (co-)annihilation cross sections and hence show that the former satisfies the requirement for a potential Dark Matter candidate. We also show that it is possible to obtain a $125 \text{ GeV}$ light Higgs in our setup.

Keywords—Split Supersymmetry, Large Volume Swiss-Cheese Calabi-Yau’s, Dark Matter, (N)LSP decays, relic abundance.

I. INTRODUCTION

OVER the past few years, producing realistic model satisfying both cosmological as well as phenomenological requirements from string compactifications has proven to be a daunting challenge. To get the phenomenological implications of B(eyond) S(tandard) M(odel)s, they must be invoked with daunting challenge. To get the phenomenological implications of B(eyond) S(tandard) M(odel)s, they must be invoked with

II. THE MODEL

Let us first briefly describe our large volume $D_3/D_7$ Swiss-Cheese setup of [1]. Type IIB compactified on the orientifold of a Swiss-Cheese Calabi-Yau in the L(arge) V(olume) S(cenarios) limit that includes non-(perturbative) $\alpha'$ corrections and non-perturbative instanton-corrections in superpotential [8]. For studying phenomenological issues, we after inflation by inelastic $2 \rightarrow 2$ scattering and $1 \rightarrow 2$-decay processes of particles from the thermal bath where abundance varies linearly with the reheating temperature $T_R$. However, in string/M theory-inspired models, this ‘standard’ way of production of Dark Matter (DM) particles is significantly altered because of the presence of moduli as decay of moduli increases the entropy which sufficiently decreases the relic abundance of gravitino. Therefore, sizable amount of gravitinos can be produced by (non-thermal) production of gravitinos (LSP) formed by decays of moduli and can dominate the thermal production of gravitinos in the early plasma, discussed in [3]. Even in particle physics models, sufficient amount of relic density of LSP can be produced by decays of N(ext-to) L(lightest) S(upersymmetric) P(article) and has been studied in literature [4]. Motivated by this approach, we had studied the abundance of gravitino produced from decay of right set of moduli (corresponding to Co-NLSP’s) in type IIB large volume $\mu$ split SUSY set up in [1].

In this paper, we summarize our study in [1] of decay width of gravitino appearing as the L(lightest) S(upersymmetric) P(article) and sleptons/squarks as N(ext-to) L(lightest) S(upersymmetric) P(article)s with (Bino/Wino-type)gaugino-dominant neutralino, emphasizing their impact on the issue of dark matter in the context of large volume $\mu$ split SUSY set up in an improvement of [5]. In addition to getting a light Higgs of $125\text{ GeV}$ and a long life time of gluino, we show that the presence of non-zero R-parity violating couplings and high squark masses help to reduce the decay width and hence lifetime of the gravitino (LSP) becomes very large (of the order or greater than age of the Universe) satisfying the requirement of potential dark matter candidate whereas life times of co-NLSP’s are small enough not to dsiturb the beautiful predictions of BBN. Relying on the non thermal production occurring through decays of sleptons/neutralino existing as co-NLSP’s in our set up, we evaluate the relic abundance of gravitino, the right amount of which helps to resolve cosmological gravitino problem while very heavy moduli masses($>> 10\text{ TeV}$) in the set-up automatically resolves the cosmological moduli problem.

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included a single mobile spacetime filling D3-brane and stacks of D7-branes wrapping the “big” divisor. Interested in
generating the possibility of distinct non-abelian gauge groups by
turning on different fluxes on the world volume of “big”
divisor, therefore requires to construct four involutively odd
harmonic distribution one-forms $A_I$ localized on the sub-localus
of $\Sigma_B$. The most non-trivial example of involutions which are
meaningful only at large volumes is mirror symmetry
implemented as three T-dualities in [6]. Interestingly, we found
in [1] that the toroidal three-cycle supporting harmonic one-
forms

$$C_3: |z_1| \sim V^{\frac{3}{2}}, \quad |z_2| \sim V^{\frac{3}{2}}, \quad |z_3| \sim V^{\frac{3}{2}}$$

