Computer Simulations of an Augmented Automatic Choosing Control Using Automatic Choosing Functions of Gradient Optimization Type

Toshinori Nawata

I. INTRODUCTION

GENERALITY, it is easy to design the optimal control laws for linear systems, but it is not so for nonlinear systems, though they have been studied for many years[1]~[8]. One of most popular and practical nonlinear control laws is synthesized by applying a linearization method by Taylor expansion truncated at the first order and the linear optimal control theory is applied to get the linear quadratic (LQ) controls[2]. These LQ controls are smoothly united by automatic choosing functions of gradient optimization type to synthesize a single nonlinear feedback controller.

This controller is of a structure-specified type which has some parameters, such as the number of division of the domain, regions of the subdomains, points of Taylor expansion, and gradients of the automatic choosing function. These parameters must be selected optimally so as to be just the controller’s fit. Since they lead to a nonlinear optimization problem, we are able to solve it by using the genetic algorithm (GA)[9] suboptimally. In this paper the suboptimal values of these parameters are selected by minimizing the Hamiltonian.

This approach is applied to a field excitation control problem of power system, which is Ozeki-Power-Plant of Kyushu Electric Power Company in Japan, to demonstrate the splendidness of the AACC. Simulation results show that the new controller using the GA is able to improve performance remarkably well.

II. AUGMENTED AUTOMATIC CHOOSING CONTROL USING ZERO DYNAMICS

Assume that a nonlinear system is given by

\[ \dot{x} = f(x) + g(x)u, \quad x \in \mathbb{D} \quad (1) \]

where \( : = \frac{d}{dt}, \ x = [x[1], \ldots, x[n]]^T \) is an \( n \)-dimensional state vector, \( u = [u[1], \ldots, u[r]]^T \) is an \( r \)-dimensional control vector, \( f : \mathbb{D} \rightarrow \mathbb{R}^n \) is a nonlinear vector-valued function with \( f(0) = 0 \) and is continuously differentiable, \( g(x) \) is an \( n \times r \) driving matrix with \( g(0) \neq 0 \), \( \mathbb{D} \subset \mathbb{R}^n \) is a domain, and \( T \) denotes transpose.

Considering the nonlinearity of \( f \), introduce a vector-valued function \( C : \mathbb{D} \rightarrow \mathbb{R}^L \) which defines the separative variables \( \{C_i(x)\} \), where \( C = [C_1 \cdots C_j \cdots C_L]^T \) is continuously differentiable. Let \( D \) be a domain of \( C^{-1} \). For example, if \( x[2] \) is the element which has the highest nonlinearity in \( f \), then

\[ C(x) = x[2] \in \mathbb{D} \subset \mathbb{R}^L \ (L = 1) \]

(see Section IV). The domain \( D \) is divided into some subdomains: \( D = \bigcup_{i=0}^{M} D_i \), where \( D_M = D - \bigcup_{i=0}^{M-1} D_i \) and \( C^{-1}(D_0) \supseteq 0 \). \( D_i(0 \leq i \leq M) \) endowed with a lexicographic order is the Cartesian product \( D_i = \Pi_{j=1}^{L} [a_{ij}, b_{ij}] \), where \( a_{ij} < b_{ij} \).

Introduce a stable zero dynamics:

\[ \dot{x}[n+1] = -\sigma x[n+1] \quad (2) \]
where $\sigma_i \zeta_i < 1$.

Eq.(1) combines with (2) to form an augmented system

$$\dot{X} = \tilde{f}(X) + \tilde{g}(X)u$$

(3)

where

$$X = \begin{bmatrix} x \\ x[n+1] \end{bmatrix} \in D \times R$$

$$\tilde{f}(X) = \begin{bmatrix} -f(x) \\ -\sigma_i x[n+1] \end{bmatrix}$$

$$\tilde{g}(X) = \begin{bmatrix} g(x) \\ 0 \end{bmatrix}.$$  

We assume a cost function being

$$J = \frac{1}{2} \int_0^\infty (X^T Q X + u^T R u) \, dt$$

(4)

where $Q = Q^T > 0$, $R = R^T > 0$, and the values of these matrices are properly determined based on engineering experience.

