Abstract—The unique structural configuration found in human foot allows easy walking. Similar movement is hard to imitate even for an ape. It is obvious that human ambulation relates to the foot structure itself. Suppose the bones are represented as vertices and the joints as edges. This leads to the development of a special graph that represents human foot. On a footprint there are point-of-contacts which have contact with the ground. It involves specific vertices. Theoretically, for an ideal ambulation, these points provide reactions onto the ground or the static equilibrium forces. They are arranged in sequence in form of a path. The ambulating footprint follows this path. Having the human foot graph and the path crossbred, it results in a representation that describes the profile of an ideal ambulation. This profile cites the locations where the point-of-contact experience normal reaction forces. It highlights the significance of these points.

Keywords—Ambulation, edge, foot, graph, vertex.

I. INTRODUCTION

The unique structural configuration found in human foot allows easy walking (see Fig. 1). Even an ape finds it hard to imitate similar movement. In fact, human foot has a lower total force in joints and muscles than do the ape feet [1]. Figure 2 is a radiograph that shows side view of a typical human foot. By inspection it has 27 bones. The calcaneus bone forms the heel which experiences contact with the ground where the normal reaction force (NRF) is at its peak upon the first step to ambulate.

It is obvious that human ambulation relates to the foot structure itself. Suppose the bones are represented as vertices and the joints as edges so that a special graph is created to represent human foot shown in Fig. 3. Figure 3 and Fig. 4 are both human foot graph but they put labels for vertex and edge respectively. The calcaneus bone is the vertex. Vertex \(v_1\) is the talus bone, and the edge that joins talus and calcaneus is \(e_{13}\). Notice that there are five trails that branch-off.

A.Y. Bani Hashim is with Department of Robotics & Automation, Faculty of Manufacturing Engineering, Universiti Teknikal Malaysia Melaka, Durian Tunggal, 76109 Melaka, MALAYSIA (Phone: 606-3316498; fax: 606-3316431, email: yusairi@utem.edu.my).
N.A. Abu Osman & W.A.B. W.A. are with the Department of Biomedical Engineering, Faculty of Engineering, University of Malaya, 50603 Kuala Lumpur, MALAYSIA (email: azuan@um.edu.my, drirwan1@gmail.com).
L. Abdul Latif is with the Department of Rehabilitation Medicine, Faculty of Medicine, University of Malaya, 50603 Kuala Lumpur, MALAYSIA (email: lydialatif@um.edu.my).

Fig. 1. The above illustration depicts a typical human ambulation. The left foot’s heel touches the ground and the reaction force accumulates. This touchdown is the beginning of the sequence in which certain bones actually have contact with the ground. This sequence creates the ambulation path on bones.

Fig. 2. The above picture is a sample human foot radiograph. The side view of this image partially shows the structural arrangement of the 27 bones. It is noted that the structure is somewhat like a leaf spring configuration and a large number of joints seem to act as shock absorbers. Similarly, the curved contact surface that is between the heel and the rest of the bones obviously maintains equilibrium while standing.

A walk from the origin can be made by connecting related vertices. If a walk is to begin from the origin vertex \(v_1\), there are at most five unique walks. Equations (1) to (5) define the walks [2]. Walk-1 begins from the talus and ends at the first digit of the foot. Similarly, walk-5 begins from the talus and end at the fifth digit of the foot.
Proposition. Fig. 1 depicts a typical human ambulation. In completing one cycle of ambulation, a foot undergoes a sequence within an exclusive path that involves thirteen points on the footprint shown in Fig. 5 (above). Point-0 is the initial point depicting the descending foot. Point-1 is when the calcaneus experiences the impulse upon touchdown. So that the sequence for ambulation on the footprint starts from point-0 and ends at point-13 as shown in Fig. 5 (below). These points are called the point-of-contacts (POCs). The intensity of ground reaction force that every POC experiences can be represented by a number or color. For example, a POC is assigned a 4 or red if the transpiring NRF acting in opposite is at the largest.

\[
\begin{align*}
S_{W1} &= \left\{ v_1, e_{12}, v_2, e_{221}, v_{121}, e_{2122}, v_{222}, e_{2222}, e_{3} \right\} \\
S_{W2} &= \left\{ v_1, e_{12}, v_2, e_{221}, v_{121}, e_{2122}, v_{222}, e_{2222}, e_{3} \right\} \\
S_{W3} &= \left\{ v_1, e_{12}, v_2, e_{221}, v_{121}, e_{2122}, v_{222}, e_{2222}, e_{3} \right\} \\
S_{W4} &= \left\{ v_1, e_{12}, v_2, e_{221}, v_{121}, e_{2122}, v_{222}, e_{2222}, e_{3} \right\} \\
S_{W5} &= \left\{ v_1, e_{12}, v_2, e_{221}, v_{121}, e_{2122}, v_{222}, e_{2222}, e_{3} \right\}
\end{align*}
\]
Figure 6 shows the bar plot for the results in 8. It explains the intensity of NRFs of all sequences for an ideal ambulation. In addition, it shows the occurring membership function with respect to the sequence. Both SEQ1 and SEQ12 experience a large NRF onto single POC. For SEQ2, SEQ3, and SEQ4 almost all POCs have contact with the ground.

Fig. 6. The bar chart of the sequence of an ideal ambulation

### III. CONSEQUENCE

By observing Fig. 3, Fig. 4, and Fig. 5 it comes by common sense that in (8) to (12) there are POCs that translate into respective NRFs. Equation (8) has \( F_{1G} \) that acts on \( v_{2x2} \) near \( e_{2x2a3} \). There is higher number of NRFs in (11) and (12). Table 1 lists the bones that relate to vertices and edges with relevant forces. Majority of NRFs are seen to converge around the phalanges. Using (7) and with some mathematical manipulations, the plot in Fig. 7 shows the proposed profile for a normal human ambulation. It is the sum of membership functions with respect to the POC. It shows that POCs 10, 11, and 12 have large concentration of NSFs during one cycle of ambulation.

Fig. 8 is the crossbreed graph that fuses the human foot graph of Fig. 3 and the graph of Fig. 5. It shows the path that exists from the start and end points of the ambulating footprint showed in numerical value 0 to 13. It is seen that every stopover point, except 0 and 13, links to a particular vertex and edge. If the foot graph has \( v_i \) decoupled, then there exists two sub graphs. One graph has two branches \( (G_1) \) while another has three branches \( (G_2) \). Sub graph \( G_1 \) contains six POCs so is \( G_2 \). Considering six NRFs on \( G_1 \), it is fair to infer that \( G_1 \) holds the foremost burden as compared to \( G_2 \).
Three-dimensional target 

The graph for foot and the sequence of the ambulating footprint have been successfully integrated. This integration has allowed the profile for an ideal human ambulation to be developed. In Fig. 7, the POC2 is the mark in the midsection of the complete ambulation profile. Accordingly, Fig. 3 and Fig. 5 are somewhat related to one another. This is true because in (8) to (12), there is the force component that implicitly cites the location of the NRFs. The ideal ambulation profile as depicted in Fig. 7 explains the significance of the POCs. The area under the curve symbolizes the footprint (see Fig. 5) itself where it has a larger area in the section towards the digits of the foot.

ACKNOWLEDGMENT

This work is funded through the Malaysian Ministry of Higher Education’s Fundamental Research Grant Scheme, and is a part of doctoral ongoing research work in the University of Malaya.

REFERENCES