(the Calabi-Yau can be thought as a $T^4$ swept out by $(argz_1, argz_2, argz_3)$-fibration over a large base
$(|z_1|, |z_2|, |z_3|)$); precisely apt for application of mirror
symmetry as three T-dualities a la Strominger Yau Zaslow
[6]) is almost a (special) Lagrangian sub-manifold because it
satisfies the requirement that $f^*J \approx 0$, $f^*\Omega \equiv e^{i\Phi}vol(C_3)$, where $f: C_3 \rightarrow CY_3$ [7]. Therefore, as discussed in [8],
the harmonic distribution one-forms can be constructed by:

$$dA_I = \left(P\Sigma_B(z_1, z_2, z_3)^\frac{1}{3}\right) dz_1 \wedge dz_2$$

In the case of harmonic distributions on $\Sigma_B$ localized on the
$D3$-brane can be written as:

$$A_I \sim \delta \left(\left|z_3\right| - \sqrt{\frac{2}{3}}\right) \delta \left(\left|z_1\right| - \sqrt{\frac{2}{3}}\right) \delta \left(\left|z_2\right| - \sqrt{\frac{2}{3}}\right) \times \omega(z_1, 2, 2),$$

Writing $A_I(z_1, z_2) = \omega_I(z_1, z_2)dz_1 + \omega_I(z_1, z_2)dz_2$ where

$$\omega(-z_1, z_2) = \omega(z_1, z_2), \quad \omega(-z_1, z_2) = -\omega(z_1, z_2)$$

and $\partial_1 \omega = -\partial_2 \omega$, one obtains:

$$A_1(z_1, z_2, z_3) \sim V^{\frac{3}{2}} \sim -z_1^{\frac{18}{3}}z_2^{\frac{19}{3}}z_3^{\frac{19}{3}},$$

$$A_2(z_1, z_2, z_3) \sim V^{\frac{3}{2}} \sim -z_1^{\frac{18}{3}}z_2^{\frac{37}{3}}z_3^{\frac{37}{3}},$$

$$A_3(z_1, z_2, z_3) \sim V^{\frac{3}{2}} \sim -z_1^{\frac{18}{3}}z_2^{\frac{37}{3}}z_3^{\frac{37}{3}},$$

$$A_4(z_1, z_2, z_3) \sim V^{\frac{3}{2}} \sim -z_1^{\frac{18}{3}}z_2^{\frac{37}{3}}z_3^{\frac{37}{3}}.$$

The $\mathcal{N} = 1$ chiral co-ordinates with the inclusion of mobile
$D3$-brane position moduli $z_1, z_2$ and D7-branes Wilson
moduli $\phi_I$ will be appropriate generalizations of $[9]
for multiple $D7$-branes (See [1]). The quadratic contribution
arising due to Wilson line moduli contribution is of the form:

$$i\kappa_3^2 \mu \Sigma B_{\nu\mu} \omega^{3\nu},$$

where $\omega \in H_{\mathcal{N} = 1}^3 (\Sigma_B).$ We can calculate the intersection matrices $C_{ij}^n$ by constructing harmonic one forms using equation (2). Also, coefficient of quadratic term

$$\omega_{\nu} \omega_{\nu} \omega^{3\nu} \sim \frac{18}{\sqrt{\lambda}}$$

arising in $T\mathcal{B}$ due to inclusion of position moduli $z_i$ can be
shown to be $O(1)$ by calculating $\langle \omega \rangle \sim \langle \omega \rangle \sim O(1)$
near $z_{1,2} \sim \frac{\sqrt{\lambda}}{\sqrt{2}}$ (See [1]). Therefore one can argue ([8], [1])
that near

$$|z_1| \sim V^{\frac{3}{2}}, |z_2| \sim V^{\frac{3}{2}}, |z_3| \sim V^{\frac{3}{2}},$$

$$|\alpha_1| \sim V^{\frac{3}{2}}, |\alpha_2| \sim V^{\frac{3}{2}}, |\alpha_3| \sim V^{\frac{3}{2}},$$

$$|\alpha_2| \sim V^{\frac{3}{2}}, |\alpha_3| \sim V^{\frac{3}{2}}, |\alpha_4| \sim V^{\frac{3}{2}},$$

one obtains a local meta-stable dS-like minimum corresponding
to the positive minimum of the potential $e^{K(T - \Omega\Sigma B) |D_T|W|^{\frac{2}{3}}}$ stabilizing

$$vol(\Sigma_B) \sim Re(\sigma_B) \sim V^{\frac{3}{2}},$$

and in the dilute flux approximation, gauge couplings corres-
tponding to the gauge theories living on stacks of $D7$-
branes wrapping the “big” divisor $\Sigma_B$ will given by:

$$Re(T_B) \sim V^{\frac{3}{2}} \sim O(1)$$

The Kähler potential relevant to all the calculations in this
paper (without being careful about $O(1)$ constant factors) is
given as under:

$$K \sim -2ln \left(\left|T_B + T_H - \mu_3^2 \right| \right)$$

The evaluation of “physical” normalized Yukawa couplings,
soft SUSY breaking parameters and various 3-point vertices
needs the matrix generated from the mixed double derivative
of the Kähler potential to be a diagonalized matrix. After
diagonalization the corresponding eigenvectors of the same
are given by:

$$\omega_1 \sim \alpha_4 + V^{-\frac{3}{2}}, \alpha_4 - V^{-\frac{3}{2}} \alpha_1 + V^{-\frac{3}{2}} \alpha_3 + V^{-\frac{3}{2}} \alpha_2 + V^{-\frac{3}{2}} \alpha_1$$

$$\omega_2 \sim -\alpha_3 + V^{-\frac{3}{2}} \alpha_4 + V^{-\frac{3}{2}} \alpha_1 - V^{-\frac{3}{2}} \alpha_3 + V^{-\frac{3}{2}} \alpha_2 + V^{-\frac{3}{2}} \alpha_1$$

$$\omega_3 \sim -\alpha_2 + V^{-\frac{3}{2}} \alpha_4 + V^{-\frac{3}{2}} \alpha_1 - V^{-\frac{3}{2}} \alpha_3 + V^{-\frac{3}{2}} \alpha_2 + V^{-\frac{3}{2}} \alpha_1$$

$$\omega_4 \sim -\alpha_1 + V^{-\frac{3}{2}} \alpha_4 + V^{-\frac{3}{2}} \alpha_1 - V^{-\frac{3}{2}} \alpha_3 + V^{-\frac{3}{2}} \alpha_2 + V^{-\frac{3}{2}} \alpha_1$$

and the effective Yukawa couplings can be calculated using

$$G_{C_1,C_2,C_3} \equiv \frac{1}{\sqrt{K_{C_1,C_2,C_3}}} C_1$$

where $C_1$ being an open string modulus which for us is $\delta Z_{1,2,3}$. $A_1$ being given by $O(\Sigma z_i)$-coefficient in the mass term $e^{K} D_{\phi} D_{\phi} W^{\frac{1}{2}} = \delta Z_{1,2,3}$ in the $N = 1$ supergravity.

By estimating in the large volume limit, all possible Yukawa
couplings corresponding to four Wilson line moduli showing
that the RG-flow of the effective physical Yukawa’s change
almost by $O(1)$ under an RG flow from the string scale down
to the EW scale [1], we see that for $V \approx 10^5, \langle z_i \rangle \approx 246 GeV$.
\( \mathcal{O}(Z_i) \) term in \( \epsilon^{\frac{2}{3}} \bar{D}_A \bar{A}_i W \sum_{\bar{Z}_i \bar{Z}_A \bar{A}_i} \equiv \tilde{Y}_{\tilde{Z}_i \bar{A}_i} \simeq 10^{-3} \times V^{\frac{1}{4}}, \)
\( \) giving \( \langle \tilde{Z}_i \rangle \tilde{Y}_{\tilde{Z}_i \bar{A}_i} \simeq M eV \) - about the mass of the electron! \( \)
\( \mathcal{O}(Z_i) \) term in \( \epsilon^{\frac{2}{3}} \bar{D}_A \bar{A}_i W \sum_{\bar{Z}_i \bar{Z}_A \bar{A}_i} \equiv \tilde{Y}_{\tilde{Z}_i \bar{A}_i} \simeq 10^{-7} \times V^{\frac{1}{4}}, \)
\( \) giving \( \langle \tilde{Z}_i \rangle \tilde{Y}_{\tilde{Z}_i \bar{A}_i} \simeq 10^8 eV \) - close to the mass of the up quark! The above shows that fermionic superpartners of \( A_1 \) and \( A_3 \) correspond respectively to first generation of left-handed \( SU(2) \) and right-handed \( U(1) \) leptons while fermionic superpartners of \( A_2 \) and \( A_4 \) correspond respectively to left-handed \( SU(2) \) and right-handed \( U(1) \) quarks.