On each $D_i$, the nonlinear system is linearized by the Taylor expansion truncated at the first order about a point $X_i \in C^{-1}(D_i)$ and $\dot{X}_i = 0$ (see Fig. 1):

$$f(x) + g(x)u \approx A_i x + w_i + B_i u \quad \text{on} \quad C^{-1}(D_i)$$

(5)

where

$$A_i = \frac{\partial f(x)/\partial x^T}{x=X_i}, \quad w_i = f(\dot{X}_i) - A_i \dot{X}_i,$$

$$B_i = g(\dot{X}_i).$$

Make an approximation of (3) by

$$\dot{X} = \tilde{A}_i X + B_i u \quad \text{on} \quad C^{-1}(D_i) \times R$$

(6)

where

$$\tilde{A}_i = \begin{bmatrix} A_i & w_i \\ 0 & -\sigma_i \end{bmatrix}, \quad B_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}.$$  

An application of the linear optimal control theory [2] to (4) and (6) yields

$$u_i(X) = -R^{-1} B_i^T P_i X$$

(7)

where the $(n + 1) \times (n + 1)$ matrix $P_i$ satisfies the Riccati equation:

$$P_i \tilde{A}_i + \tilde{A}_i^T P_i + Q - P_i B_i R^{-1} B_i^T P_i = 0.$$  

(8)

Introduce an automatic choosing function of gradient optimization type:

$$I_i(x) = \prod_{j=1}^L \left\{ 1 - \frac{1}{1 + \exp \left( 2N_i (C_j x - a_{ij}) \right)} \right\}$$

(9)

where $N_i$: positive real value, $-\infty \leq a_{ij}$, $b_{ij} \leq \infty$. $I_i(x)$ is analytic and almost unity on $C^{-1}(D_i)$, otherwise almost zero (see Fig. 2).

**Fig. 1** Sectionwise linearization

**Fig. 2** Automatic Choosing Function ($N_i=3.0, 6.0$)

Unifying $\{u_i(X)\}$ of (7) with $\{I_i(x)\}$ of (9), we have an augmented automatic choosing control

$$u(X) = \sum_{i=0}^M u_i(X) I_i(x).$$

(10)

**III. PARAMETER SELECTION BY GA**

The Hamiltonian for Eqs.(3) and (4) is given by

$$H(X, u, \lambda) = \frac{1}{2} (X^T Q X + u^T R u)$$

$$+ \lambda^T (f(X) + g(X) u).$$

(11)

Assume that the adjoint vector $\lambda \in R^{n+1}$ is

$$\lambda = \sum_{i=0}^M P_i X I_i(x).$$

(12)

The necessary condition of the optimality is $\partial H/\partial u = 0$ or $u = -R^{-1} g(X)^T \lambda$, which derives Eq.(10) using Eq.(12) and

$$H(X, u, \lambda) = \frac{1}{2} X^T Q X - \frac{1}{2} u^T R u + f(X) \lambda$$

(13)

using Eq.(11).

Thus we can define a performance

$$PI = \int_D |H(X, u, \lambda)|/X^T X dX.$$  

(14)

A set of parameters included in the control of Eq.(10) is

$$\Omega = \{ M, N_i, a_{ij}, b_{ij}, \dot{X}_i \}.$$  

(15)
which is suboptimally selected by minimizing $PI$ with the aid of GA[9] as follows.