### III. Mass Scales

The gravitino mass

\[ m_{3/2} = \epsilon^{\frac{2}{3}} W M_p \sim V^{\frac{1}{4}} \]

in the context of gravity mediation and for \( n^* = 2 \) and \( V = 10^5 \), turns out to be around \( 10^8 \) GeV. Hereafter, using diagonal metric matrix and calculating \( F \)-terms corresponding to bulk moduli \( T_{B,S,G} \), soft SUSY breaking parameter \( \epsilon \) position moduli mass (to be identified with Higgs) \( Z_i \) come out to be (see [1])

\[ m_{Z_i} \sim V^{\frac{1}{2}} m_{3/2} \]

and Wilson line moduli masses(to be identified with scalar masses), come out to be very heavy :

\[ m_{A_i} \sim \sqrt{V} m_{3/2} \]

for all Wilson line moduli at string scale and changes only by \( \mathcal{O}(1) \) under the RG solution down to EW scale as shown in [11], giving one of the signatures of \( \mu \)-split SUSY. In addition to these, SUSY breaking trilinear couplings \( A_{Z_i \gamma} \) and supersymmetric Higgsino mass parameter \( \hat{\mu}_{Z_i} \) also turns out to be very large of the order

\[ \hat{\mu}_{Z_i} \sim V^{\frac{1}{2}} m_{3/2} \]

Now, to realize split SUSY as in [2], we calculated in [1], decay life time of gluino comes out to be high (in the range \( 10^{-5} s (\tilde{g} \rightarrow q \bar{q} \chi_1^0), 10^2 s (\tilde{g} \rightarrow g g \chi_1^0), 10^4 s (\tilde{g} \rightarrow q \bar{q} \chi_1^0), 10^{11} s (\tilde{g} \rightarrow g g \psi \mu) \) see), thus showing the stability of Gluino and hence giving another concrete evidence of \( \mu \) split SUSY in our set up.

### IV. Life Time of N(LSP) Decay Channels

The very important constraint that the hadronic/electromagnetic energy released from decay products of next-to-lightest supersymmetric particle (NLSP) must not alter the observed abundance of light elements in the universe essentially fixed by average lifetime around \( \tau \sim 10^{20} sec \) referred to as the B(bi)g(B(ang)) N(ucleosynthesis) constraint, is satisfied by NLSP candidates if decay of same occurs before BBN era [12]. In addition to this, taking R-parity violating couplings into account, the (lightest) neutralino might decay into leptons/quarks rather than gravitino and hence elude the relic abundance of gravitino coming from decay of neutralino (Co-NLSP) if life time for the former decay is less than the latter; via explicit calculations, we ensure that this does not happen. For the same one needs to calculate the decay widths of all important 2- and 3-body decay channels for which we will be using the following terms (written out in four-component notation or their two-component analogs and utilizing/generalizing results of [9]) in the \( N = 1 \) gauged supergravity action of Wess and Bagger [13] with the understanding that \( n_{moduli/modulini} << m_{KK} \sim \frac{M_{KK}}{\sqrt{\epsilon}} \sim 10^{14} \text{GeV}, M_s = \frac{M_{KK}}{\sqrt{\epsilon}} \sim 10^{15} \text{GeV}, \) and that for multiple \( D7 \)-branes, the non-abelian considering a small fine tuning i.e \( 0.03 + \delta_1 m_0^2 \sim -0.06 S_0 \) (\( S_0 \) is hypercharge weighted sum of squared soft scalar mass having value around \( m_0^2 \)) and

\[ \hat{\xi} \sim 2 + \frac{1}{8} \frac{m_{3/2}^2}{m_0^2}, \]

we obtain one light Higgs (corresponding to the negative sign of the square root) of order \( 125 \) GeV and one heavy Higgs (corresponding to the positive sign of the square root) whereas the squared Higgsino mass parameter \( \hat{\mu}_{Z_i} \) then turns out to be heavy with a value, at the EW scale of around \( \sqrt{V} m_{3/2} \), thus showing the possibility of realizing \( \mu \) split SUSY scenario in the context of LVS \( D3 \sim D7 \) set up.

In fact, in addition to being able to generate the mass-scales relevant to (in this paper, first generation) quarks/leptons, using the RG-flow arguments of [10], one can also show that the Weinberg-type dimension-five Majorana-mass generating operator: \( \mathcal{O}(\bar{Z}_i^2) \) coefficient in \( \frac{\sqrt{2} \mathcal{D}_i \bar{A}_i W}{\sum_{\bar{Z}_i \bar{A}_i} \equiv \tilde{Y}_{\bar{Z}_i \bar{A}_i} \simeq 10^{-2} \times V^{\frac{1}{4}}, \)
\( \) in fact \( \epsilon^{\frac{2}{3}} \bar{D}_A \bar{A}_i W \sum_{\bar{Z}_i \bar{Z}_A \bar{A}_i} \equiv \tilde{Y}_{\bar{Z}_i \bar{A}_i} \simeq 10^{-7} \times V^{\frac{1}{4}} \)

produces the correct first-generation neutrino mass scale of slightly less than \( 1eV \) for \( \langle \hat{\xi} \rangle \sim \mathcal{O}(1) \) results.
gauged isometry group, corresponding to the killing vector $6i\kappa_2^{\mu T}(2\pi\alpha')Q_B\partial_{\mu T}Q_B = (2\pi\alpha')\int_{\Sigma_B} i^*\omega_B\wedge F_3$ arising due to the elimination of the two-form axions $D_B^{(3)}$ in favor of the zero-form axions $\rho_B$ under the KK-reduction of the ten-dimensional four-form axiom [9] (which results in a modification of the covariant derivative of $T_B$ by an additive shift given by $6i\kappa_2^{\mu T}(2\pi\alpha')Tr(Q_BA_\mu)$) can be identified with the SM group (i.e. $A_\mu$ is the SM-like adjoint-valued gauge field [13]):

\[
\mathcal{L} = g_{YM}g_{T_B}Tr\left(X^{T_B}\chi_\lambda^T\chi_\lambda - J_R\right) +g_{T_B}Tr\left(\chi^T_{\lambda L} + \Gamma^i_{MJ}\phi^M_{\lambda L} + \frac{1}{4} (\partial_\alpha M K\partial_{\alpha M} - c.c.) \chi_\lambda^T\right)
\]

\[
+g_{T_B}Tr\left(\partial_\alpha M K\partial_{\alpha M} - c.c.) \chi_\lambda^T\right)
\]

\[
+g_{T_B}Tr\left(\partial_\alpha M K\partial_{\alpha M} - c.c.) \chi_\lambda^T\right)
\]

\[
+g_{T_B}Tr\left(\partial_\alpha M K\partial_{\alpha M} - c.c.) \chi_\lambda^T\right)
\]

\[
+Tr\left(\partial_\alpha M K\partial_{\alpha M} - c.c.) \chi_\lambda^T\right)
\]

\[
+h.c.
\]

The gaugino mass obtainable from bulk $F$-terms comes out to be $\mathcal{V}^2m_{3/2}$ [1]. The smallest eigenvalue of neutralino corresponding to eigenvector

\[
\tilde{\chi}_0^0 \sim -\lambda^0 + \tilde{f} \left(\tilde{H}_1^0 + \tilde{H}_2^0\right)
\]

(\begin{matrix} \tilde{\chi}_0^0 \\ \tilde{\chi}_1^1 \\ \tilde{\chi}_2^2 \\ \tilde{\chi}_3^3 \\ \tilde{\chi}_4^4 \\ \tilde{\chi}_5^5 \\ \tilde{\chi}_6^6 \\ \tilde{\chi}_7^7 
\end{matrix})

where $\tilde{H}_1^0,\tilde{H}_2^0$ are the Higgsinos formed by solving neutralino mass matrix for the four-Wilson-line-moduli setup similar to [5] comes out to almost same as gaugino mass [1]. The dominant decay channels (See Figs. 1 - 4) of gaugino/neutralino into gravitino include

\[
\tilde{B}/\tilde{\chi}_0^0 \rightarrow \tilde{\psi}_1^0 \gamma, \tilde{\psi}_2^0 \gamma, \tilde{\chi}_3^1 \rightarrow \tilde{\psi}_3^1 \text{ and } \tilde{\psi}_4^1 \gamma
\]

and R-parity violating (Fig. 7)

\[
\tilde{\chi}_0^0 \rightarrow \tilde{d}\tilde{e}^{-}
\]

decay while dominant decay modes of sleptons into gravitino’s (See Figs. 5 and 6) are

\[
\tilde{\ell} \rightarrow \ell'\tilde{\psi}_1^0 V, \text{ and } \tilde{\ell}/\tilde{q} \rightarrow \ell/q\tilde{\psi}_1^0.
\]