<ALGORITHM>
step1:Apriori: Set values $\Omega_{apriori}$ appropriately.
step2:Parameter: Choose $\Omega \subset \Omega$ to be improved and rewrite
$$\Omega = \{N_i, a_k, b_k \cdots\} = \{a_k : k = 1, \cdots, K\}.$$
step3:Coding: Represent each $a_k$ with a binary bit string of $\tilde{L}$ bits and then arrange them into one string of $\tilde{L}K$ bits.
step4:Initialization: Randomly generate an initial population of $\tilde{q}$ strings
$$\{\Omega_p : p = 1, \cdots, \tilde{q}\}.$$
step5:Decoding: Decode each element $a_k$ of $\Omega_p$ by
$$a_k = (a_{k,\text{max}} - a_{k,\text{min}})A_k/(2^{\tilde{L}} - 1) + a_{k,\text{min}},$$
where $a_{k,\text{max}}$: maximum, $a_{k,\text{min}}$: minimum, and $A_k$: decimal values of $a_k$.
step6:Control: Design $u = u(X)|_p$ ($p = 1, \cdots, \tilde{q}$) for $\Omega_p$ by using Eq.(10).
step7:Adjoint: Make $\lambda = \lambda(X)|_p$ ($p = 1, \cdots, \tilde{q}$) for $\Omega_p$ by using Eq.(11).
step8:Fitness value calculation: Calculate
$$PI_p = \int_{D} \frac{1}{2} X^T Q X - \frac{1}{2} u(X)_p^T R u(X)_p$$
$$+ f^T(X) \lambda(X)|_p / X^T X_d X$$
by Eqs.(13) and (14), or fitness $F_p = -PI_p$. Integration of (16) is approximated by a finite sum.
step9:Reproduction: Reproduce each of individual strings with the probability of
$$F_p / \sum_{j=1}^{\tilde{q}} F_j.$$ 
step10:Crossover: Pick up two strings and exchange them at a crossing position by a crossover probability $P_c$.
step11:Mutation: Alter a bit of string (0 or 1) by a mutation probability $P_m$.
step12:Repetition: Repeat step5~step11 until prespecified $G$-th generation. If unsatisfied, go to step2.
As a result, we have a suboptimal control $u(X)$ for the string with the best performance over all the past generations.

IV. NUMERICAL EXAMPLE

Consider a field excitation control problem of power system. Fig. 3 is a diagram of Ozeki-Power-Plant of Kyushu Electric Power Company in Japan. This system is assumed to be described[8] by

$$\dot{\theta} = \frac{d^2 \delta}{dt^2} + \frac{P_d}{M} \frac{d}{dt} \frac{dE_{d}}{dt} + P_c = P_{in},$$
$$P_e = E_d^2 Y_{11} \cos \theta_{11} + E_I \tilde{V} Y_{12} \cos(\theta_{12} - \delta),$$
$$E_I + T_{0} \frac{dE_I}{dt} = E_{I0},$$
$$E_d = E_{q0} + (X_d - X_d') I_d,$$
$$I_d = -E_I Y_{11} \sin \theta_{11} - \tilde{V} Y_{12} \sin(\theta_{12} - \delta),$$
$$\tilde{D} = \tilde{V} \left\{ \begin{array}{c} T_{0}^{\prime \prime}(X_d' - X_d'') \\ -2 \sin^2 \delta \\ \frac{T_{0}^\prime (X_d - X_d')}{(X_q + X_e)^2} \cos^2 \delta \end{array} \right\},$$

where $\delta$: phase angle, $\dot{\delta}$: rotor speed, $\tilde{M}$: inertia coefficient, $\tilde{D}(\delta)$: damping coefficient, $P_{in}$: mechanical input power, $P_c(\delta)$: generator output power, $\tilde{V}$: reference bus voltage, $E_I$: open circuit voltage, $E_{I0}$: field excitation voltage, $X_d$: direct axis synchronous reactance, $X_d'$: direct axis transient reactance, $X_e$: external impedance, $Y_{11}(\theta_{11})$: self-admittance of the network, $Y_{12}(\theta_{12})$: mutual admittance of the network, and $I_d(\delta)$: direct axis current of the machine. Put $x=[x[1], x[2], x[3]]^T=[E_I - \tilde{E}_I, \delta - \tilde{\delta}_0, \dot{\delta}]^T$ and $u = E_{I0} - \tilde{E}_{I0},$ so that

$$\begin{bmatrix} \dot{x}[1] \\ \dot{x}[2] \\ \dot{x}[3] \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} + \begin{bmatrix} g_1(x) \\ 0 \\ 0 \end{bmatrix} u$$