Utilizing the general expression of decay width for each different channel (See [14]), life time of $\tilde{B} \rightarrow \tilde{\psi}_1^0 \gamma$ was shown in [1] to be extremely small for $m_B \sim m_Z \sim \mathcal{V}^2m_{3/2}$ and $m_{3/2} \sim \mathcal{V}^{-2}m_P$ (n = 2), life time of $\tilde{B} \rightarrow \tilde{\psi}_2^0 \gamma$ to be around $10^{-30}s$, for wino decay $W^0 \rightarrow \tilde{\psi}_1^0 W^+ W^-$, around $10^{-8} s$ and for three body decays $\tilde{B} \rightarrow Z\tilde{\psi}_1^0 u\tilde{u}$ and $W \rightarrow \tilde{\psi}_1^0 u\tilde{u}$, it would be $10^{-13}s$ and $10^{-12}s$ respectively. In case of sleptons, using extensively the analytical expressions given

\[
\text{As explained in [9], one of the two Pecci-Quinn/shift symmetries along the RR two-form axions c}^0\text{ and the zero-form axion } \rho_B \text{ gets gauged due to the dualization of the Green-Schwarz term } \int_{\mathbb{R}^{1,3}} dD_B^{(3)} \wedge A \text{ coming from the KK reduction of the Chern-Simons term on } \Sigma_B \cup \sigma(\Sigma_B) - D_B^{(3)}\text{ being an RR two-form axion. In the presence of fluxes for multiple } D_B\text{-branes, the aforementioned Green-Schwarz is expected to be modified to } Tr\left(Q_B \int_{\mathbb{R}^{1,3}} dD_B^{(3)} \wedge A \right), \text{ which after dualization in turn modifies the covariant derivative of } T_B \text{ and hence the killing isometry.}
\]
in the references in [1], life time of $\tilde{t} \to l'\psi_\mu V$ comes out to be around $10^{-28}$s and for $l/q \to l/q\psi_\mu$, around $10^{-25.5}$s.

Since life time of all aforementioned co-NLSP decay channels is smaller than $10^2$ sec (onset of BBN era), one is justified to argue that NLSP decays into gravitino do not disturb the cosmological BBN constraint. Thereafter, we calculated the decay channels corresponding to R-parity violating neutralino three-body decays to ordinary particles in [1], i.e., $\chi_3^0 \to ud\nu_e$ - life time comes around 10 sec - more than the lifetime of neutralino decays into the gravitino thereby ensuring that the gravitino relic abundance is not spoilt.

Fig. 6 Three-body slepton decays

The viable dark matter particle should have life time of the order or greater than the age of the universe. Unlike assuming R-parity to be conserved and hence stability of LSP, we first calculate the contribution of possible trilinear R-parity violating couplings $\lambda_{ijk}$, $\lambda'_{ijk}$ and $\lambda''_{ijk}$:

$W_R = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c + \mu_i H L_i$

in the effective $\mathcal{N} = 1$ gauged supergravity action [1]. Evaluating the same, we explicitly calculate life time for gravitino which comes out in the range $10^{20} - 10^{21}$ sec (of the order or greater than the age of the universe).

Similarly, the evaluation of two-body decays ($\psi_\mu \to \gamma\nu_e, Z\nu_e, \nu_e h$) also give the life time respectively of the order $10^{21}$ sec for $\psi_\mu \to \gamma\nu_e, Z\nu_e$ and $10^{17}$ sec for $\psi_\mu \to \nu_e h$.

V. RELIC ABUNDANCE OF GRAVITINO

For gravitino to be an appropriate potential dark matter candidate, the contribution of gravitinos to the energy density of the universe must not exceed the closure limit, i.e., $\Omega = \rho_G/\rho_c < 1$. If the gravitino LSP produced by decay of Co-NLSP’s is to account for all the gravitinos, the relic abundance of gravitino is given as

$$\Omega_G h^2 = \Omega h^2 \times \frac{m_\chi}{m_G}$$  \hspace{1cm} (6)
if Co-NLSP’s freeze out with appropriate thermal relic density ($\Omega_0 h^2$) before decaying and then eventually decay into the gravitino [4]. The freeze out condition depends on thermal cross-section $\sigma v_{\text{Møl}}$ of such particles which in partial wave expansion approach, is given as:

$$\langle \sigma v_{\text{Møl}} \rangle \equiv a + bx + O(x^2)$$

where analytical expression of $a$ and $b$ are given for each annihilation channel in [15]. To evaluate these for important annihilation channels possible in our set up (Figs. 14 - 16):

$$\chi^0_3 \chi^0_3 \rightarrow hh, \chi^0_3 \chi^0_3 \rightarrow ZZ, \chi^0_3 \chi^0_3 \rightarrow ff$$
in case of neutralino annihilation and (Figs. 17 - 19)
\[ \tilde{\ell}_a \tilde{\ell}_b \rightarrow ZZ, \tilde{\ell}_a \tilde{\ell}_b \rightarrow Zh, \tilde{\ell}_a \tilde{\ell}_b \rightarrow \gamma \gamma, \]
\[ \tilde{\ell}_a \tilde{\ell}_b \rightarrow \gamma h, \tilde{\ell}_a \tilde{\ell}_b \rightarrow ll \]
in case of slepton annihilation, we had calculated in [1] the
volume suppression factors corresponding to each interaction
vertex making use of \( N = 1 \) gauged supergravity action.
Utilizing the same and thereafter solving for partial wave
coefficient of each channel, we found in [1]:
\[ a_{\chi^0_1 \chi^0_1 \rightarrow f_1 f_2} = \tilde{a}_{hh} + \tilde{a}_{ZZ} + \tilde{a}_{ff} \equiv O(10)^{-29} GeV^{-2} \]
and
\[ b_{\chi^0_1 \chi^0_1 \rightarrow f_1 f_2} = \tilde{b}_{hh} + \tilde{b}_{ZZ} + \tilde{b}_{ff} \equiv O(10)^{-29} GeV^{-2}. \]
Similarly
\[ a_{\chi^0_1 \chi^0_1 \rightarrow f_1 f_2} = \tilde{a}_{ZZ} + \tilde{a}_{hh} + \tilde{a}_{\gamma \gamma} + \tilde{a}_{ll} \equiv O(10)^{-9} GeV^{-2} \]
and
\[ b_{\chi^0_1 \chi^0_1 \rightarrow f_1 f_2} = \tilde{b}_{ZZ} + \tilde{b}_{hh} + \tilde{b}_{\gamma \gamma} + \tilde{b}_{ll} \equiv O(10)^{-9} GeV^{-2}. \]
Integrating \( \langle \sigma v_{\text{Mol}} \rangle(x) \) in limits from 0 to \( x_f \) (value of this
comes out to be around 1/33 by solving numerically the
equation
\[ x_f^{-1} = \ln \left( \frac{m_X}{2\pi^2} \sqrt{\frac{45}{2g_s G_N}} \langle \sigma v_{\text{Mol}} \rangle(x_f) x_f^{1/2} \right), \]
using the analytical expression of relic abundance [16]
\[ \Omega_{\chi^0 h^2} = \frac{1}{\mu^2 \sqrt{g_s J(x_f)}} \]
VI. Conclusion

To summarize, in the framework of L(arge) V(olume) “D3/D7μ- split SUSY” scenario including four Wilson line moduli on the world volume of space-time filling D7-branes wrapped around the “big divisor” and two position moduli of a mobile space-time filling D3-brane restricted to (nearly) a special Lagrangian sub-manifold, we show that fermionic superpartners of $A_1$ and $A_3$ get identified, respectively with $e_L$ and $e_R$, and the fermionic superpartners of $A_2$ and $A_4$ get identified, respectively with the first generation quarks: $u/d_L$ and $u/d_R$. The scenario is very appealing on the cosmology side; explicit life times calculation of co-N(LSP) candidates’ possible decay channels verify that decay of NLSP into gravitino do not disturb primordial abundance i.e BBN and life time of gravitino comes out to be around the order or greater than the age of the universe and hence satisfies the requirement of an appropriate dark matter candidate in the context of $\mathcal{N} = 1$ gauged supergravity. The numerical estimates of various N(LSP) decay lifetimes are provided in table I. Next, inspired from non-thermal production mechanism of gravitino, the calculated value of relic abundance equal to 0.16 from neutralino annihilation, is almost in agreement with the value suggested by WMAP 7-year CMB anisotropy observation [17].
Fig. 18 Feynman diagrams for $\tilde{\ell}_a\tilde{\ell}_b \rightarrow Zh$ via $s$-channel Higgs exchange and t-channel $\tilde{\ell}_c$ exchange.

Fig. 19 Feynman diagrams for $\tilde{\ell}_b\tilde{\ell}_b \rightarrow \gamma\gamma$ via point interaction and t-channel $\tilde{\ell}_c$ exchange.