where
$$f_1(x) = -\frac{1}{kT_{d0}} (x[1] + \tilde{E}_I - \tilde{E}_{I0})$$
$$\quad + \frac{(X_d - X_d') \tilde{V} Y_{12}}{k} X_3 \cos(\theta_{12} - x[2] - \tilde{\delta}_0),$$
$$f_2(x) = x[3],$$
$$f_3(x) = \frac{\tilde{V} Y_{12}}{M} (x[1] + \tilde{E}_I) \cos(\theta_{12} - x[2] - \tilde{\delta}_0)$$
$$\quad - \frac{Y_{11} \cos \theta_{11}}{M} (x[1] + \tilde{E}_I)^2 - \frac{\tilde{D}}{M} x[3] + \frac{P_c}{M}$$
$$g_1(x) = \frac{1}{kT_{d0}}, \quad k = 1 + (X_d - X_d') Y_{11} \sin \theta_{11}.$$
We have studied an augmented automatic choosing control using the automatic choosing functions of gradient optimization type for nonlinear systems. This approach was applied to a field excitation control problem of power system to demonstrate the splendidness of the AACC. Simulation results have shown that this controller could improve performance remarkably well.

V. CONCLUSIONS

We have studied an augmented automatic choosing control using the automatic choosing functions of gradient optimization type for nonlinear systems. This approach was applied to a field excitation control problem of power system to demonstrate the splendidness of the AACC. Simulation results have shown that this controller could improve performance remarkably well.

REFERENCES


Toshinori Nawata received his B.S degree in Computer Science from Kyushu Institute of Technology in 1990 and his Dr. Eng. degree in System Information Engineering from Kagoshima University in 2003. He is currently an Associate Professor at the Department of Human-Oriented Information Systems Engineering, Kumamoto National College of Technology. His research interests include the nonlinear system control theory. Dr. Nawata is a member of IEEJ, IIIEE and ISCI.
### TABLE I
**PERFORMANCES**

<table>
<thead>
<tr>
<th>Method</th>
<th>$x^T(0)$ : initial point</th>
<th>LOC</th>
<th>AACC($N_i$: fix)</th>
<th>AACC($N_i$: GA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$[0, 0.4, 0]$</td>
<td>×</td>
<td>0.95873</td>
<td>0.94224</td>
</tr>
<tr>
<td></td>
<td>$[0, 0.5, 0]$</td>
<td>×</td>
<td>1.03947</td>
<td>1.23581</td>
</tr>
<tr>
<td></td>
<td>$[0, 1.0, -5]$</td>
<td>×</td>
<td>7.69293</td>
<td>7.40167</td>
</tr>
<tr>
<td></td>
<td>$[0, 1.2, 0]$</td>
<td>×</td>
<td>2.31948</td>
<td>2.84883</td>
</tr>
<tr>
<td></td>
<td>$[0, 1.295, 0]$</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

$\times$ : very large value

---

**Fig. 6** Responses of LOC, AACC($N_i$: fix), AACC($N_i$: GA) ($x^T(0) = [0, 1.2, 0]$)

**Fig. 7** Responses of AACC($N_i$: fix), AACC($N_i$: GA) ($x^T(0) = [0, 1.0, -5]$)

**Fig. 8** Responses of AACC($N_i$: fix), AACC($N_i$: GA) ($x^T(0) = [0, 1.0, -5]$)

**Fig. 9** Responses of AACC($N_i$: fix), AACC($N_i$: GA) ($x^T(0) = [0, 1.0, -5]$)
Fig. 10 Responses of AACC($N_i$ : fix), AACC($N_i$ : GA) ($x^T(0) = [0, 1.298, 0]$)

Fig. 11 Responses of AACC($N_i$ : fix), AACC($N_i$ : GA) ($x^T(0) = [0, 1.298, 0]$)

Fig. 12 Responses of AACC($N_i$ : fix), AACC($N_i$ : GA) ($x^T(0) = [0, 1.298, 0]$)